EUROPEAN CENTRAL BANK
EUROSYSTEM

WORKING PAPER SERIES
NO 1717 / AUGUST 2014

SHORT-TERM DETERMINANTS OF THE IDIOSYNCRATIC SOVEREIGN RISK PREMIUM

A REGIME-DEPENDENT ANALYSIS FOR EUROPEAN CREDIT DEFAULT SWAPS

Giovanni Calice, RongHui Miao,
Filip Štěrba, and Bořek Vašíček

In 2014 all ECB publications feature a motif taken from the €20 banknote.

MACROPRUDENTIAL RESEARCH NETWORK

Macroprudential Research Network

This paper presents research conducted within the Macroprudential Research Network (MaRs). The network is composed of economists from the European System of Central Banks (ESCB), i.e. the national central banks of the 27 European Union (EU) Member States and the European Central Bank. The objective of MaRs is to develop core conceptual frameworks, models and/or tools supporting macro-prudential supervision in the EU.

The research is carried out in three work streams: 1) Macro-financial models linking financial stability and the performance of the economy; 2) Early warning systems and systemic risk indicators; 3) Assessing contagion risks.

MaRs is chaired by Philipp Hartmann (ECB). Paolo Angelini (Banca d'Italia), Laurent Clerc (Banque de France), Carsten Detken (ECB), Simone Manganelli (ECB) and Katerina Šmídková (Czech National Bank) are workstream coordinators. Javier Suarez (Center for Monetary and Financial Studies) and Hans Degryse (Katholieke Universiteit Leuven and Tilburg University) act as external consultants. Fiorella De Fiore (ECB) and Kalin Nikolov (ECB) share responsibility for the MaRs Secretariat.

The refereeing process of this paper has been coordinated by a team composed of Gerhard Rünstler, Kalin Nikolov and Bernd Schwaab (all ECB).

The paper is released in order to make the research of MaRs generally available, in preliminary form, to encourage comments and suggestions prior to final publication. The views expressed in the paper are the ones of the author(s) and do not necessarily reflect those of the ECB or of the ESCB.

Acknowledgements

This work was supported by Czech National Bank Research Project No. C4/2012. We are grateful for helpful comments and suggestions from Tomáš Adam, Gianni Amisano, Julian Idier, Iulian Obreja, and Flemming Würtz and for comments from participants of the Capital Markets Division seminar series at the ECB. The views expressed in this paper are those of the authors and not necessarily those of the Czech National Bank.

Giovanni Calice

University of Birmingham; e-mail: g.calice@bham.ac.uk

RongHui Miao

University of Bath; e-mail: ronghuimiao@gmail.com

Filip Štěrba

RSJ Algorithmic Trading; e-mail: FSterba@seznam.cz

Bořek Vašíček

Czech National Bank; e-mail: borek.vasicek@cnb.cz

Address	Kaiserstrasse 29, 60311 Frankfurt am Main, Germany
Postal address	Postfach 16 03 19, 60066 Frankfurt am Main, Germany
Telephone	+49 69 1344 0
Internet	http://www.ecb.europa.eu

ISSN	1725-2806 (online)
ISBN	978-92-899-1125-2 (online)
EU Catalogue No	QB-AR-14-091-EN-N (online)

Abstract

This study investigates the dynamics of the sovereign CDS term premium for five European countries. The CDS term premium can be regarded as a forward-looking measure of idiosyncratic sovereign default risk as perceived by financial markets. Using a Markov-switching unobserved component model, we decompose the daily CDS term premium into two components of statistically different nature and link them in a vector autoregression to various daily observed financial market variables. We find that such decomposition is vital for understanding the short-term dynamics of this premium. The strongest impacts can be attributed to CDS market liquidity, local stock returns, and overall risk aversion. By contrast, the impact of shocks from the sovereign bond market is rather muted. Therefore, the CDS market microstructure effect and investor sentiment play the main roles in sovereign risk evaluation in real time. Moreover, we also find that the CDS term premium response to shocks is regime-dependent and can be ten times stronger during periods of high volatility.

JEL Codes: G01, G15, G21, G24.

Keywords: Credit default swaps, Markov switching model, sovereign risk, State space model, term premium.

Nontechnical Summary

The use of sovereign sovereign credit default swaps (CDS) has increased dramatically during the last decade. They represent key instruments for credit risk transfer related to sovereign exposures. The CDS buyer aims to insure against default or credit event on reference entity, in this case sovereign, and in turn pays to the seller the CDS spread as a price for such insurance. Specifically, the CDS spread is the annual amount paid the protection buyer must pay the protection seller over the length of the contract, expressed as a percentage of the notional amount.

This paper examines the dynamic behavior of the (CDS) term premium for a group of European countries. We define the CDS term premium a difference between CDS spreads at two different maturities, specifically 5 and 10 years. Therefore, the CDS term premium can be interpreted as a forward-looking measure of idiosyncratic sovereign credit risk as perceived by financial markets; in particular, it tracks investors' evaluation that a country might suffer a financial crisis. We attempt to identify the main determinants of the high-frequency dynamics of this premium. While the CDS premium (or spread) at a certain maturity (e.g., 10 years) has been found to be significantly affected by common risk factors, the CDS term premium (i.e., the slope of the CDS credit curve) allows tracking primarily of the idiosyncratic part of the sovereign risk premium. Therefore, the focus of our analysis is on the time rather than the cross-country dimension of sovereign credit risk. In particular, we focus on the high-frequency drivers of this idiosyncratic constituent of sovereign risk as perceived by financial markets in real time.

Our econometric approach reflects an empirical observation that while the CDS term premium should be mean-reverting, it often shows a nonstationary pattern and significant heteroskedasticity. Therefore, we use a framework that allows us to distinguish between the nonstationary and stationary components of the CDS term premium and their associated volatility regimes. As such, we can estimate the differential effects of a set of financial market variables on the two components in each volatility regime separately. Our central argument here is that the evolving pattern of the perceived sovereign default risk can be understood by performing an in-depth examination of the sovereign CDS term premium, which consists in identifying their unobserved components and their short-term determinants.

Our empirical evidence uncovers dissimilar behavior for the two components of the CDS term premium across time. We also find that decomposing the CDS term premium into two components is vital for understanding its short-term dynamics. Specifically, major increases in the CDS term premium are driven by abrupt changes in its nonstationary component, and this component is affected by shocks to several financial market variables. The strongest impacts can be attributed to CDS market liquidity, local stock returns, and overall risk aversion. Our results also suggest that the response to shocks to these variables is regime-dependent and can be ten times stronger during periods of high volatility. By contrast, the impact of shocks originating in the domestic sovereign bond market and other sovereign CDS markets is rather muted. The less volatile stationary component of the CDS term premium (unlike its nonstationary counterpart) is largely unaffected by shocks to selected financial variables, suggesting a potential link with long-term fundamental factors (e.g., government debt, fiscal stance, macroeconomic performance).

All in all, our results also suggest a major disconnection between sovereign CDS and bond markets and limited scope for cross-country spillovers when slope effects (i.e., the term premium rather than the premium at a single maturity) are taken into account. On the other hand, CDS market microstructure effects and investor sentiment play a role in sovereign risk evaluation in real time. The results in this paper might have important policy implications, especially given the recent events related to the eurozone sovereign crisis. First, we believe that our analysis can provide monetary policy authorities with more detailed information on financial market perceptions of vulnerabilities present in sovereign debt markets as well as on the sources of propagation of those vulnerabilities. Second, it might shed some new light on the potential effect of some regulatory initiatives, such as the ban on the use of "naked" CDS contracts on European sovereign entities, which will arguably reduce the liquidity of the sovereign CDS market and in turn change the perceived risk valuation of single sovereigns.

1. Introduction

Tensions in the euro area sovereign debt market represent the most recent form of the global financial crisis. The succession of events following the beginning of the European sovereign debt crisis in May 2010 has clearly underscored that excessive systemic sovereign credit risk can lead to detrimental real macroeconomic effects and financial instability. Indeed, it is because of the risk of macroeconomic shocks and financial contagion that regulators and governments are currently so concerned about sovereign-specific credit risk. Given the massive size of the sovereign debt market in Europe, it is clear that understanding the systemic nature of sovereign credit risk is of fundamental importance. However, there is little theoretical basis on how to interpret the evolution of sovereign risk premia and how it relates to economic cycles, asset prices, and changes in policy regimes.

The use of sovereign CDS has increased dramatically during the last decade. They represent key instruments for credit risk transfer related to sovereign exposures. However, since the onset of the U.S. subprime crisis they have become very controversial and many commentators have blamed them for exacerbating the credit crunch by allowing excessive leverage and risk-taking by financial institutions and even market manipulation (see Stulz, 2010, for a discussion).

In 1998, the global CDS market was estimated at about $300bn. The notional outstanding amount grew to over $2.2tn in 2002, peaked at around $62tn in 2007, and subsequently fell to $30tn in 2009 and below $15tn in early 2013. Originally, CDS were developed to mitigate corporate credit risk in bank's balance sheets. However, with the onset of the Asian crisis in 1997, CDS started to be used as a means of protection against default risk on sovereign debt of emerging countries and ten years later also developed countries. This trend has become even more apparent during the last five years. In fact, as of today (according to ISDA data as of March 2013), five out of the ten biggest notional outstanding CDS positions measured in gross notional as well as in net notional terms include CDS written on sovereign debt of developed countries. At nearly $413bn gross notional, CDS written on Italian sovereign debt are the single biggest CDS position. This is followed by nearly $219bn written on Spanish debt, $179bn on French debt, and $156bn on German debt. The fifth to ninth biggest outstanding amounts belong to the sovereigns of Brazil, Turkey, Russia, and Mexico, which just demonstrates the importance of sovereign debt to existing CDS markets. The current outstanding amount of sovereign CDS is $3tn, which is still rather modest as compared to the approximately $50tn in outstanding government bonds (IMF, 2013).

Conceptually, CDS spreads have some advantages for empirical analysis over the more commonly used bond yield spreads. They are deemed to be a more direct measure of default risk, as they are not distorted by other risks unrelated to defaults and market microstructure (Longstaff et al., 2005). Unlike cash bonds, positions in CDS contracts do not require up-front funding. Therefore, CDS spreads are less distorted by liquidity dry-up during crisis periods (Chen et al., 2007). There is no need to set a risk-free rate, whose variation can distort the variation of the spread itself. CDS premia are deemed to be more responsive to information about the underlying credit quality of the issuer, which may lead to short-term deviation from bond prices (Blanco et al., 2005; Zhu, 2006). However, there is still disagreement about the nature of the price discovery process in the case of sovereign bonds and CDS.[1]

Our article has two main innovations relative to the existing literature. First, we formalize our intuition of proposing the term premium of a sovereign CDS as a useful forward-looking benchmark for idiosyncratic sovereign credit risk. Second, we explore empirically the real and financial market-related factors and the magnitude of their effects on a country's CDS term premium. Our tests contribute to our understanding of sovereign credit markets by providing direct evidence about the role of the sovereign CDS term premium, the two channels (components) into which it can be partitioned, and the differential effects of a variety of exogenous variables on the two components in two distinct volatility regimes. Furthermore, we present evidence that is consistent with the view of a major disconnection between the aggregate behavior of sovereign CDS and debt markets and limited scope for cross-country spillovers when slope effects are taken into account.

The term premium of a sovereign CDS, which, in this paper, is measured as the difference between the CDS 10-year and the CDS 5-year maturities, can be viewed as representing the default risk uncertainty over a 5-year time horizon. Therefore, the CDS term premium of a sovereign can be interpreted as a forward-looking market indicator of sovereign credit risk for 5 years hence.[2] In particular, the CDS term premium tracks investors' evaluation of the likelihood that the country will suffer an immediate financial crisis. This CDS term premium is generally positive, which corresponds to an upward-sloping yield curve for government bonds, although sovereign financial distress results typically in a negative term premium (i.e., as yield curve inversion). More importantly, the term premium features some interesting statistical features such as trends and cycles and time-varying volatility, which underlines the importance of using appropriate statistical tools for its analysis

Our paper is mainly related to the empirical literature on sovereign credit risk, which is proxied by sovereign CDS spreads. In this study, we focus nonetheless on its *idiosyncratic constituent* and its *high-frequency drivers* rather than on *common factors* or measures of *contagion*. Therefore, our focus is on the time rather than the cross-country dimension. A number of articles, such as Berndt and Obreja (2010) and Dieckmann and Plank (2012), directly address the modeling of credit spreads through the use of a set of explanatory variables. However, these studies use a regression framework that by definition neglects possible non-linear relationships between CDS spreads and their determinants. Our work also has a resemblance to Pan and Singleton (2008) and Longstaff et al. (2011), who attempt to estimate default risk using the entire credit curve of sovereign CDS premia. However, we depart from them in that we take a pure time-series perspective.

Specifically, we posit that the economy-wide forward-looking default risks embedded in the CDS term premium can be disaggregated in a way similar to trend-cycle decomposition. In general, the term premium resembles the behavior of the yield curve. It follows a mean-reverting process,

[1] See the International Monetary Fund's April 2013 Global Financial Stability Report for a comprehensive review of this strand of the literature.

[2] If an investor perceives the difference between the 5-year index premia and the 10-year index premia as being too steep, in other words, that the implied probability of default between 5 and 10 years is higher than that implied from fundamentals, but he/she expects the slope to flatten, then this investor could buy 5-year protection and sell 10-year protection on the CDS index. From a theoretical perspective, the credit curves of sovereigns with high credit quality should be upward sloping, whereas those of sovereign entities having very poor credit quality do exhibit negative slopes.

despite short-term spikes during periods of financial turmoil. Therefore, we can reasonably assume that the term premium can be decomposed by means of the unobserved component model into two factors. The first is a stationary factor, which is probably driven by fundamental forces (see also Garratt et al., 2006). The second factor, which is modeled as a driftless random walk process, represents a seemingly unpredictable component in the term premium. Essentially, this factor captures market uncertainty, which induces random walk behavior in the term premium. The apparent heteroskedasticity will also be accounted for. We do this by means of a Markov-switching model that allows for two different volatility regimes for each CDS term premium subcomponent.

The decomposition enables us to understand the evolution of the sovereign CDS term premium in terms of its subcomponents of statistically different nature in structurally different periods and their links to observable financial market variables that might in turn be affected by the market view of country riskiness. In particular, we use vector autoregression to analyze how the CDS term premium is affected (via its subcomponents) by shocks to: (i) the reference asset (the yield curve slope of domestic sovereign bonds), (ii) the liquidity of the sovereign CDS market (the bid-ask spread), (iii) other observed domestic financial variables (short-term interest rate, stock market returns, domestic banking sector CDS term premium), and (iv) international factors (European CDS term premium, VIX).

Our study focuses on selected European countries whose CDS term premia experienced the most notable swings during the global financial crisis period, which in turn resulted in nonstationary patterns and abrupt changes in the volatility of these premia. Our central argument here is that the evolving pattern of the sovereign CDS term premium can provide the relevant monetary authorities with detailed information on financial market perceptions of the vulnerabilities in sovereign debt markets as well as on the sources of propagation of those vulnerabilities. A better and deeper understanding of these forces will in turn serve as a useful tool for the identification of systemic and contagion risks and will also potentially enable authorities to respond effectively in advance in order to mitigate shocks jeopardizing financial stability.

A number of important empirical results emerge from this analysis. First, we show that the decomposition of the CDS premium of a sovereign entity is relevant, as its two components show very dissimilar behavior and major increases in the CDS term premium (with both positive and negative sign) are driven mainly by spikes in the nonstationary (random walk) component. Second, decomposing the CDS term premium proves useful in understanding its short-term dynamics. Most selected financial market variables, observed at high frequency, significantly affect the dynamics of the nonstationary component, which is a seemingly unpredictable random walk. Conversely, the stationary component seems largely unaffected by such short-term financial shocks and therefore the low-frequency dynamics of the sovereign CDS term premium might be driven by macroeconomic fundamentals (e.g., government debt, fiscal deficit, nominal GDP), which are not considered in our analysis. Third, the CDS term premium shows, via its nonstationary component, very pronounced regime-dependent behavior. In particular, the response of the CDS term premium to normalized shocks to some financial variables can be ten times stronger during periods of high volatility. The strongest impacts are due to CDS market liquidity, local stock returns, and overall risk aversion. By contrast, the slope of the bond yield curve has a small and transient effect on this component and in turn on the CDS term premium, signaling an important disconnection between CDS and bond markets when slope effects are taken into account.

Our paper addresses several questions of MARS WS3 (the special initiative on sovereign contagion risk. *i) How significant is/was sovereign contagion/spillovers in Europe?* While on purpose we aim at the CDS term premium (i.e. slope of CDS credit curve) as rather idiosyncratic measure of sovereign default risk, i.e. a measure that shall be less prone to international contagion/spillover, we indeed confirm that when slope effects are taken into account (i.e. when looking at CDS term premium defined as 10Y CDS spread – 5Y CDS spread), the scope of sovereign contagion/spillover is rather limited. *ii) Is there evidence of non-linearities or amplification effects?* We find significant non-linearities in short-term sovereign risk evaluation (on CDS market). However, most of this driven rather by CDS market microstructure (liquidity) and investor sentiment (as represented e.g. by local stock market return and VIX) than cross-country contagion.

The remainder of the paper is organized as follows. Section 2 discusses the related literature. Section 3 provides some theoretical considerations on the economic determinants of the sovereign CDS term premium and describes the data used in the analysis. Section 4 presents our methodology. Section 5 reports the results from the empirical analysis. Section 6 summarizes the results and makes concluding remarks. All proofs of the basic equations of our model are given in the Appendix.

2. Related Literature

Starting with Edwards (1984) there has been extensive research on the determinants of sovereign credit risk premia. This research has traditionally focused on emerging economies. Attention has turned only recently to advanced economies, in particular those within the eurozone. Sovereign risk premia have been commonly proxied by sovereign yield spreads vis-à-vis risk-free rates such as the U.S. treasury yield of corresponding maturity. While the low-frequency movements are usually attributed to macroeconomic variables (typically available at monthly or quarterly frequency), the financial variables (available at high frequency) are deemed to determine the high-frequency dynamics. This distinction has given rise to two different strands of research: (i) cross-country panel studies with low-frequency macroeconomic data, and (ii) studies using high-frequency financial data and financial econometrics.

A number of articles, especially in the first strand of the empirical literature, have primarily considered heterogeneous panels of emerging countries. Most of them point to an increasing role of global factors (Uribe and Zue, 2006) as major determinants of sovereign risk premia. These studies claim that domestic factors such as fiscal and political ones (Baldacci et al., 2008) are important. Recently, as a result of the rapidly worsening situation in the eurozone, the focus has changed dramatically and a number of empirical papers have addressed the issue of sovereign risk in the euro area (Mody, 2009; Shuknecht et al., 2010). The interest of this strand is on investigating the role of idiosyncratic and global factors in the determination of sovereign bond yield spreads in the euro area during the financial crisis, as well as on discriminating between credit and liquidity premia. The recent literature touches upon deviations from idiosyncratic fundamentals in terms of possible mispricing of sovereign credit risk (de Grauwe and Ji, 2012).

The literature about CDS premia has been expanding considerably in recent years. Berndt and Obreja (2010) show that European daily corporate CDS returns are significantly related to a factor which captures what the authors call “economic catastrophe risk.” They seek to explain the residual common factor found by Collin-Dufresne et al. (2001), building a "catastrophic factor" as the difference between the spreads of tranches with different seniority in CDO products. Cecchetti et al. (2010) document several fiscal indicators against CDS spreads for advanced economies. They find correlations across countries with substantial heterogeneity. Longstaff et al. (2011) examine the sovereign CDS of 26 emerging countries at monthly frequency. They find that sovereign spreads are more associated with global factors (U.S. stock, treasury, and high-yield markets) than local factors (stock return, exchange rate, and foreign reserves). This evidence is corroborated by a study by Fender et al. (2012) using daily data. They argue that in the post-2007 period the impact of global factors even increased. Dieckmann and Plank (2012), using a panel of 18 European sovereign CDS (weekly frequency), find a significant positive association between stock market volatility and sovereign CDS spreads. They also show that the relative importance of a country's financial system before the euro debt crisis is the main reason for this association.

Some recent studies investigate specifically the relation between sovereign bond yields and sovereign CDS (Fontana and Scheicher, 2010; Palladini and Portes, 2011; Delatte et al., 2012). There emerges a consensus that bond and CDS markets seem to be driven by common factors and that the CDS market can lead price discovery under certain conditions. Finally, given the recent feedback loop between sovereign and banking credit risk some studies investigate the relationship between sovereign and banking CDS (Acharya et al., 2011; Alter and Schüler, 2012). They find evidence that the linkages strengthened as a result of the bail-out program. Furthermore, the authors highlight significant time and space heterogeneity.

Indeed, there is great uncertainly about these determinants, as variables derived from structural credit risk models are unable to explain the entire spread variation (Eom et al., 2004). Despite a sizeable literature on credit risk, empirical studies on CDS that involve modeling of the entire credit curve are still rare. A major reason for this is that data on sovereign CDS premia for a wider range of maturities have only recently become available. Indeed, although CDS contracts on some sovereign issuers are extensively traded, the market is still rather illiquid. Consequently, there is a paucity of empirical work regarding their CDS term structure, with studies focused mainly on U.S. synthetic corporate indices such as the CDX (see Longstaff et al., 2008; Calice et al., 2012). Pan and Singleton (2008) explores the nature of default arrival and recovery implicit in the term structure of the sovereign spreads of Korea, Mexico, and Turkey. The authors find strong comovement of risk premia across countries and with indicators of global risk appetite such as the VIX.

Against this background suggesting that sovereign CDS premia are driven by common factors, our primary goal is to explore the fundamental connection between a set of selected domestic and international financial variables and the sovereign CDS term premium. Therefore, we are interested in both the quantitative predictions and the qualitative implications of such a connection. As compared to the existing empirical literature, we use an entirely novel empirical setting. We make use of the CDS term premium, since it provides a much more direct measure of idiosyncratic credit risk for particular sovereign issuers. Furthermore, by identifying two distinct statistical components driving the term premium of a sovereign CDS, we provide evidence on how the fundamental and volatility components of the sovereign CDS are determined by daily observed variables over a sample period surrounding the 2009–2010 euro sovereign debt crisis. The use of high-frequency data produces accurate estimates of price volatility, which is often not the case when weekly or monthly aggregate data are used. The country-level VAR framework also seems appropriate for revealing (potentially multidirectional) linkages among the financial variables.

3. CDS Term Premium

The CDS term premium is measured as the difference between the CDS 10-year and the CDS 5-year maturities. The CDS term premium can be considered a preferable measure of *idiosyncratic* sovereign credit risk to CDS spreads or sovereign bond yields of certain maturity, as it is less prone to contagion. Indeed, if the forces of international contagion are in place there is in principle no reason to believe that they might have a differential impact on 5-and 10-year maturities and affect the term premium. This is evident from simple correlation measures, which are substantially higher for pairs of sovereign CDS at certain maturity (5 or 10 years) than between corresponding CDS term spreads. Similarly, it is relatively straightforward to extract a single informative factor from a sample of CDS quotes than from a sample of CDS term spreads. Therefore, looking at the premium/yield at a particular maturity implies the basic identification strategy of isolating idiosyncratic from common factors. This challenge has recently been tackled by several papers aimed at examining contagion, especially in the European context. By contrast, analysis of the slope of the sovereign CDS credit curve has been largely ignored in the literature.

3.1 Economics of the CDS Term Premium

To motivate our empirical strategy and to guide our empirical tests, we begin with a brief discussion of the theoretical properties of the CDS term premium. This is not intended as an exhaustive summary, but is simply meant to illustrate the economic foundations of the CDS term premium and give a specific example of the mechanisms our model predicts.

We extend the canonical formulation of deriving forward rates from the term structure of default-free interest rates (e.g., Harrison and Kreps, 1979[3]) to a country's CDS term premium. Hence, the analysis that follows is in the spirit of deriving forward rates from the term structure of default-free interest rates.[4]

Consider a unit of time t that denotes quarters. Suppose that $m_{t,t+1}$ denotes a stochastic discount factor and $\chi_{t,t+1}$ is an indicator function that takes the value of 1 if a country is solvent over the interval $\left[t,t+1\right]$ and the value 0 otherwise. This can in practice (e.g., by rating agencies) be approximated by the marginal default probability (MDP) and the cumulative probability of default (CPD). Then, the premium paid on 10Y sovereign CDS solves (in a risk-free world with complete and arbitrage-free markets):

$$CDS_t^{10}\sum_{s=0}^{40}E_t\left[m_{t,t+s}\chi_{t,t+s}\right]=\sum_{s=0}^{40}E_t\left[m_{t,t+s}\chi_{t,t+s}\left(1-\chi_{t+s,t+s+1}\right)L_{t+s+1}\right] \tag{1}$$

where L_s is the loss in the event of default between *s-1* and *s*. The right-hand side of the equation can be rewritten as the sum of two terms, A and B, where

[3] Interested readers can refer to their article for additional modeling details.

[4] We are very grateful to Iulian Obreja for his suggestion on this framework.

$$A = \sum_{s=0}^{20} E_t \left[m_{t,t+s} \chi_{t,t+s} \left(1 - \chi_{t+s,t+s+1}\right) L_{t+s+1} \right] \tag{2}$$

$$B = \sum_{s=20}^{40} E_t \left[m_{t,t+s} \chi_{t,t+s} \left(1 - \chi_{t+s,t+s+1}\right) L_{t+s+1} \right] \tag{3}$$

The previous two equations can be rewritten as follows:

$$A = CDS_t^5 \sum_{s=0}^{20} E_t \left[m_{t,t+s} \chi_{t,t+s} \right] \tag{4}$$

$$B = E_t \left[m_{t,t+20} \chi_{t,t+20} CDS_{t+20}^5 E_{t+20} \left[m_{t+20,t+20+s} \chi_{t+20,t+20+s} \right] \right] \tag{5}$$

where A is the solution for 5Y CDS bought at time *t=0* and CDS_{t+20}^5 in the term B is the *forward* CDS spread of 5Y CDS at time *t=20* (i.e., after 5 years, that is, when a 5Y CDS contract priced in A matures). Combining (1) and (3) we obtain

$$CDS_t^{10} - CDS_t^5 = \frac{E_t \left[m_{t,t+20} \chi_{t,t+20} \left(CDS_{t+20}^5 - CDS_t^{10} \right) \sum_{s=0}^{20} E_{t+20} \left[m_{t+20,t+20+s} \chi_{t+20,t+20+s} \right] \right]}{\sum_{s=0}^{20} E_t \left[m_{t,t+s} \chi_{t,t+s} \right]} \tag{6}$$

A strong link can be seen between the sign of the CDS term premium $CDS_t^{10} - CDS_t^5$ and the sign of $CDS_{t+20}^5 - CDS_t^{10}$, which is the difference between the *forward* 5-year CDS premium and the current 10-year CDS premium. Therefore, the CDS term premium is negative when a decrease is expected in the demand for default protection in the future. For example, if a country is currently facing a financial crisis but it is expected to be out of the crisis within 5 years the probability of imminent default (in 5 years from now) is higher than a default at a longer time horizon (after 5 years).

Therefore, the sign of the CDS term premium is strongly related to investors' predictions about the timing of a country entering a crisis, which in turn determines the probability of default. Of course, this is in general dependent on the state (and evolution) of the fundamentals of the country. However, CDS spreads and the CDS term premium (the cost of external funding) are both subject to substantial *short-term variation*. This is clearly observable in Figure 1, which plots the evolution of the term premium for several EU sovereigns. Indeed, we can see very abrupt switches between positive and negative values. Furthermore, the CDS term premium is characterized by trends and heteroskedasticity. Therefore, our objective is to explore empirically the factors underpinning the structural disconnection between the aggregate behavior of the market in the short term and the fundamentals of the economy.

3.2 Data and Statistical Properties

Since our main empirical focus is on the short-term dynamics of the CDS term premium, we use daily market data. Indeed, with daily data we can explore the richness in the variation of the observations. In fact, monthly frequency time series would exhibit less volatile dynamic behavior since the short-term fluctuations would simply average out. As a result, the interaction between the observed market variables and the CDS term premium would show different patterns.[5]

Our study focuses on selected European countries whose CDS term premia experienced the most notable swings between positive and negative territory, which in turn resulted in nonstationary patterns and abrupt changes in volatility. As a rule-of-thumb *we focus on those countries for which the CDS term premium amounted to at least 30–40 basis points (positive or negative) for a period in excess of a single trading day*. On the contrary, we disregard smaller deviations, which in our view can be attributed primarily to market microstructure factors.[6]

Figure 1 clearly shows that an economically meaningful deviation of the CDS term premium from zero (as defined above) can be observed for only two groups of countries: (i) the EMU periphery (Spain, Portugal, and Ireland; Italy is excluded due to data constraints), and (ii) the CEE countries (the Czech Republic and Poland; Hungary is excluded due to data constraints), which were adversely affected as the global financial crisis hit the region in early 2009. Our data sample spans from September 2007 (for some countries slightly later) to February 2012. Given that the CDS market for European sovereigns was practically nonexistent prior to the onset of the global financial crisis, for most of the sovereign CDS the quotes are available only from 2007 onwards. The main source of data is Bloomberg LP.

We calculate *the sovereign CDS term premium* as the difference between the 10Y and the 5Y sovereign CDS quotes (mid-price). These series, along with CDS liquidity, as defined below, are plotted for all the available European countries in Figure 1. We can see that the time evolution of the CDS term premium shows very similar behavior for the most vulnerable sovereigns. For instance, for the IIPS the term premium exhibits positive values over the 2007–2008 period, then fluctuates considerably in 2009 and 2010, and turns negative in mid-2011 (for Ireland and Portugal in mid-2010). By contrast, the CDS term premium of EU core countries such as Germany and the Netherlands (reported in Figure 1), perceived as "safe," is rarely negative, i.e., following an inverse pattern vis-à-vis IIPS. Finally, the term premium for the CEE countries (the Czech Republic, Hungary, and Poland) clearly reveals the changing perception of the "safety" of that region. The premium is initially positive, then moves into negative territory for several months towards the end of 2008, and has been positive since then (turning negative in late 2011 for Hungary). Indeed, it seems that the market episodes of a negative term premium, suggesting an increasing probability of sovereign default in the short term, are the most remarkable because they reflect idiosyncratic elements of sovereign risk.

On a technical note, it is useful to evaluate the statistical properties of the CDS term premium. We noted that the salient features of some series include switches between positive and negative territory as well as trends and time-varying volatility. Therefore, as described above, we use an empirical framework that is able to track these different time series properties. In particular, we use a framework that enables time-series decomposition into a stationary and a nonstationary (random walk) component as well as changes in their respective volatility regimes. Consequently,

---

5 In addition, the most flexible model for CDS term premium decomposition contains 16 parameters. This requires a large sample of data to achieve estimation robustness. For example, if weekly frequency was used (instead of daily) the sample of four years would shrink to only 200 data points.

6 Although our analysis focuses primarily on financial market determinants of sovereign CDS term spreads, we also consider measures of market microstructure such as CDS market liquidity as measured by the average bid-ask spread (see also Calice et al., 2013). By contrast, data on the volume of trading in the sovereign CDS market is unavailable.

the original CDS term premium should be a nonstationary variable and display a pattern of time-varying volatility. Indeed, visual inspection suggests that the CDS term premium shows both nonstationary and time-varying volatility behavior. Standard unit root tests confirm that all the CDS term premium series (with the exception of Sweden) are first-order integrated. A possible explanation for this nonstationarity and heteroskedasticity is sovereign CDS market segmentation as a result of the global financial crisis.

Similarly to other markets, CDS prices are driven by demand and supply forces. However, one needs to think carefully about the price formation process to identify the key leading factors. This is especially the case for the derivative market, whose dynamics depend on the evolution of the underlying assets. A deeper understanding of the microstructure of the CDS market requires taking into account other markets that are most relevant for every single price move in the CDS premia. As such, our focus of interest here is the underlying security (government bonds in our case). Another major candidate for our examination is the equity market. Furthermore, since at least at some stages the eurozone sovereign debt crisis has been very closely interconnected with the implementation of monetary policy as well as with structural changes in money markets, we also include short-term interest rates in the analysis. Moreover, the existing cross-country and public debt and financial institutions' balance sheet interlinkages justify the inclusion of additional variables in our analysis.

As we are interested in the sources of the short-term dynamics, we collect several financial market variables which are observable at daily frequency (see Figure 3):

(i) *Sovereign CDS market liquidity* calculated as the average of the bid-ask spread of 10Y and 5Y CDS. The effect of CDS market liquidity on the CDS term premium is not clear-cut (see Calice et al., 2012). However, we can basically discriminate between two sets of countries (Figure 2, dashed line). On the one hand, we can observe decreasing CDS term premia accompanied by falling liquidity in CDS markets in the first group of countries (i.e., Spain, Italy, Portugal, and Ireland). On the other hand, for the second group of countries, the general pattern is an increase in the CDS term premia accompanied by a drop in liquidity in the sovereign CDS market (e.g., Germany, France, and the Netherlands, which are displayed in Figure 2). This divergent pattern between the CDS term premium and CDS market liquidity in these two groups of countries is presumably an indication of how market participants differentiate between periphery and core countries.
(ii) The *slope of the bond yield curve* of each sovereign, which is calculated as the difference between the 10Y and the 5Y government bond yield (bid-close). This slope is the bond market counterpart of the CDS term premium. The expected effect is not obvious given the ambiguous evidence about (CDS vs. bond) price discovery in the case of European sovereign issuers (Ammer and Cai, 2011; Calice et al., 2013; IMF, 2013).
(iii) The *short-term interest rate* is proxied by the 3M money market interest rate for each country (3M Euribor for the euro area countries). It tracks monetary policy as well as liquidity conditions in the money market.
(iv) The *stock index return*, calculated as the daily return (in percentage points) of the local major stock market index.
(v) The *CDS term premium of the banking* sector, which is computed as the difference between the 10Y and the 5Y CDS quotes (mid-price) of the two largest banks by asset in each country.[7]

[7] The CDS quote was always available for the largest bank. In the few cases where no quote was available for the second largest bank, either the third largest bank was used instead, or, if it was not available, only the first

This variable encompasses the potential transfer of credit risk between sovereign debt and the domestic banking sector.

(vi) *International sovereign spillover/contagion*, which is proxied by a common factor derived from the CDS term premia of other European countries (i.e., for each of the five countries considered here a factor is derived by applying the principal factor method to the CDS term premium of all the countries). Following Longstaff et al. (2011), we use only the first factor, which accurately captures most of the variance.

(vii) *Stock market volatility*, as measured by the Chicago Board of Options Exchange S&P500 Volatility Index (VIX). This variable reflects the overall market sentiment or the degree of risk aversion, which can have disturbing effects on sovereign risk premia.[8] It is worth pointing out that while variables (i)–(v) denote a set of key domestic variables tracking developments in the sovereign CDS market itself (liquidity) as well as other markets (sovereign bond market, money market, banking CDS market, stock market), variables (vi) and (vii) identify two potentially relevant international variables.

---

largest bank quotation was considered. For CE countries these data are not available, as their major banks are foreign-owned. However, it does not seem appropriate to proxy the credit risk of CE subsidiaries by the CDS quotes of their mostly Western European parents given that the subsidiaries are subject to local banking supervision that impedes the direct transfer of credit risk from parent company to local subsidiary.

[8] A corresponding measure of the implied volatility of stock options is not available for most EU stock indices. Such measures do exist, for example, for the German DAX (VDAX) stock market index and for the pan-European Euro Stoxx 50 (V2X) index. However, these are almost perfectly correlated with the VIX. The historical volatilities of each stock market index could be calculated, but as backward- rather than forward-looking measures, they are arguably worse proxies for current market sentiment given that the participants are forward-looking.

Figure 1: Sovereign 5Y and 10Y CDS Premium (Maximum Available Time Span)

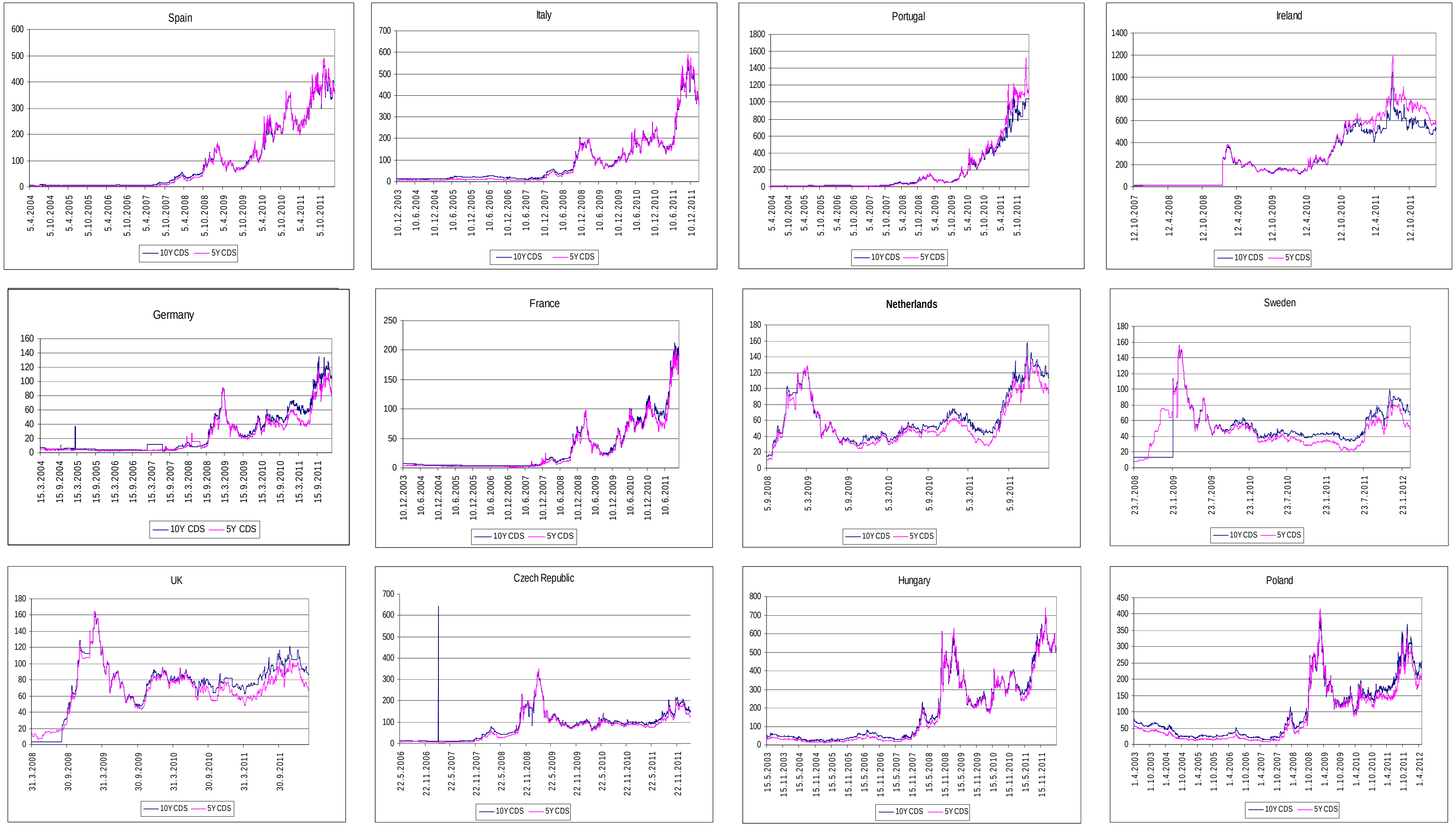


Figure 2: Sovereign CDS Term Premium and Sovereign CDS Market Liquidity

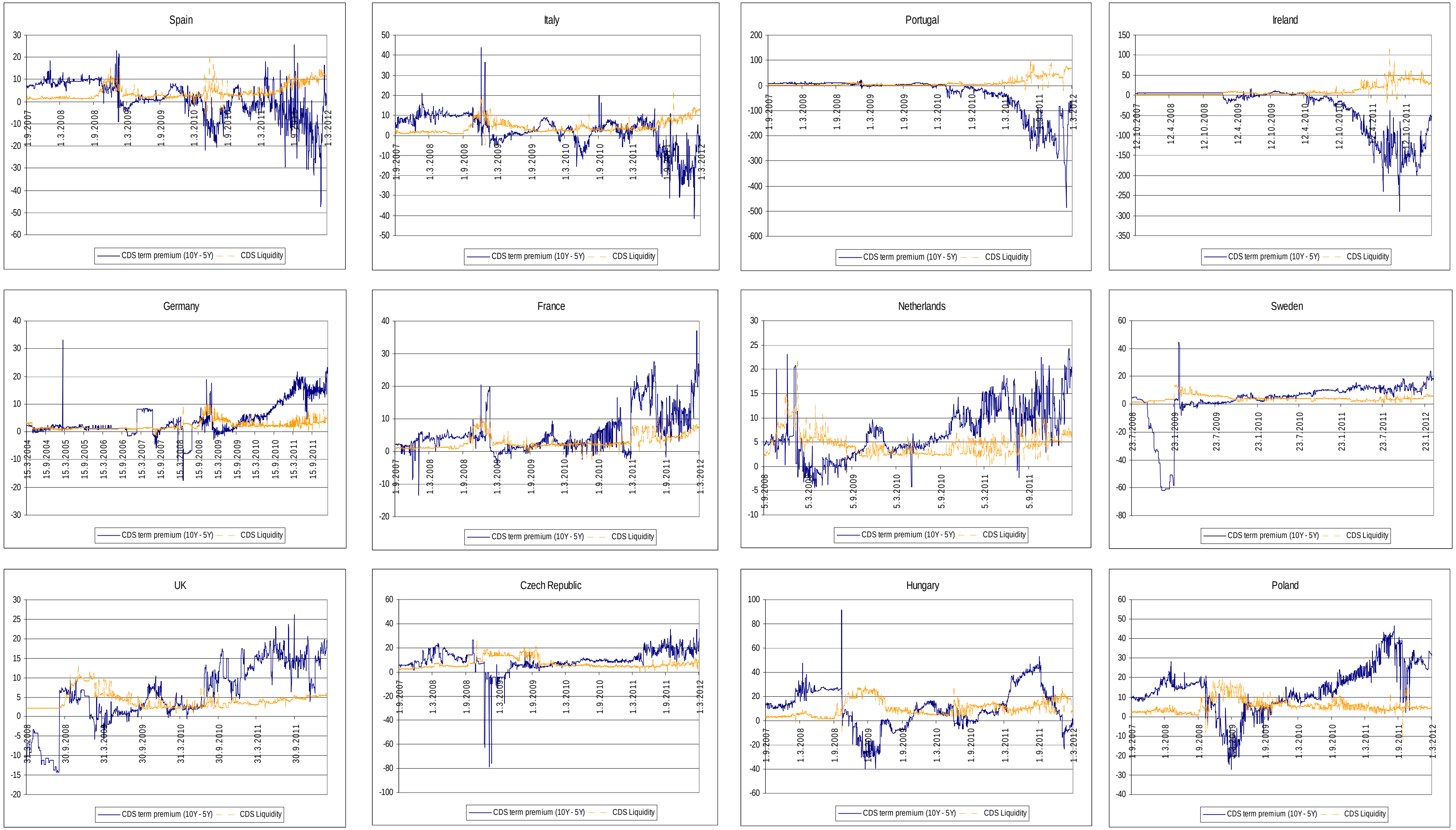
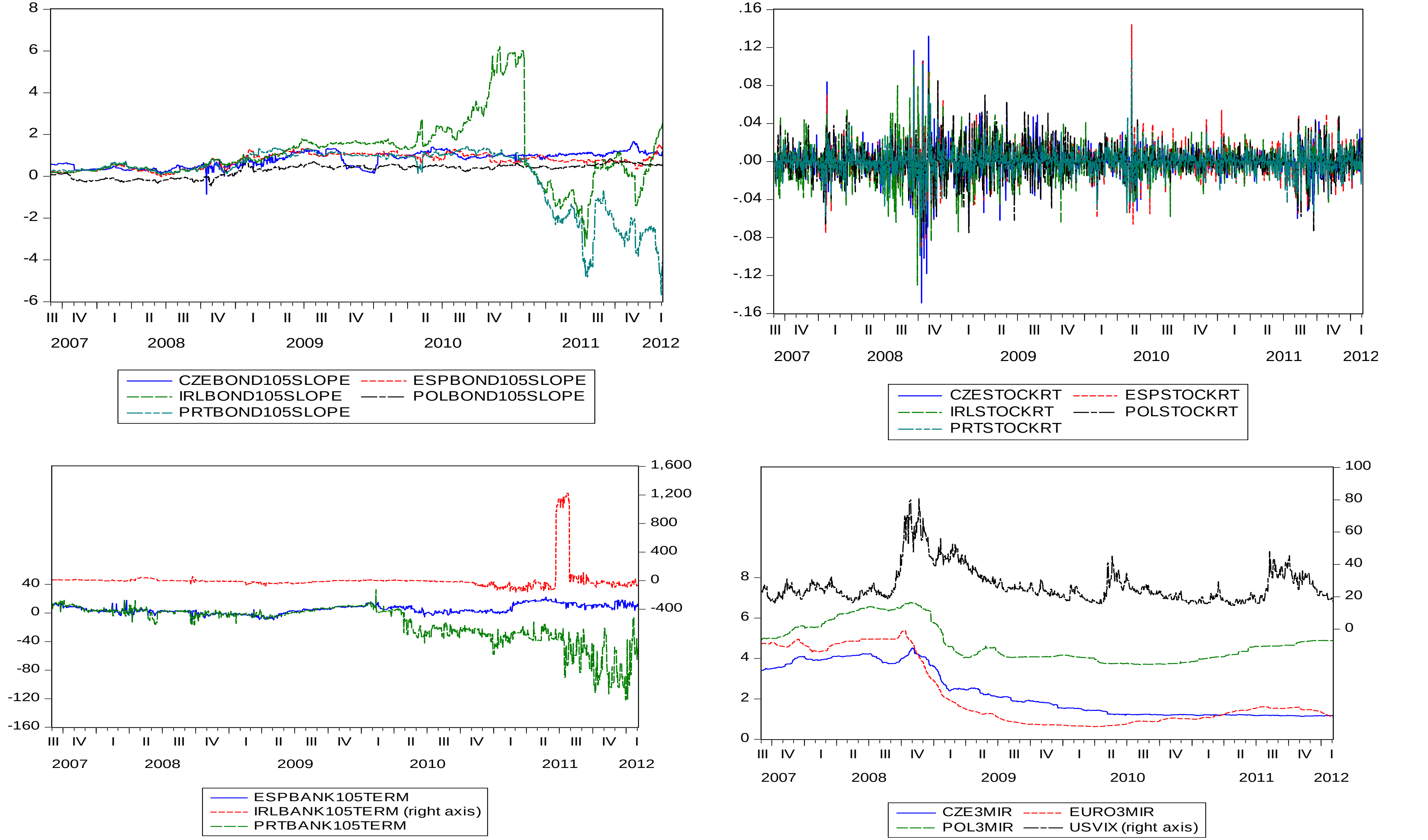


Figure 3: Financial Market Variables Observed at Daily Frequency



4. Methodology

In this paper, we investigate the univariate time series of the sovereign CDS term premium on a selection of European countries. Several approaches to decomposing univariate time series have been proposed in the econometric literature. A well-established methodology is the unobserved components approach, postulated in separate contributions by Harvey (1985), Watson (1986), and Clark (1987). The econometric methodology employed in this paper relies upon the statistical approach developed initially by Nerlove, Grether, and Carvalho (1979) and extended by Harvey (1989) and Harvey and Shephard (1993). The essential element of this methodology is to estimate a model which considers the observed time series as being the sum of a permanent (nonstationary) and a transitory (stationary) component. It seems natural to consider an economic time series in terms of these two components. The decomposition of a univariate time series into these two components is a primary tool for analyzing business cycles, with these two components often used as measures of the unobserved trend (permanent component) and cycle (transitory component). Researchers also use unobserved component models to study the mean reversion in stock prices. Fama and French (1988) find a stationary mean-reverting component in addition to a permanent component in the U.S. stock price dynamics. Poterba and Summers (1988) test for the existence of a stationary component, although they do not perform a formal decomposition of stock prices into stationary and permanent components. These components capture the salient features of the series that may be unobserved and are useful in explaining and predicting its time evolution. In terms of our decomposition of the CDS term premium, the stationary (mean-reverting) component underscores the fundamental driving forces in the economy, while the nonstationary (random walk) component captures the overall uncertainty underpinning the evolution of the fundamentals.[9]

As evidenced by the sharp increase in sovereign risk premia and their volatilities during the recent financial crisis, sovereign risk premia behave differently in distinct regimes. Traditionally, a sudden shift in the mean and volatility level of a time series is modeled as a "structural break" in which this shift is due to some permanent change in the economy's structure. One can either pre-select the break points based on a prior or let the data itself determine the break points endogenously (data-driven approach). However, the issue of identifying a structural break within a finite sample is a subtle one. A criticism of pre-selecting the break points is that this may lead to data-snooping.[10] The data-driven approach of testing for structural breaks is also subject to a well-known criticism. A long time span of data is normally required to obtain consistent parameters, yet structural break tests require these parameters to be estimated by splitting the finite sample into even smaller subsamples. The search for structural breaks over small subsamples, as argued in Lo and MacKinlay (1990), can bias the inference toward mis-identification, especially in a very persistent covariance stationary time series.

[9] Use of the terms "permanent" and "transitory" would be slightly confusing in our case. Whereas in business cycle analysis, the GDP series have a permanent (nonstationary) trend and there is some temporary (stationary) cyclical fluctuation around the trend, in our case the CDS term premium is a mean-reverting variable. Therefore, the fundamental part is mean-reverting and stationary as well, while the short-term spikes are nonstationary. Therefore, the economic meaning of the two components is different.

[10] In addition, this approach assumes these shifts in the structure of the economy are deterministic and give no guidance about their recurrence.

An alternative method for modeling shifts in the CDS term premium is to assume that those changes are recurrent. By allowing for endogenous regime switches in volatility, one does not have to explicitly set a switching threshold value, but the data endogenously identify the switching to a different regime. By adding Markov-switching disturbance terms into the two unobserved components (stationary and nonstationary), one can explicitly model high- and low-volatility regimes over different time periods. Although it complicates the estimation procedures – since additional filters must be employed to make inference on the hidden Markov chain process – allowing the two components to depend on different states of the economy provides an alternative approach to dealing with the potential heteroskedastic variance in the daily risk premia series.[11]

4.1 Modeling the Unobserved Factors that Drive the Term Premia

Let $X_{1,t}$ represent the stationary component (STAT) that drives the term premium, and assume that $X_{1,t}$ is an Ornstein-Uhlenbeck process whose dynamic evolution can be described by the stochastic differential equation

$$dX_{1,t} = k\left(\delta - X_{1,t}\right)dt + \tilde{\sigma}_1 dZ_{1,t} \tag{7}$$

where δ is the target equilibrium or mean value supported by fundamentals; $\tilde{\sigma}_1 > 0$ is the scale of volatility that exogenous shocks can transmit to the dynamics of $X_{1,t}$; $dZ_{1,t}$ is the standard Brownian motion with zero mean and unit variance that generates random exogenous shocks; $k > 0$ is the rate at which these shocks dissipate; and the variable $X_{1,t}$ reverts back to its mean. Therefore, it is a *mean-reversion process*.

The econometric modeling, however, emphasizes the discrete-time representation of stochastic processes. Consequently, the exact discrete time model corresponding to Eq. (7) is given by the following AR(1) process:

$$X_{1,t} = \delta\left(1 - e^{-k\Delta t}\right) + e^{-k\Delta t} X_{1,t} + \sigma_1 \Delta Z_{1,t} \tag{8}$$

where $\Delta t = 1/250$ is the sampling interval and $\sigma_1 = \tilde{\sigma}_1 \sqrt{\dfrac{\left(1 - e^{-k\Delta t}\right)}{2k}}$. It is easy to see that $k > 0$ implies $e^{-k\Delta t} < 1$ and hence stationarity, $k \to 0$ or $\Delta t \to 0$ implies $e^{-k\Delta t} \to 1$, and the model converges to a unit root model.

Now, let $X_{2,t}$ be the second component that drives the term premium. We assume that it follows a driftless random walk (RW) process as shown in Eq. (9):

$$dX_{2,t} = \sigma_2 dZ_{2,t} \tag{9}$$

[11] The more conventional way of testing for financial time series heteroskedasticity is to consider ARCH-type volatility models, which allow constant unconditional volatility but time-varying conditional volatility. However, neglecting possible regime shifts in the unconditional variance, as shown in Lamoureux and Lastrapes (1990), would overestimate the persistence of the variance of a time series.

where σ_2 is the scaled volatility parameter and $dZ_{2,t}$ is the standard Brownian motion, which can be assumed to be either dependent on or independent of $dZ_{1,t}$.The discrete time version of Eq. (9) yields

$$X_{2,t} = X_{2,t-1} + \sigma_2 \Delta Z_{2,t} \tag{10}$$

The RW process has long been a popular choice for modeling the price dynamics of financial assets. In continuous time financial models, the price of stocks and stock indexes are modeled as geometric Brownian motions. It is relatively straightforward to show that the geometric Brownian motion of the price dynamics is equivalent to an RW path followed by the logarithm of the price in discrete time. The efficient market hypothesis in fact states that the financial asset's price follows an RW process, which literally assumes that the asset's price at time t is determined by the price in the previous time period and the instantaneous price impact of the new flow of information. Although an RW process, such as the one described in (10), has infinite unconditional mean and variance, the conditional mean and variance can be measured as

$$\begin{aligned} E_t\left(X_{2,t}\right) &= X_{2,t-1} \\ Var_t\left(X_{2,t}\right) &= \sigma_2^2 \end{aligned} \tag{11}$$

where the conditional expectation of the process at the current time t depends only on the observation in the previous time period.

Given the two unobserved components constructed using Eq. (7) through Eq. (10), we estimate the parameter space as given by the system in Eq. (12), with the dynamics of the two components updating in a Bayesian manner, namely, the Kalman filter algorithm based on a state space system. State space representation is usually applied in dynamic time series models that involve unobserved variables (e.g., Engle and Watson, 1981; Hamilton, 1994; Kim and Nelson, 1989). A typical state space model consists of two equations. One is a state equation that describes the dynamics of the unobserved variables, as shown below in Eq. (12); and the other one is a measurement equation that describes the relation between the measured variables and the unobserved state variables, as shown in Eq. (13).

$$\begin{gathered} \begin{bmatrix} X_{1,t} \\ X_{2,t} \end{bmatrix} = \begin{bmatrix} \delta\left(1-e^{-k\Delta t}\right) \\ 0 \end{bmatrix} + \begin{bmatrix} e^{-k\Delta t} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} X_{1,t-1} \\ X_{2,t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix}, \\ \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix} \sim \mathrm{N}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_1^2 & \sigma_1\sigma_2\rho_{12} \\ \sigma_2\sigma_1\rho_{21} & \sigma_2^2 \end{bmatrix} \Delta t \right) \end{gathered} \tag{12}$$

$$Y_t = X_{1,t} + X_{2,t} \tag{13}$$

In Eq. (12), the covariance terms $\sigma_1\sigma_2\rho_{12}$ and $\sigma_2\sigma_1\rho_{21}$ will be zero under the assumption of independence between the two disturbance terms (the correlation between the two disturbance terms – ρ_{12} – is zero). In compact form, Eq. (12) can be rewritten as

$$\begin{aligned} X_t &= C + FX_{t-1} + \Sigma_t, \\ \Sigma_t &\sim \mathrm{N}\left(0, Q\right) \end{aligned} \tag{14}$$

where $X_t = \begin{bmatrix} X_{1,t} \\ X_{2,t} \end{bmatrix}, C = \begin{bmatrix} \delta\left(1-e^{-k\Delta t}\right) \\ 0 \end{bmatrix}, F = \begin{bmatrix} e^{-k\Delta t} & 0 \\ 0 & 1 \end{bmatrix}, \Sigma_t = \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix}$ and $Q = \begin{bmatrix} \sigma_1^2 & \sigma_1\sigma_2\rho_{12} \\ \sigma_2\sigma_1\rho_{21} & \sigma_2^2 \end{bmatrix}\Delta t$.

The measurement equation, as described by Eq. (13), links linearly the CDS term premium to the STAT and RW components. Rewriting this expression in compact form, Eq. (13) reduces further to give

$$Y_t = HX_t \tag{15}$$

where Y_t is the term premium series and $H = \begin{bmatrix} 1 & 1 \end{bmatrix}$ represents the weights of the two components in the term premium.

4.2 Markov-Switching Disturbances

An additional feature of our model is that it allows each component's disturbance term to depend on different states of the economy. In practice, we let the volatilities of the disturbance terms switch between high- and low-volatility regimes. Formally, we assume that σ_1^2 and σ_2^2 in Eq. (12) are driven by two discrete-valued, independent unobserved first-order Markov chain processes $S_{1,t} = \{0,1\}$ and $S_{2,t} = \{0,1\}$ given by

$$\begin{aligned} \sigma_1^2 &= \left(1-S_{1,t}\right)\sigma_{1H}^2 + S_{1,t}\sigma_{1L}^2 \ , \ \sigma_{1H}^2 > \sigma_{1L}^2 \\ \sigma_2^2 &= \left(1-S_{2,t}\right)\sigma_{2H}^2 + S_{2,t}\sigma_{2L}^2 \ , \ \sigma_{2H}^2 > \sigma_{2L}^2 \end{aligned} \tag{16}$$

When both $S_{1,t}$ and $S_{2,t}$ are zero, the two components will be in the high-volatility state, as $\sigma_1^2 = \sigma_{1H}^2$ and $\sigma_2^2 = \sigma_{2H}^2$; similarly, if both $S_{1,t}$ and $S_{2,t}$ equal 1, the two components will be in the low-volatility state, since $\sigma_1^2 = \sigma_{1L}^2$ and $\sigma_2^2 = \sigma_{2L}^2$.
However, it is also possible for one component to be in the high-volatility state while the other is in the low-volatility state. This is a Markov chain process, which means that the current value of the process at time t depends only on its previous value at time $t-1$. The likelihood of the process remaining at the previous value or changing to the alternative depends on the probabilities of transition from one state to the other, which are shown below as

$$\begin{aligned} p_{1,00} &= \Pr\left[S_{1,t} = 0 \mid S_{1,t-1} = 0\right] \\ p_{1,11} &= \Pr\left[S_{1,t} = 1 \mid S_{1,t-1} = 1\right] \\ p_{2,00} &= \Pr\left[S_{2,t} = 0 \mid S_{2,t-1} = 0\right] \\ p_{2,11} &= \Pr\left[S_{2,t} = 1 \mid S_{2,t-1} = 1\right] \end{aligned} \tag{17}$$

To estimate the transition probabilities as shown above, we need to choose the appropriate functional forms of the probability functions that govern the Markov chain variables. Since the transition probabilities have to be bounded within $[0,1]$ the usual choice is to adopt the logistic transformation on the probability terms as

$$p_{1,00} = \Pr\left[S_{1,t} = 0 \mid S_{1,t-1} = 0\right] = \frac{\exp\left(d_{1,0}\right)}{1+\exp\left(d_{1,0}\right)}, p_{1,01} = 1 - p_{1,00}$$

$$p_{1,11} = \Pr\left[S_{1,t} = 1 \mid S_{1,t-1} = 1\right] = \frac{\exp\left(d_{1,1}\right)}{1+\exp\left(d_{1,1}\right)}, p_{1,10} = 1 - p_{1,11}$$

$$p_{2,00} = \Pr\left[S_{2,t} = 0 \mid S_{2,t-1} = 0\right] = \frac{\exp\left(d_{2,0}\right)}{1+\exp\left(d_{2,0}\right)}, p_{2,01} = 1 - p_{2,00} \tag{18}$$

$$p_{2,11} = \Pr\left[S_{2,t} = 1 \mid S_{2,t-1} = 1\right] = \frac{\exp\left(d_{2,1}\right)}{1+\exp\left(d_{2,1}\right)}, p_{2,10} = 1 - p_{2,11}$$

Where $d_{1,0}, d_{1,1}, d_{2,0}$, and $d_{2,1}$ are the unconstrained parameters.

To estimate the state space Markov-switching model described previously, we use Kim's filter (Kim, 1994), which is a numerical algorithm that combines the Kalman filter in estimating state space models and the Hamilton filter (Hamilton, 1989) in estimating Markov-switching models. Specifically, we use the estimation procedures developed in Calice et al. (2012).

4.3 VAR Analysis

Once we decompose the term premia into the unobserved STAT and RW components we can test for the impact of observed economic and financial variables on these components within a VAR setting. In particular, we assume that this *propagation can be non-linear depending on the volatility regime of each component*. Therefore, central to our analysis is whether the observed economic and financial variables have a different impact on STAT and RW (as opposed to the whole CDS term premium) and whether the impacts on STAT and RW differ in the low- and high-volatility regime. Therefore, after obtaining the aggregate results for the whole CDS term premium (on the whole time sample) we estimate two quasi-threshold VARs, one for STAT and another one for RW. We postulate that the threshold variable for each VAR is the MS probability of being in the high-volatility regime obtained from the univariate decomposition. In addition, we assume that the threshold value is 0.5, i.e., at lower probability values the component is in the low-volatility regime, and otherwise it is in the high-volatility regime. The two VAR(p) models can be written as follows:

$$Y_t = c + \sum_{t=0}^{p} \pi_i Y_{t-i} I\left[s_{STATt} > \gamma_{STAT}\right] + \varepsilon_t \tag{19}$$

$$Y_t = c + \sum_{t=0}^{p} \pi_i Y_{t-i} I\left[s_{RWt} > \gamma_{RW}\right] + \varepsilon_t \tag{20}$$

where Y_t is the vector of p endogenous variables including the stationary component (STAT), the difference of the random walk (RW) component as well as six financial variables (defined below) observed at daily frequency, and I is an indicator function that takes value 1 when the threshold variable s_t, in our case the estimated MS probability of being in the high-volatility regime, exceeds the threshold value γ (set to 0.5) , and 0 otherwise.

One can naturally observe that the only difference between these two VAR models is in the threshold variable within the indicator function. As we impose two independent first-order Markov chain processes, we attempt to capture the differential effect of each volatility regime on each subcomponent. Thus, we compute the generalized impulse response functions that are invariant to any ordering specification to trace out the responsiveness of the dependent variables (each component of the term premium) to one unit generalized shock to each of the variables. This approach is useful to evaluate the relative impact of several factors (macroeconomic and financial) on the systemic credit risk or "health" of a domestic economy as measured by the sovereign CDS term premium.

It will be noted that the first step (the decomposition of the CDS premium into the two components and the estimation of the volatility regime for each of them) is subject to uncertainty, which also conditions the results obtained in the second step (VAR analysis). Unfortunately, as joint estimation in one step is empirically unfeasible, the uncertainty cannot be completely avoided. Still, we take a number of steps to at least reduce it. First, besides VAR analysis based on the STAT and RW subcomponents, we also consider the whole CDS term premium (without decomposition). Second, we adopt a simplification consisting in using moving averages of the estimated switching probabilities, which avoids using the exact value estimated for each point in time and instead relies on their smoothed average on a window of one month, which also eliminates some erratic developments (i.e., very frequent switches).[12]

5. Empirical Results

Using the methodology described in Section 4, we estimate for each country a series of nested Markov-switching unobserved component models. Furthermore, we run a battery of tests on the model specification to determine the preferred model to use in the empirical analysis. We present the results of the model selection tests in the Appendix.

5.1 Model Selection Tests

It is well known that for Markov-switching models the standard likelihood ratio test of the null hypothesis of linearity does not have the usual χ^2 distribution. The reason is that there are nuisance parameters which cannot be identified under the null hypothesis. As a result, the scores evaluated at the null hypothesis are identically zero.[13] We use the Hansen (1992) procedure, which provides an upper bound on the $p-$value for linearity, to determine the significance of the improvement for allowing Markov-switching disturbance terms in the two components. In addition, we consider more conventional ways of selecting models based on the Akaike Information Criterion (AIC) and Schwarz Bayesian Information Criterion (BIC). Finally, we verify our model selection results by running a series of residual diagnostic tests to establish whether the selected model is able to infer serial correlation and heteroskedasticity in the data series.

[12] Another possible extension would be the use of a threshold VAR (e.g., Balke, 2000), which allows estimation of the unknown threshold (for a selected threshold variable, which in our case is the estimated probability of the high-volatility regime) as well as inference of its relevance, rather than assuming that the threshold is equal to a certain value (in our case 0.5). However, our use of the moving average of the estimated probabilities makes the identification of the regime "rougher," which in our view avoids the need for a very precise threshold estimation method.

[13] Hansen (1992) and Garcia (1998) introduce alternative tests of linearity against regime switching.

To implement the Hansen (1992) procedure, we need to evaluate the constrained likelihood under the null hypothesis over a grid of values for the nuisance parameters. Defining the restricted model under the null hypothesis of no regime switching of the two components' disturbance terms as described in Eq. (12) with $\rho_{12} = \rho_{21} = 0$, and the alternative model under the assumption of Markov-switching disturbance terms (as shown in Eq. 16–18), the nuisance parameters are denoted as $\{\sigma_{1H}, \sigma_{2H}, p_{1,00}, p_{1,11}, p_{2,00}, p_{2,11}\}$.[14]

Further, we test whether a model allowing correlated disturbance terms performs better than a model with restrictions to zero correlations (see, for example, the estimates for Spain reported in Tables A.1–A.4 in the Appendix). From Table A.1, we can clearly see that the models with correlated disturbance terms generally produce higher likelihood values and lower AIC and BIC statistics.[15] We verify this result with the residual diagnostic tests (see Table A.4), where we test the overall randomness of the residuals of the models (the summation of the disturbance terms of the two components) with the null hypothesis of assuming randomness.[16] It is important to stress that although the most flexible model (Model 8 in Table A.1) is not a powerful autocorrelation measure in the residuals (like all the other alternative models) it nonetheless does a relatively good job in capturing the ARCH effects in the residuals.

5.2 Estimation of the Markov-Switching Unobserved Component Model

Table 1 reports the maximum likelihood estimates from the most flexible and best performing model (Model 8, see the Appendix) for the five countries (Spain, Portugal, Ireland, the Czech Republic, and Poland). As is evident, there is a significant regime-dependent long-term equilibrium of the stationary component for Spain, Portugal, Poland, and Ireland, but not for the Czech Republic. The two regimes, which are defined in our model as low- and high-volatility regimes of the term premium series, are strongly associated, respectively, with a positive and negative long-term equilibrium level of the stationary component for all countries with the exception of Poland.

In normal market conditions, the CDS term premium is generally upward sloping, which suggests that the market is not factoring in imminent default risks but expectations about protection costs are increasing with the tenor of the CDS contract. On the contrary, the term premium could turn negative if market conditions worsened in the immediate future. Since a negative long-term equilibrium level of the term premium is in general interpreted as the result of a short-term deterioration in credit markets, the coincidence of this with high-volatility regimes of the term

---

[14] The grids that we use for σ_{1H} and σ_{2H} in the case of Spain (the grids used on the volatility for each country are guided by the min and max of the volatilities calculated in an overlapping window of 30 days) are $[0.005, 0.15]$, each with an incremental step of 0.05. The grids for $n+1$ and $\{p_{1,11}, p_{2,11}\}$ vary from 0.4 to 0.9 with an increment step of 0.15. The Hansen test yields for all countries conservative p-values significantly below 0.05, which provides strong evidence of rejection of linearity in favor of our Markov-switching formulation.

[15] The likelihood ratio test (see Table A.2) confirms that models 5–8 in general outperform models 1–4. Specifically, the likelihood ratio tests within each nested group of models (see Table A.3) show that Model 8 is the most flexible model.

[16] We report two Ljung-Box Q statistics for each model: one is the autocorrelation Q statistic based on the standardized residuals up to 20 lags. The other one is the ARCH effect Q statistic based on the squared standardized residuals up to 20 lags.

premium is not a surprise. In other words, a worsening of credit market conditions brings about a surge in volatility as well as an automatic correction of the term premium to its long-term equilibrium.

Figure 4 provides the decomposition of the CDS term premium into the STAT and RW components (left panels) as well as the estimated probabilities of each component switching to the high-volatility regime (right panels). A visual inspection of Figure 4 tells us that the stationary component for countries like Spain, Portugal, and Ireland turns negative in early 2011 at the peak of the euro sovereign debt crisis. As can be seen from the plot of the Spanish stationary component, the slope of the credit curve is positive until early 2011. This simply implies that the compensation for default risk in 5 years' time is positive. The situation dramatically changes in January 2011, when the slope of the credit curve turns negative, leading to an increase in default risk. This, to a large extent, reflects the markets' reactions to the European sovereign debt crisis, when banks' asset write-downs and diminishing liquidity in funding markets raised the degree of uncertainty about future credit events. In particular for Spain, worries about the government's ability to repay its debt, as well as the negative state of the economy[17] (nominal GDP contracted by 3.7% and 0.1% in 2009 and 2010, respectively), further intensified the strains in financial markets. The inversion of the credit curve, as embedded in a negative Spanish term premium, vividly captures this deteriorating outlook.

Another notable feature is that the decompositions for Portugal and Ireland appear to be surprisingly similar. Interestingly, we can observe that prior to summer 2010, the CDS term premium for Portugal remains above zero, with a quite low-volatility impact of the RW and stationary components. The cut of two notches in Portugal's sovereign bond rating by Moody's is the key determinant of the steady decline of its term premium in the latter part of the sample period. The RW component seems to have been leading this negative trend since the crisis, whereas the stationary component reverts to negative territory only in summer 2011, when the crisis intensified, leading the EU to implement a series of financial support measures such as the European Financial Stability Facility (EFSF) and the European Stability Mechanism (ESM). As for Ireland, the initial negative term premium around 2008–2009 is certainly a concrete manifestation of the global financial crisis. The analysis shows also that, throughout late 2009 and early 2010, the RW and stationary components both start to fall. This is consistent with the market's concerns over Ireland's debt spiral, which intensified in 2011 when Moody's downgraded Irish sovereign bonds to junk status.

The Central European countries exhibit different decomposition results from Ireland and Portugal. The term premium series for these two countries is positive for most of the sample period, with the notable exception of 2008. For most of the 2009–2010 period, both the RW and stationary components for the Czech and Polish term premia experience a relatively "mild" regime. This could possibly be explained by improving conditions in credit markets and a better outlook for the CE region. Although both countries' banks belong to global financial groups that have been severely hit by the "credit crunch," their activities are mainly inward oriented. The tendency for generating profits mainly through dynamically expanding retail banking activities has ensured a

[17] As Spain is one of the largest eurozone economies (larger than Greece, Portugal, and Ireland combined) the condition of its economy is of particular concern to international observers. Under pressure from the United States, the IMF, other European countries, and the European Commission, the Spanish government eventually succeeded in trimming the deficit from 11.2% of GDP in 2009 to an expected 5.4% in 2012.

high level of balance sheet liquidity for Czech and Polish banks and has avoided a strong dependence on funds from foreign markets, unlike in Spain, Portugal, and Ireland.

The estimation results also reveal that for all these countries the mean reversion speed has an inverse relationship with the volatilities, i.e., a high speed of mean reversion materializes when the term premium is in a relative stationary state, whilst it takes longer for the term premium to revert to its long-term mean when the market enters the high-volatility regime. During non-crisis periods, asset prices are less likely to stay high or low period-to-period, but mean revert quickly to their long-term equilibrium values. In other words, mean-reverting asset prices imply a low probability of ending up in the tail of the distribution.[18] Portugal and Ireland show similar inverse relationships between the mean-reverting speed parameter and the volatility regimes.[19]

As for the Czech Republic and Poland, the rising profile of the term premium generates considerable volatilities in the market. The transition probabilities, plotted in Figure 4, clearly show that the term premium enters the high-volatility regime in early 2011 for both countries.[20] Although the Czech Republic and Poland have more favorable credit market conditions than Portugal and Ireland, the spikes in the transition probabilities of both components switching to the high-volatility regime after mid-2011 may be an indication of potential spillover effects, as volatility shocks quickly transmitted to the Central European countries' capital markets.

---

[18] Our estimate of the Spanish mean-reverting speed (k) is 22.4741 in the low-volatility regime, which translates into a first-order autocorrelation of -0.9140. The speed in the high-volatility regime, on the other hand, falls to 0.6438 or -0.9974 in terms of first-order autocorrelation, revealing very persistent behavior of the stationary component in the high-volatility regime but less persistent behavior in the low-volatility regime.

[19] Our estimate of the mean-reverting speed is 66.2939 (126.4403) in the low-volatility regime, which translates into a first-order autocorrelation of -0.7671 (-0.6030) for Portugal (Ireland). The speed in the high-volatility regime, on the other hand, falls to 0.6822 (0.4118) or -0.9972 (-0.9983) in terms of first-order autocorrelation, which suggests very persistent behavior of the stationary component in the high-volatility regime.

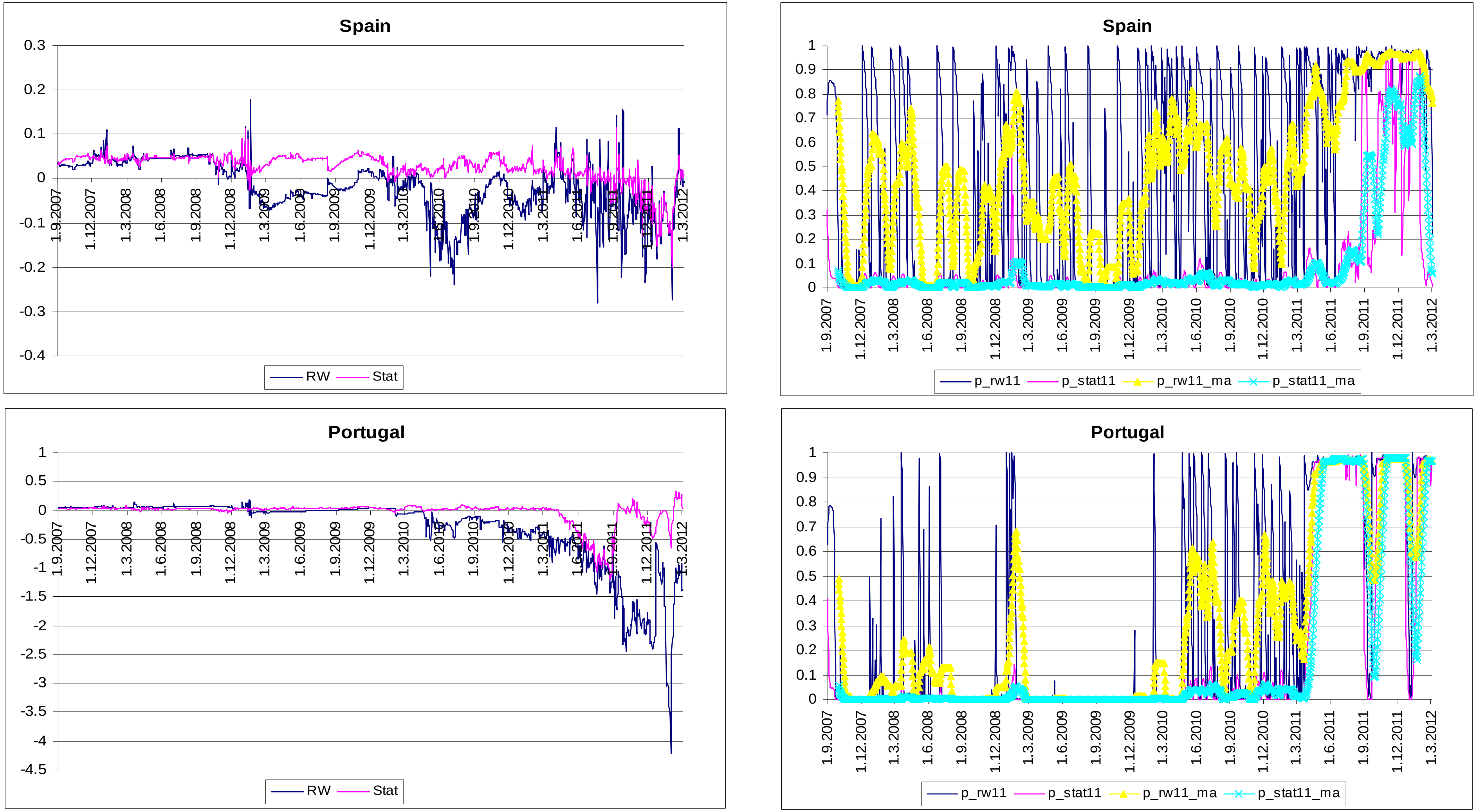
[20] Particularly for Poland, the estimate of the high-volatility regime long-term equilibrium (0.3477) is much higher than the low-volatility regime one (0.1433).

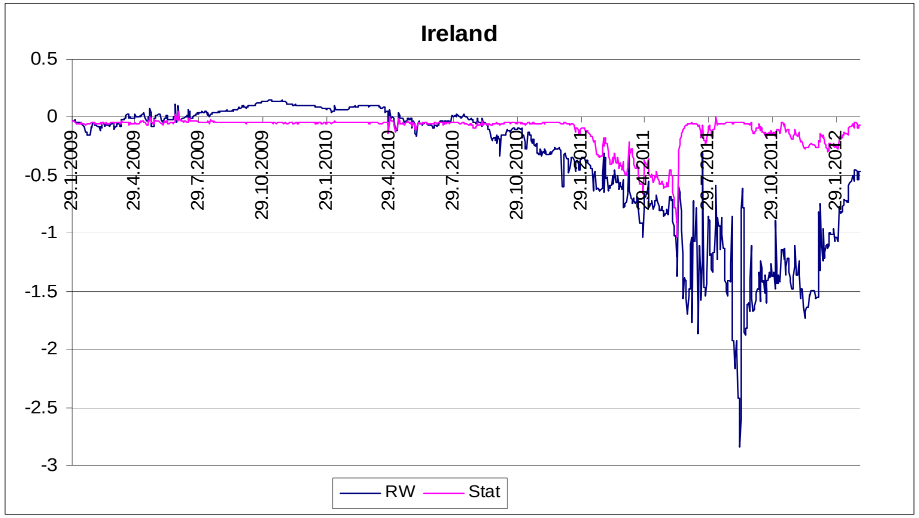
Table 1: Estimation Results

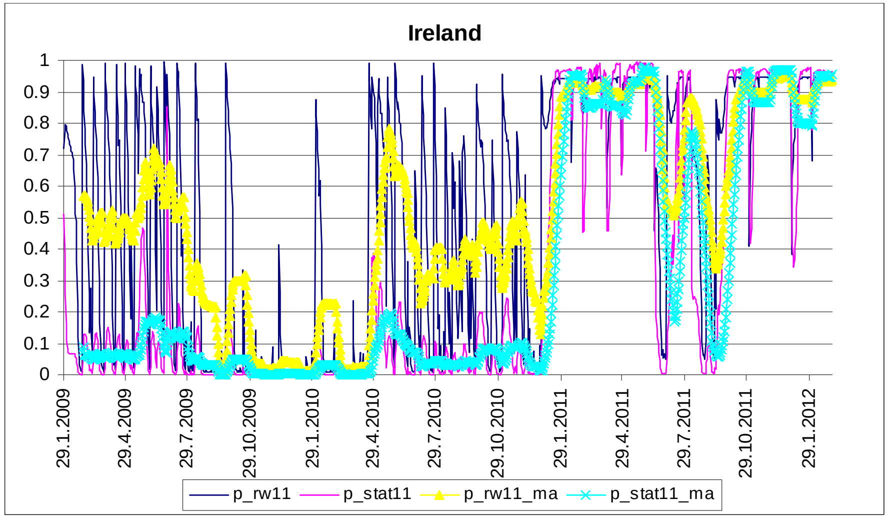
Parameters	Spain	Portugal	Ireland	Czech	Poland
δ_L	0.0485 (3.1056E-05)	0.0197 (1.0804E-05)	-0.0475 (1.4887E-05)	0.0225 (5.8769E-05)	0.1433 (0.0312)
δ_H	-0.0190 (2.1885E-05)	-3.0668 (1.2516E-04)	-1.4020 (0.2443)	-0.0105 (0.1668)	0.3477 (0.0227)
k_L	22.4741 (0.0012)	66.2939 (0.0014)	126.4403 (8.5057E-03)	0.1425 (1.5520E-03)	0.4226 (0.1297)
k_H	0.6438 (4.9187E-05)	0.6822 (1.9017E-04)	0.4118 (0.0324)	0.0783 (0.7619)	0.3477 (0.0227)
$\sigma_{1,L}$	0.0320 (1.0230E-05)	0.2502 (1.7560E-05)	0.0378 (1.7379E-05)	0.0763 (8.5528E-07)	0.0010 (2.0738E-03)
$\sigma_{1,H}$	0.5147 (1.9946E-05)	1.6820 (9.9406E-05)	0.5189 (0.0002)	0.4892 (2.2147E-05)	0.0109 (0.0522)
$\sigma_{2,L}$	0.0315 (1.2994E-05)	0.0221 (7.6557E-06)	0.0711 (1.1732E-05)	0.1033 (1.3741E-07)	0.2457 (3.8638E-03)
$\sigma_{2,H}$	0.5534 (6.8113E-05)	3.2845 (3.5630E-06)	0.7310 (6.7586E-06)	0.7047 (2.6688E-06)	0.8019 (0.0100)
$\rho_{1L,2L}$	0.6073 (3.2774E-04)	0.6317 (9.2002E-04)	0.6300 (4.8726E-04)	-0.0839 (1.4499E-03)	-0.8520 (1.6914)
$\rho_{1H,2L}$	0.8248 (4.0280E-04)	0.8010 (5.0474E-05)	0.7360 (0.3688)	0.1917 (4.0405E-04)	-0.8043 (1.2358)
$\rho_{1L,2H}$	0.7743 (8.3614E-05)	0.8228 (6.6015E-04)	-0.7710 (0.5146)	0.9897 (0.3076)	-0.9964 (0.2025)
$\rho_{1H,2H}$	0.7472 (1.6348E-04)	0.7882 (1.0329E-04)	-0.8316 (2.1876E-04)	-0.3593 (9.6442E-04)	0.8811 (0.4177)
$p_{1,LL}\left(p_{1,00}\right)$	0.9863 (9.0534E-07)	0.9712 (1.8354E-06)	0.9430 (4.8348E-06)	0.9626 (1.0508E-06)	0.9682 (1.7403E-03)
$p_{1,HH}\left(p_{1,11}\right)$	0.9831 (4.1444E-06)	0.9847 (1.8719E-05)	0.9735 (1.3860E-05)	0.9898 (2.8870E-06)	0.9926 (0.0014)
$p_{2,LL}\left(p_{2,00}\right)$	0.9827 (1.1462E-06)	0.9894 (1.6175E-06)	0.9506 (4.2208E-06)	0.9974 (5.8977E-08)	0.9957 (7.7948E-04)
$p_{2,HH}\left(p_{2,11}\right)$	0.9879 (8.0015E-07)	0.9831 (2.8821E-06)	0.9531 (4.6609E-06)	0.9719 (1.0084E-06)	0.9707 (0.0018)
$\ln L$	3964.227	2888.165	1687.581	3866.381	3496.308

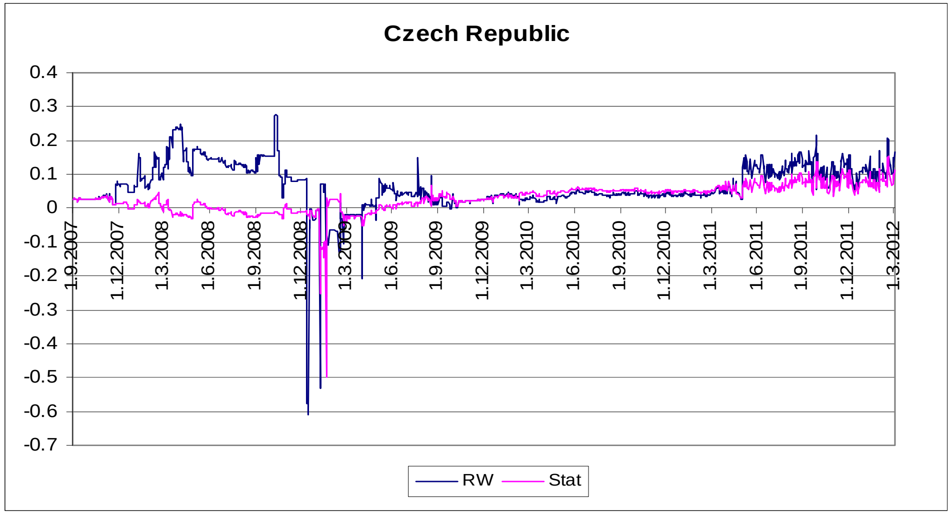
Note: The standard errors of the estimates are in parentheses.

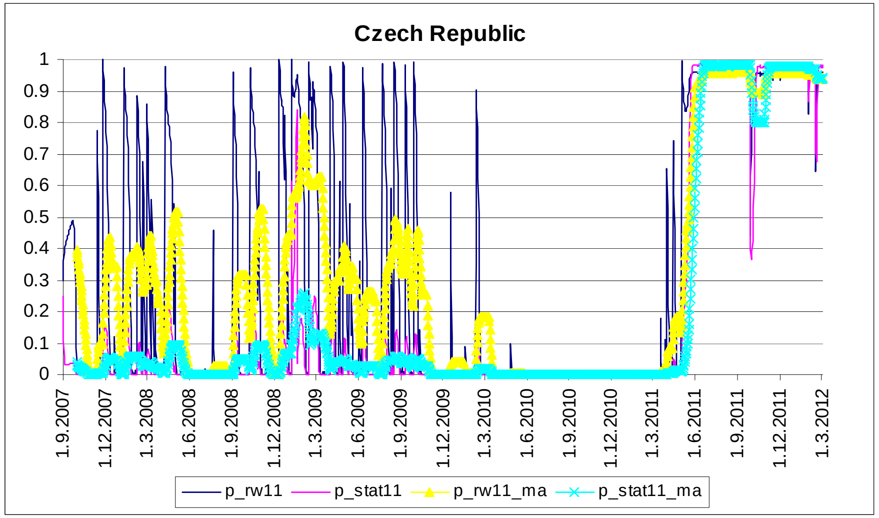
Figure 4: CDS Term Premium Decomposition (Left) and Probabilities of Switching to High-Volatility Regime (Right)

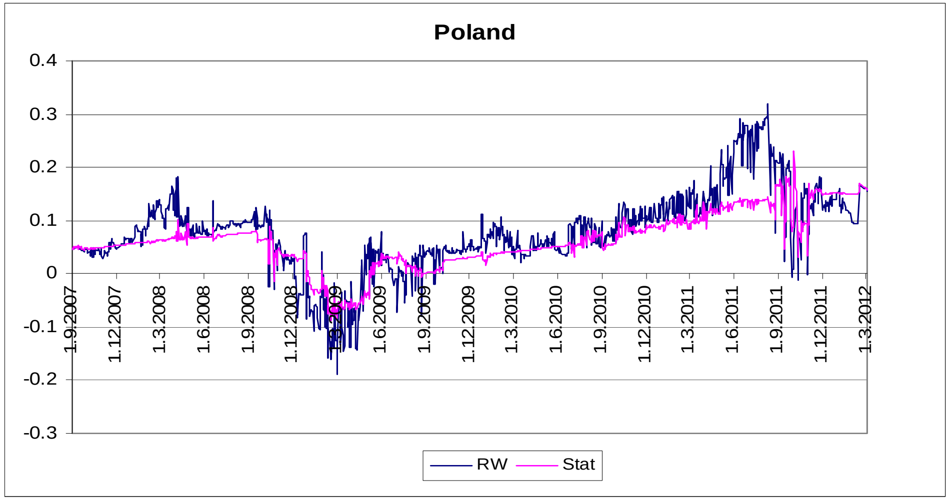


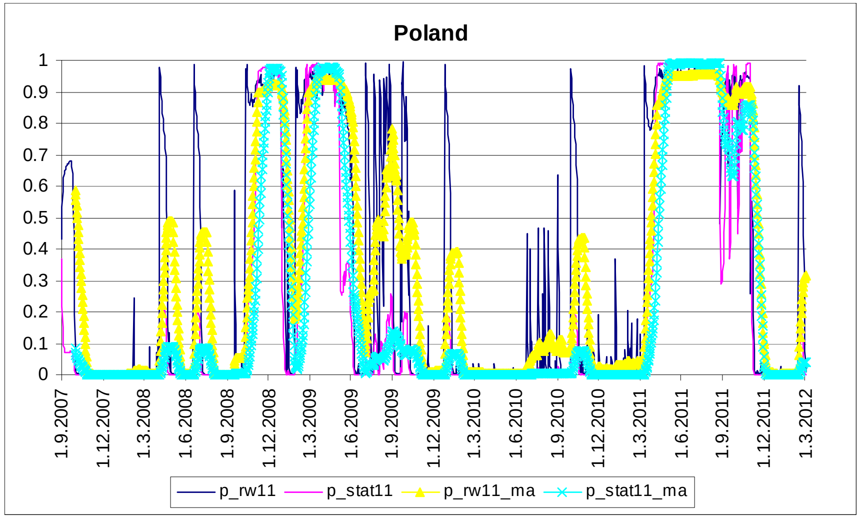

Ireland
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
29.1.2009
29.4.2009
29.7.2009
29.10.2009
29.1.2010
29.4.2010
29.7.2010
29.10.2010
29.1.2011
29.4.2011
29.7.2011
29.10.2011
29.1.2012
RW
Stat


Ireland
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
29.1.2009
29.4.2009
29.7.2009
29.10.2009
29.1.2010
29.4.2010
29.7.2010
29.10.2010
29.1.2011
29.4.2011
29.7.2011
29.10.2011
29.1.2012
p_rw11
p_stat11
p_rw11_ma
p_stat11_ma


Czech Republic
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
1.9.2007
1.12.2007
1.3.2008
1.6.2008
1.9.2008
1.12.2008
1.3.2009
1.6.2009
1.9.2009
1.12.2009
1.3.2010
1.6.2010
1.9.2010
1.12.2010
1.3.2011
1.6.2011
1.9.2011
1.12.2011
1.3.2012
RW
Stat


Czech Republic
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1.9.2007
1.12.2007
1.3.2008
1.6.2008
1.9.2008
1.12.2008
1.3.2009
1.6.2009
1.9.2009
1.12.2009
1.3.2010
1.6.2010
1.9.2010
1.12.2010
1.3.2011
1.6.2011
1.9.2011
1.12.2011
1.3.2012
p_rw11
p_stat11
p_rw11_ma
p_stat11_ma





Note: RW is the nonstationary unobserved component of the CDS term premium, STAT is the stationary component of the CDS term premium, p_rw11 is the filtered probability of the high-volatility regime for the RW component, p_stat11 is the filtered probability of the high-volatility regime for the STAT component, p_rw11_ma is the moving average of p_rw11, and p_stat11_ma is the moving average of p_stat11.

5.3 Determinants of the CDS Term Premium – VAR Analysis

Figure 5 illustrates the results of the VAR models for each country. Overall, we can clearly see that the CDS term premium is affected by both domestic and international variables. This impact is mostly short-lived and materializes within one or two days. Furthermore, note that, in some cases, there is some indication of overshooting, i.e., the response in one direction one day is corrected in the opposite direction the next day.

First, a notable domestic driver of the term premium is CDS market liquidity (first column), although the magnitude of its impact differs somewhat across countries. Indeed, whilst for most countries (Spain, Italy, Portugal, and Ireland) a shock to market liquidity (i.e., an increase in the bid-ask spread and therefore a decrease in liquidity) drives down the term premium, for the Czech Republic the opposite pattern emerges. Note, however, that this effect is short-lived (only one day). At first sight, this finding may seem puzzling. However, we offer an intuitive explanation. If financial market conditions are stable we should see an increase in the CDS term premium. This effect became even stronger as the eurozone sovereign crisis deepened (July 2011 through September 2012). Obviously, the market perception of the imminent default of "severely stressed" countries (Spain, Portugal) soared dramatically in that period. Thus, during periods of financial distress, the CDS term premium normally tends to flatten (i.e., 5Y CDS spreads increase more rapidly than 10Y CDS spreads). On the other hand, for those countries less exposed to the risk of default on government debt, such as the Czech Republic, the CDS term premium tends to exhibit a steepening profile around crisis times. It is worth noting that this interpretation is also consistent with the claim that explicit and implicit government backing for peripheral European countries depresses the 5-year maturity sovereign CDS spreads of the core sovereign debt issuers (Germany, Netherlands) to levels below where they would otherwise be in the absence of government support.

Second, overall, the response to shocks to the slope of the sovereign bond yield curve (second column) is significant and again heterogeneous across countries. Noticeably, only for Spain and Portugal do we find an immediate increase in the CDS term premium following a steepening of the slope, which seems to suggest that in this case the government bond market leads price discovery (this is confirmed by the IRFs of the bond slope response to a shock to the CDS term premium).[21] This effect can once again be attributed endogenously to credit market states. Indeed, at the height of the sovereign debt crisis, we typically observe a flattening of the curve (i.e., the 5Y yield rising more than the 10Y yield) in peripheral countries (Spain, Portugal), whereas when markets are in extremely good states the reverse is true, namely, a steepening of the curve (i.e., the 5Y yield falling more than the 10Y yield) occurs in these countries. These yield curve moves contribute to CDS term premium decreases during bear markets and to CDS term premium increases during booms. Most notably, the remaining countries do not show a similar pattern. The lack of a robust response in either direction for the other countries suggests that these two markets are rather disconnected, which is not surprising. For example, Germany has been considered a safe haven country throughout the sovereign debt crisis and the majority of the vast sell-offs in peripheral government bond markets were accompanied by a buying spree of German government bonds. Furthermore, such vast trading activity did not strain the creditworthiness of German sovereign debt. Additionally, in the case of the CE countries this is in line with the fact that the

[21] The IRFs are available upon request.

sovereign CDS market, as opposed to the bond market, is still substantially underdeveloped, thereby restraining the potential for arbitrage opportunities.

Third, the response to the 3M money market interest rate (third column) is significant only for the Czech Republic and Poland, although the sign of the response is ambiguous. This finding provides evidence that the eurozone common monetary policy (proxied by the 3M interbank rate) is unable to influence the relative risk of default of its members. This result is not too surprising because the 3M Euribor is a common money market rate for 17 different countries, whereas the 3M Pribor and 3M Wibor are country-specific. Consequently, the latter provide better guidance and explanatory power for country-specific market variables such as CDS premia.

Fourth, by contrast, the response to a shock to stock market returns (fifth column) is almost uniformly significant and positive. That a positive mood on the stock market is reflected in decreased perceptions of sovereign default risk is to be expected. The effect is observable only within one day, which merely confirms that the markets are highly interconnected, with information from one market and one asset class spilling over very quickly to other markets and other asset classes.

Fifth, the response to a steepening of the banking CDS term spread is significant for Spain, Portugal, and Ireland (sixth column),[22] although it is contradictorily negative for Ireland, suggesting that sovereign and banking default risk are substitutes rather than complements as commonly believed. This finding underscores the different nature of the problems in Ireland in comparison to Spain and Portugal. The sovereign debt crisis in Ireland originated primarily in structural weaknesses in the domestic banking sector. As a consequence of this, Irish policy makers had to deploy liquidity assistance measures for the banking sector. This effort strengthened the resilience of the Irish banking system (steepening Irish banks' CDS term premium) but obviously led to a severe deterioration in the financial position of the public sector (flattening the Irish sovereign CDS term premium). In contrast, the risks stemming from the negative spiral of economic downturn, austerity measures, and further economic downturn in Spain and Portugal spilled over to local banks. As a result, banks and sovereign CDS premia have been tracking each other closely throughout the crisis.

The international factors are represented by the European CDS common factor (fourth column) and the U.S. VIX (seventh column for Spain, Portugal, and Ireland; sixth column for the Czech Republic and Poland). Our empirical evidence shows a significant response to a shock to the European CDS factor for only a few countries, which lends support to our main argument that the CDS term premium is to some extent a measure of idiosyncratic risk. By contrast, the response to the VIX is negative. This result parallels the findings of Alexander and Kaeck, 2008, and is in line with the original model of Merton (1974), suggesting that higher volatility implies a higher probability of default, which in turn induces a significant reduction in the CDS term premium.

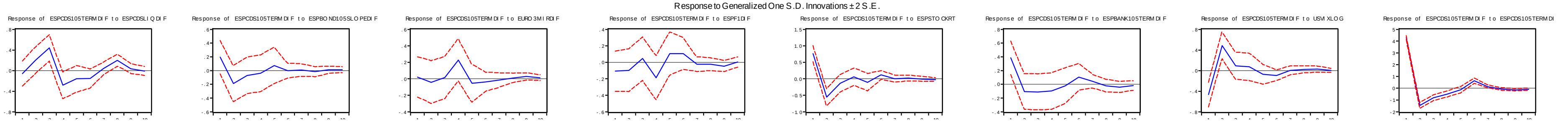
The analysis of the entire CDS term premium based on the whole sample yields compelling empirical evidence on the determinants of the idiosyncratic sovereign risk premium. Indeed, as we have shown above, the term premium seems to embody two components of very different

[22] This variable is not available for the Czech Republic and Poland, as major domestic banks in those two countries are controlled by foreign banking groups. As such, there are no CDS contracts written on these institutions.

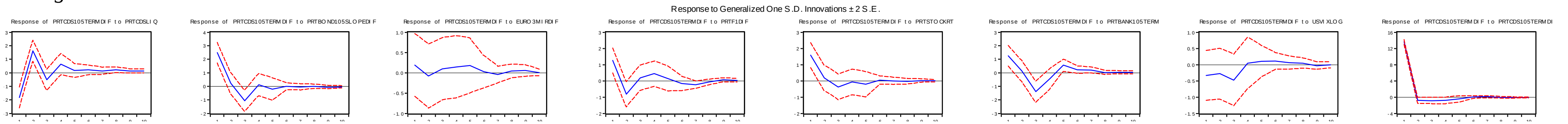
statistical nature, which in turn might even have different determinants according to each of the possible volatility regimes. Calice et al. (2012) have already explored this issue for the corporate risk premium, providing evidence of regime-dependence of its determinants. Consequently, regime-dependent analysis can provide more accurate results even for the sovereign risk premium.

Figure 5: Generalized Impulse Response Function of the VAR model – Comparison Across Countries (Response of the CDS Term Premium to All the Variables)

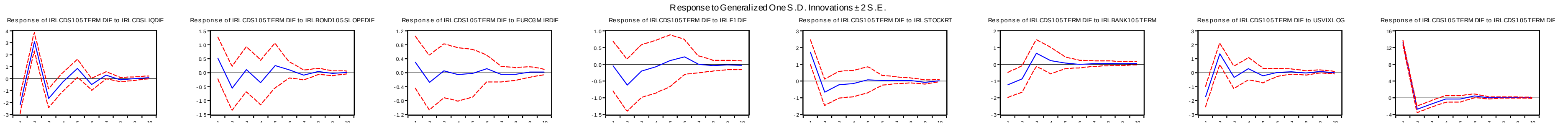
Spain



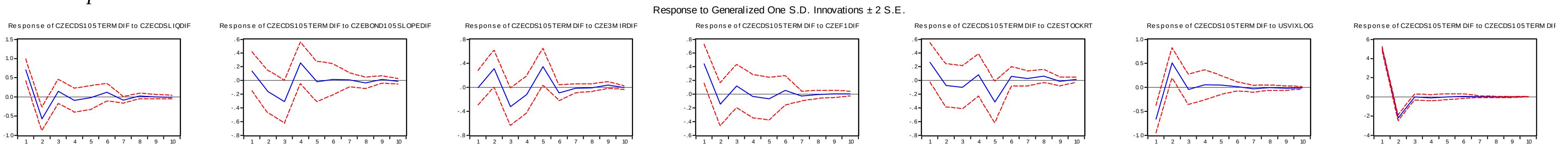
Portugal



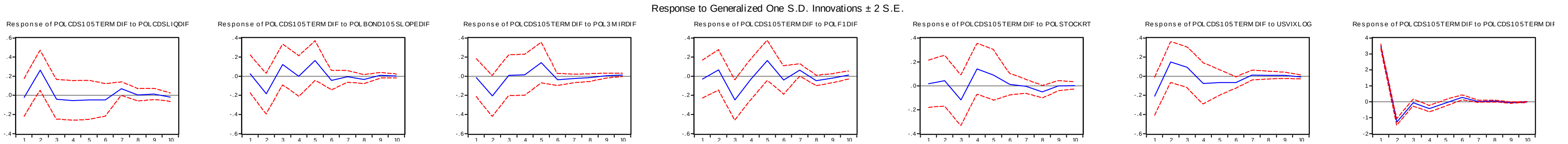
Ireland



Czech Republic



Poland



5.4 Determinants of the CDS Term Premium Components – Regime-Dependent VAR Analysis

To shed some light on the relative contribution of the key determinants of the sovereign CDS term premium, we perform a regime-dependent VAR analysis of the CDS term premium subcomponents. Therefore, we try to establish a link between the unobserved components STAT and RW and the observed market variables. Since here we adopt a two-step estimation procedure, it is again worth acknowledging that some degrees of estimation uncertainty would be inevitably carried over to the second step of the estimation of the VAR model. Alternatively, a macro-finance setting, such as the model of Ang and Piazzesi (2003), could substantially reduce the estimation errors. However, the restrictive formulation of the observed variables in a typical macro-finance setting could overshadow the economically meaningful interpretation of the interactive market variables. Our goal, in this paper, is to test for an economically meaningful relationship between the unobserved components and a set of observed information that is available to both market participants and policy makers.

Table 2 summarizes the forecast error variance decomposition (FEVD) of RW and STAT over a time horizon of 10 days. The results reveal significant heterogeneity in the responses. This is somewhat puzzling since this contradicts the results when the entire CDS term premium is considered. In general, the RW component is affected by the observed financial variables to a greater extent than the STAT component. The major impact can be attributed to the liquidity of the CDS market. This finding, combined with the rather limited effect attributable to the slope of the yield curve, suggests that sovereign CDS and bond markets are rather disconnected and the arbitrage is limited when we assess the term premium (the slope of the yield curve) rather than spreads at single maturity. Indeed, most studies aiming at single maturity find a strong relationship between the bond and CDS spread, although the nature of the price discovery process can change across heterogeneous market conditions (see Delatte et al., 2012).

Interestingly, the importance of other domestic variables varies substantially according to component, volatility regime, and country. According to our findings, the impact of short-term interest rate shocks is strong for the Czech Republic and Spain, stock market returns play a major role for Spain, and the banking variable matters the most for the CDS term premium of Ireland. Finally, the response to the VIX is relevant for several countries, and the share of the dynamics attributable to the common European factor is somewhat limited, offering further evidence that the CDS term premium is, to some extent, idiosyncratic.

Table 2: FEVD (Cholesky) of the VAR model at the 10-Day Horizon (Response of RW/STAT in Each Regime to All the Variables) – Comparison Across Countries

Spain	S.E.	CDSLIQDIF	BONDSLDIF	3MIRDIF	F1DIF	STOCKRT	BANKTRDIF	VIXLOG	PRWDIF	STAT
RW (pr_rw > 0.5)	4.799134	3.122572	0.910119	0.394297	0.734076	**6.605756**	0.629217	0.430128	85.32413	1.849703
RW (pr_rw < 0.5)	0.777795	**7.392406**	1.537259	0.805343	1.851821	1.93506	1.714591	1.085539	82.51127	1.16671
STAT (pr_stat > 0.5)	5.161283	3.118661	2.566985	8.922478	3.953085	**18.06567**	4.522141	11.26717	43.38364	4.200171
STAT (pr_stat < 0.5)	1.793107	1.076536	0.31923	0.686407	0.261555	**1.771092**	0.225835	0.321473	43.43816	51.89971
Portugal										
RW (pr_rw > 0.5)	18.61273	**4.599346**	3.371069	0.22666	2.986204	2.63687	1.862217	1.950671	82.05522	0.311741
RW (pr_rw < 0.5)	2.929016	**5.440617**	0.855406	0.650121	3.722335	0.706017	2.245269	1.426892	81.70073	3.252613
STAT (pr_stat > 0.5)	17.29297	1.718047	1.931909	0.69018	**3.947095**	**3.876447**	1.320468	**3.825645**	79.06818	3.622032
STAT (pr_stat < 0.5)	2.767482	1.190514	1.854128	0.453532	0.51607	0.336487	**2.238994**	1.234901	23.16723	69.00814
Ireland										
RW (pr_rw > 0.5)	17.18126	**11.30898**	0.189897	0.265044	0.459697	3.591715	0.619687	1.731614	81.72007	0.113289
RW (pr_rw < 0.5)	6.481057	**14.60815**	**11.02513**	2.075606	2.006875	2.652629	**10.86177**	1.37691	47.47026	7.922667
STAT (pr_stat > 0.5)	12.06632	3.342739	0.358827	0.498776	0.04361	0.393445	1.329662	**6.379667**	32.45871	55.19457
STAT (pr_stat < 0.5)	1.879222	1.936934	**5.965701**	1.350934	0.425909	1.692821	0.448872	2.930404	3.148307	82.10012
Czech Republic										
RW (pr_rw > 0.5)	6.816741	**4.116408**	1.964265	1.40734	1.09464	1.074442	-	2.150548	84.1246	4.067754
RW (pr_rw < 0.5)	0.817574	**4.292823**	0.197389	0.601914	2.373946	0.684727	-	0.327088	90.86817	0.653942
STAT (pr_stat > 0.5)	24.0379	0.609457	0.299014	14.47985	0.371493	3.188395	-	**3.486406**	63.26015	14.30523
STAT (pr_stat < 0.5)	2.487897	0.569055	0.22653	1.32437	3.890726	0.427979	-	**4.026812**	5.153673	84.38085
Poland										
RW (pr_rw > 0.5)	3.755478	**1.173805**	0.789466	0.632573	0.830709	0.396611	-	0.770076	94.93905	0.46771
RW (pr_rw < 0.5)	1.857994	**7.525892**	1.235702	1.186442	2.796114	1.989723	-	0.483973	84.26174	0.520415
STAT (pr_stat > 0.5)	3.288833	0.96421	**0.992727**	0.072711	0.434016	0.659136	-	0.455477	34.80622	61.6155
STAT (pr_stat < 0.5)	1.685425	0.383139	0.722546	0.271312	1.578659	**1.819298**	-	0.173996	33.13244	61.91861

Figures 6 to 10 illustrate the detailed results for each country. Four VARs are run for each country, dividing the sample according to the volatility regime of STAT and RW (using moving averages of the filtered probabilities in Figure 4). Overall, it appears that there is relevant heterogeneity of the responses across the CDS term subcomponents and their volatility regimes. Indeed, the responses of the overall CDS term premium depicted in Figure 5 are driven very often by the responses of one subcomponent and/or one volatility regime. As expected, the responses are more significant for the RW component, in particular in its high-volatility regime. Remarkably, even where there is a response in both volatility regimes, the magnitude of the RW response in the high-volatility regime is sometimes as much as *ten times* higher than in the low-volatility regime.[23]

First, a shock to CDS market liquidity (first column) affects at least one subcomponent in all countries. The typical pattern is that a shock to CDS market liquidity (i.e., an increase in the bid-ask spread and a decrease in liquidity) is accompanied by an immediate decrease of the CDS term premium, which corrects to positive territory the next day. The latter suggests that during very messy risk-off days, the market participants are even more negative than what would correspond to the negative news flow. Therefore, the prices tend to overshoot on that day and this overreaction is very often corrected the following day. The response is sharpest in the case of RW in high volatility. Therefore, the analysis suggests that when the CDS market dries up it becomes more costly to insure against short-term default. This is in line with market observations, because any time there is a big economic event that impacts the markets, the market participants widen the spreads until the price discovery process is finished.

Second, a shock (steepening) to the sovereign yield curve (second column) initially significantly increases the entire sovereign CDS term spread, which is driven by the RW component. A steepening of the bond yield curve in normal times (i.e., the low-volatility regime for RW) indicates an expected future increase in short-term rates. Further transmission to the RW component of the CDS term premium is detected for Spain, Ireland, and Poland. By contrast, in periods of distress (high volatility for RW), a steepening of the yield curve might reflect short-term liquidity provision by the central bank, which is reflected in an increase in the RW subcomponents of Spain and Portugal. Interestingly, for these two countries we do not detect a significant response to a short-term interest rate shock (third column). Therefore, it seems that that for the eurozone countries the liquidity conditions on the money market or monetary policy action are unable to steepen the CDS term spread (its RW component) directly and the effect has to be intermediated by a steepening of the bond yield curve. On the contrary, for the two CE countries that have retained autonomous monetary policy a shock to the 3M interest rate is reflected in the RW subcomponent, although the expected positive sign is recorded only in the Czech Republic. Finally, the overall irresponsiveness of the STAT component suggests that short-term

[23] Note that the magnitudes of the IRFs will not be automatically compared in the low- and high-volatility regimes given that the size of the shocks (the depicted shock corresponds to one standard deviation of each endogenous variable) might differ across these regimes. However, since we define the regimes in terms of the volatility of RW and STAT the variability of the other variables in the VAR might be independent of these regimes. Indeed, the standard deviations of the bond yield slope, short-term interest rate, stock returns, banking CDS term spread, and VIX are very similar in both volatility regimes. Therefore, one can reasonably compare the magnitude of the response of RW (and STAT) in each regime. In contrast, the standard deviations of CDS market liquidity vary substantially across these regimes, as this variable is more directly linked to the volatility regimes of the CDS term premium components.

developments in sovereign bond or money markets do not affect this fundamental part of the CDS term premium.

Third, the response to stock market returns (fifth column) is almost uniformly positive. This is consistent with the argument that an increase in stock returns is at any time a sign of optimism about the country's economy, which in turn steepens the CDS term premium. This holds for both regimes and components. Interestingly, especially in the high-volatility regime the original positive response in the first period is subsequently corrected in the second one. This points to some kind of overshooting response that is consequently corrected. Interestingly, for RW the response in the high-volatility regime is much stronger than that in the low-volatility regime, for example, ten times stronger in the case of Spain and five times stronger in the case of Ireland. This confirms the existence of a very strong link between the stock markets and the sovereign debt market

Fourth, the response of the banking sector to the CDS term premium (sixth column, not available for CE countries) is positive and significant for both regimes and subcomponents for Spain and Portugal. This variable seems to be closely linked to investor optimism, as its IRFs are practically the same as those of stock market prices. Therefore, a decrease in the immediate credit risk of the country banking sector (i.e., an increase in the bank term premium) steepens the CDS term premium as well. An interesting aspect is the change in the magnitude of the response along the volatility regime and subcomponents. For example, a more detailed look at Spain suggests that in the high-volatility regime the response of the sovereign CDS term premium to the banking CDS term premium is by far the major driver of the sovereign term premium. This is logically related to the fact that the European sovereign debt crisis represents the major part of this high-volatility period and Spain was at the epicenter of it. Similar developments can be found for Portugal, where, like in Spain, both the RW and STAT components get affected. The puzzling negative response detected for the whole CDS term premium for Ireland is confirmed when one performs a regime-dependent analysis for its subcomponents. Indeed, there is no clear economic intuition to explain why a decrease in the risk of the banking sector represented by an increase in the banking CDS term premium should significantly increase the idiosyncratic sovereign risk, i.e., reduce the sovereign CDS term premium and its subcomponents.

Fifth, the response to a shock to the overall EU CDS term premium (fourth column), which is obtained by the principal factor method using the CDS term premium of the other 11 sovereigns depicted in Figure 1 and is aimed at tracking international spillover on the sovereign CDS market, is limited and practically nonexistent in the high-volatility regime. As noted, the sovereign CDS term premium, unlike a CDS on a particular maturity, arguably measures the idiosyncratic sovereign default risk.[24] Therefore, the prevalence of its domestic drivers becomes evident especially in turbulent times (the high-volatility regime for RW). By contrast, in calmer periods (low volatility for RW) we find a significant response for more countries.

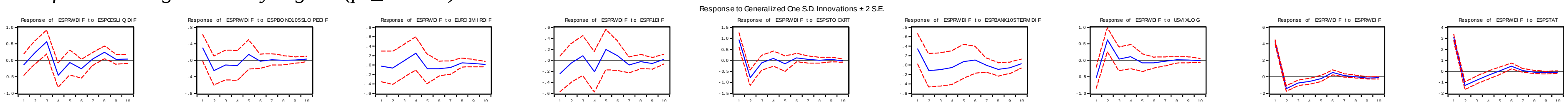
[24] This becomes evident when one compares the results of factor analysis with the CDS premium at 5Y/10Y maturity with the result related to the CDS term premium (10Y – 5Y). Indeed, although the first factor tracks most of the variance in the system, its importance is smaller in the second case. Also, the different size and sign of the factor loading in the second case suggest there are much more idiosyncratic movements in the CDS term premium.

Sixth, the response to overall market sentiment as proxied by the VIX index (seventh column, sixth for CE countries) is often significant and negative. Therefore, an increase in risk aversion significantly flattens the CDS term premium, i.e., increases the short-term credit risk premium by increasing the perceived probability of financial crisis and therefore also of sovereign default. For some countries, such as Spain and Ireland, we again find (as in the case of stock returns) a response several times higher during turbulent periods. As in the case of stock prices we note a pattern of an overshooting reaction in the first period that in general was corrected the following day. Moreover, for the EMU periphery a negative response can also be found for the STAT component, postulating that an increase in risk aversion can have a more fundamental impact on the perceived riskiness of these countries. The inverse relationship between the two components and the VIX is broadly consistent with previous econometric evidence, as illustrated by Campbell and Taksler (2003) and Alexander and Kaeck (2008). In the theoretical framework of Merton (1974), higher equity volatility means a higher probability of hitting the default barrier, which induces higher compensation on holding the bond in the form of a larger credit spread.

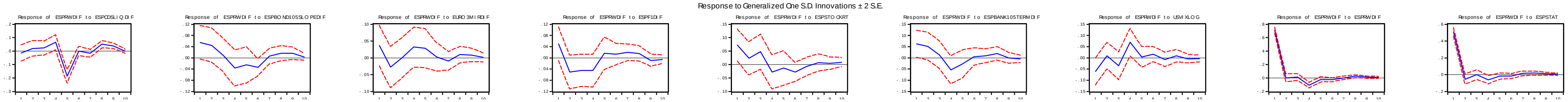
Finally, the last two figures in each row represent the IRFs of each subcomponent, RW and STAT, vis-à-vis its own shock as well as the shock from the other. An interesting feature is that the two components do affect each other, even though their statistical properties are by definition different. The response of RW to a shock to STAT is usually rather short-lived. By contrast, shocks originating from RW take much longer to dissipate in the STAT component. Notably, we can see that the volatile part of the CDS term premium represented by RW, which, as noted earlier on, is in turn affected by other financial variables, does have a significant impact even on the STAT component (essentially macroeconomic fundamentals).

Figure 6: Generalized Impulse Response Function of the VAR Model for Spain (Response of RW/STAT to All the Variables)

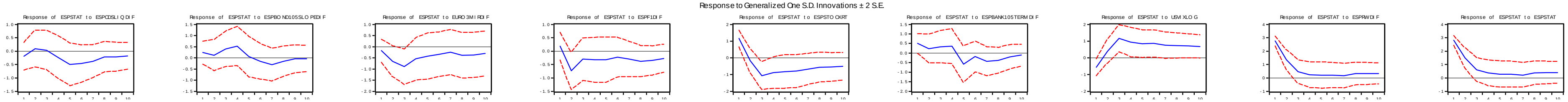
IRFs for RW in high-volatility regime (pr_rw>0.5)



IRFs for RW in low-volatility regime (pr_rw<0.5)



IRFs for STAT in high-volatility regime (pr_stat>0.5)



IRFs for STAT in low-volatility regime (pr_stat<0.5)

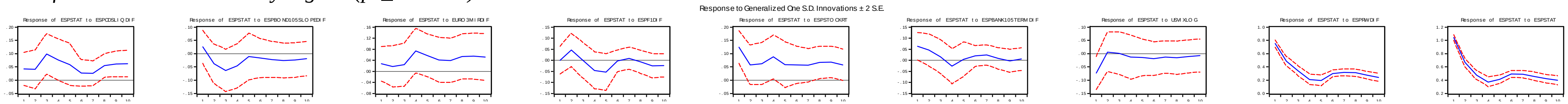
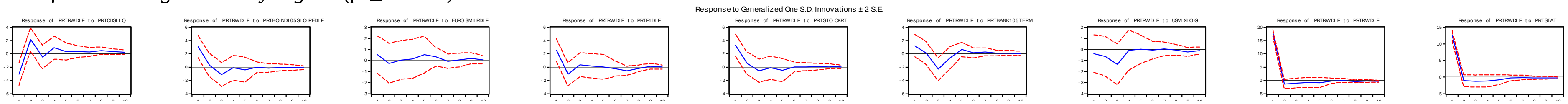
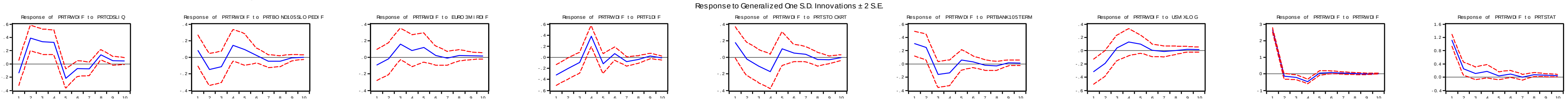


Figure 7: Generalized Impulse Response Function of the VAR Model for Portugal (Response of RW/STAT to All the Variables)

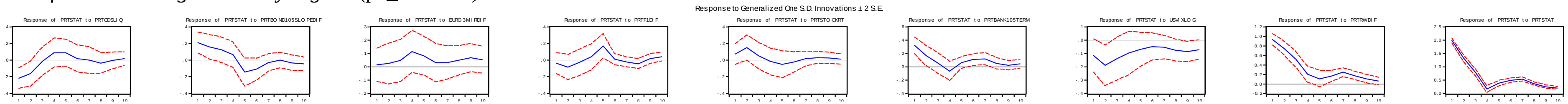
IRFs for RW in high-volatility regime (pr_rw>0.5)



IRFs for RW in low-volatility regime (pr_rw<0.5)



IRFs for STAT in high-volatility regime (pr_stat>0.5)



IRFs for STAT in low-volatility regime (pr_stat<0.5)

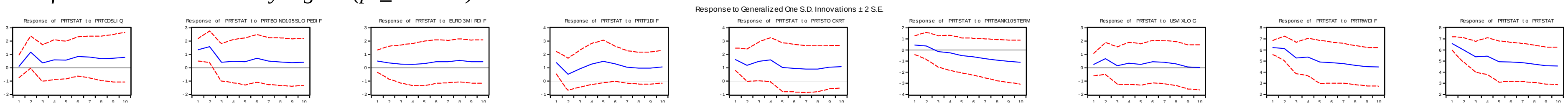
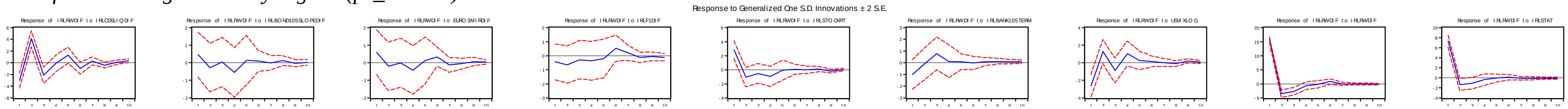
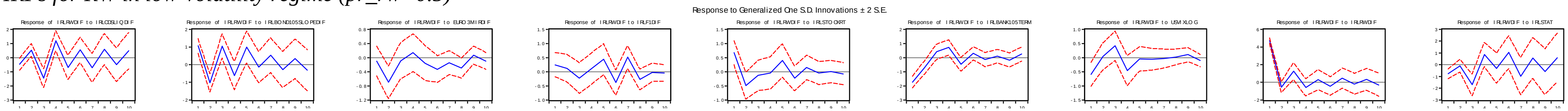


Figure 8: Generalized Impulse Response Function of the VAR Model for Ireland (Response of RW/STAT to All the Variables)

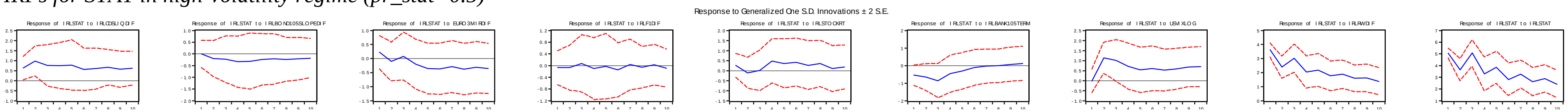
IRFs for RW in high-volatility regime (pr_rw>0.5)



IRFs for RW in low-volatility regime (pr_rw<0.5)



IRFs for STAT in high-volatility regime (pr_stat>0.5)



IRFs for STAT in low-volatility regime (pr_stat<0.5)

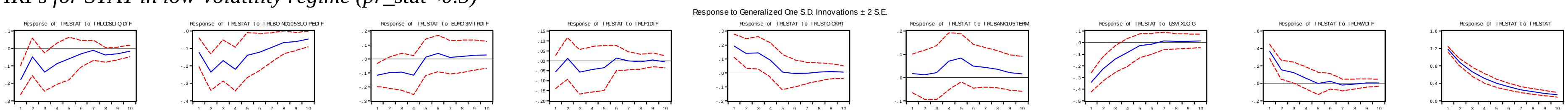
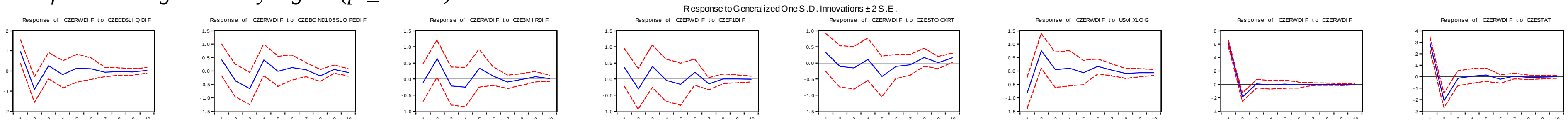
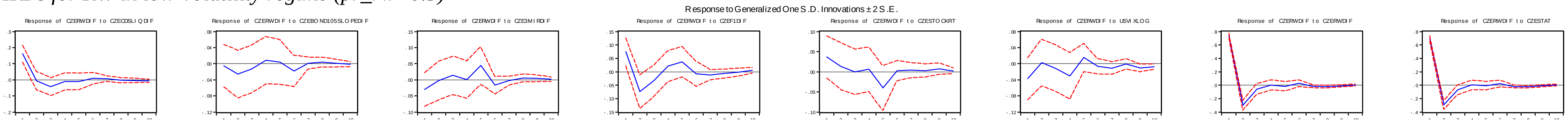


Figure 9: Generalized Impulse Response Function of the VAR Model for the Czech Rep. (Response of RW/STAT to All the Variables)

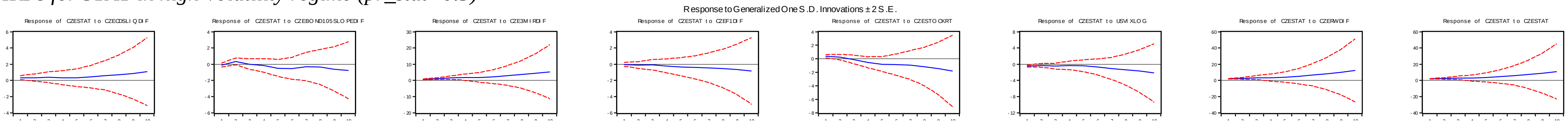
IRFs for RW in high-volatility regime (pr_rw>0.5)



IRFs for RW in low-volatility regime (pr_rw<0.5)



IRFs for STAT in high-volatility regime (pr_stat>0.5)



IRFs for STAT in low-volatility regime (pr_stat<0.5)

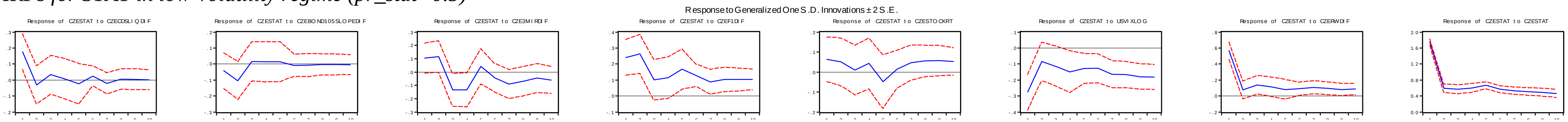
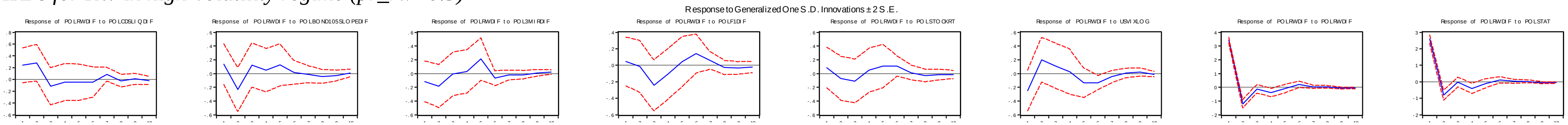
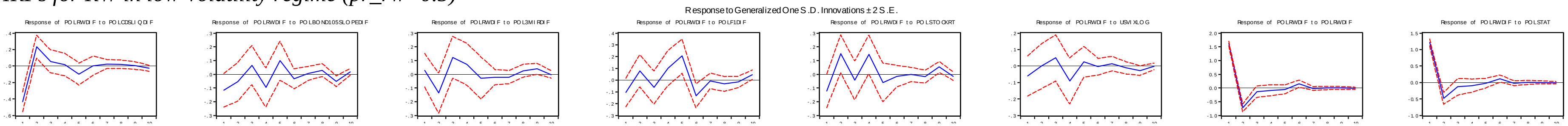


Figure 10: Generalized Impulse Response Function of the VAR Model for Poland (Response of RW/STAT to All the Variables)

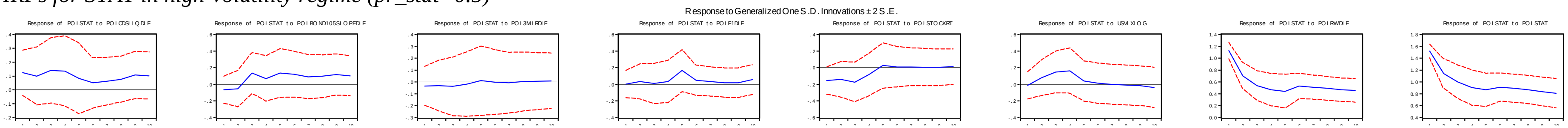
IRFs for RW in high-volatility regime (pr_rw>0.5)



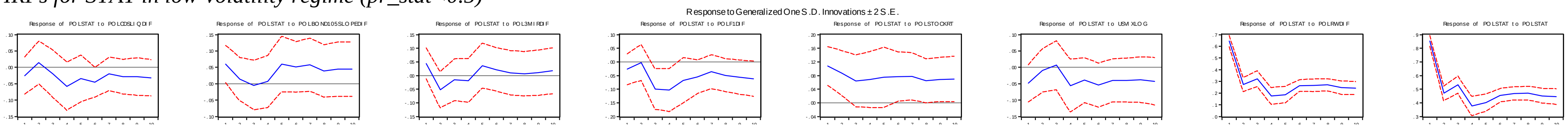
IRFs for RW in low-volatility regime (pr_rw<0.5)



IRFs for STAT in high-volatility regime (pr_stat>0.5)



IRFs for STAT in low-volatility regime (pr_stat<0.5)



6. Conclusions

This study was designed to examine one specific measure of sovereign risk, specifically the CDS term premium, and its dynamic behavior. Following the logic of forward-rate derivation from the term structure, the CDS term premium might be seen as the market's evaluation of the probability that immediate financial turmoil will hit a country. We focus on a selected group of European sovereigns (the EMU periphery and CE countries) over the financial crisis period, when the use of CDS contracts dramatically increased. The CDS term premia of these sovereigns (Spain, Portugal, Ireland, the Czech Republic, and Poland) recorded substantial swings between positive and negative territory and also featured nonstationary and regime-dependent behavior. In order to explain the short-term dynamics of the CDS term premia of these countries, we estimate a Markov-switching unobserved component model. The model allows the CDS term premium to be decomposed into two unobservable components which are of different statistical nature and as such will be affected by different shocks. Our interest is on how these sovereign risk components are affected by observed financial market variables that mostly express the optimism of the market (e.g., local stock prices) in structurally different periods, i.e., in periods of low and high volatility. Unlike most recent research on sovereign default risk we aim at its time dimension rather than its cross-country dimension.

The evolution of the sovereign CDS market at different maturities is an important signal of the perceived "health" of a sovereign. A sudden inversion indicates a sharp deterioration in the current economic conditions and a perceived increased probability of default. We show that the decomposition of this CDS premium is statistically and economically important. Its two unobserved components (stationary and nonstationary) exhibit rather dissimilar behavior, and major increases in the CDS term premium (with both positive and negative sign) are driven mainly by spikes in its nonstationary component. Consequently, we find that the sovereign CDS term premium is significantly affected (mainly through this nonstationary component) by a number of financial market variables in a nonlinear, regime-dependent fashion. On the other hand, the smoother stationary component might rather be linked to slow movements in fundamentals. The magnitude of the response of the nonstationary component to other financial market variables, especially in periods of elevated volatility, when the CDS term premium becomes negative, seems to indicate that under financial distress, the perception of sovereign risk can be exaggerated by shocks from other markets, even those that are not directly exposed to sovereign risk, such as the stock market. Therefore, although the sources of instability can be global or regional (as evidenced by the temporal coincidence of the volatility regimes across countries), the response in these periods can be related rather to domestic factors and vulnerabilities. A notable example is the impact of the banking CDS term premium, tracking the market view of the likelihood of immediate banking turmoil, on the sovereign CDS term premium of the euro area periphery.

It is also interesting to note that the impact of financial market variables is quick and short-lived, as it fully materializes within one or two trading days on average. This implies that market mood (arguably disconnected from the fundamentals) can also affect sovereign risk evaluation. In some cases, there is an indication of market overshooting, i.e., the original response is corrected in the opposite direction the next day. One common driver of the CDS term premium is domestic CDS market liquidity, which suggests that market microstructure and imperfections both matter in pricing sovereign default risk, together with fundamental factors and shocks generated by other markets. By contrast, our results reveal a mutual response between the sovereign CDS and the bond market only for a few countries, suggesting a certain disconnection in the dynamics of the two markets when slope effects are taken into account. The generalized positive response to moves in stock prices provides further evidence of persistent transmission of shocks across markets. By contrast, the response of the national CDS term premia to a pan-European risk factor is quite contained, demonstrating its relevance as an idiosyncratic measure of sovereign risk.

The short-term factors of the CDS term premium dynamics feature some cross-country differences. One notable one is the response to the money market interest rate, which tracks short-term liquidity conditions similarly to monetary policy actions. In particular, while the sovereign risk premium responds to the money market rate in the CE countries, which have kept autonomous monetary policy, no response to money market rates can be detected in the three EMU periphery countries. In other words, it seems that the direct effect of the common monetary policy on the sovereign CDS term premium is limited and has to be "intermediated" by changes in the sovereign bond yield curves. Another notable difference between the CE countries and the EMU periphery is that the perceived sovereign risk increases as the overall level of risk aversion (as tracked by the VIX) increases for the latter but not the former. More importantly, for the EMU periphery we also find evidence that an increase in risk aversion can have a more persistent effect on the perceived riskiness of these countries (by also affecting the stationary component of the CDS term premium). In countries such as Spain or Ireland we also (unsurprisingly) document a very strong link between sovereign and banking credit risk.

The results in this paper might have important policy implications, especially given the recent events related to the eurozone sovereign crisis. For instance, the ban on the use of "naked" CDS contracts on European sovereign entities might reduce the liquidity of the sovereign CDS market through these microstructure affects and in turn change the perceived risk valuation of single sovereigns. While corporate – including banking – CDS are not included in this regulation, our and other evidence suggests that some of the most dramatic movements in sovereign risk dynamics were indeed driven by shocks to the riskiness of domestic banking sectors.

This article is aimed to be a step toward the development of a full-fledged consistent framework to gain greater insight into the dynamics of the sovereign CDS curve across different parts of the credit cycle and into the relationship between the shape of the term structure and macro/financial variables. Interesting possibilities for further research include the consideration of an extended number of maturities and the nexus between fundamental, financial, and microstructure factors of sovereign risk premia. These extensions, along with a complementing examination of liquidity risks and the risk of spillovers, will enhance our understanding of the dynamics of sovereign risk from the systemic viewpoint.

References

ACHARYA, V., I. DRECHSLER, AND P. SCHNABL (2011): "A Pyrrhic Victory? Bank Bailouts and Sovereign Credit Risk." NBER Working Paper No. 17136.

ALEXANDER, C. AND A. KAECK (2008): "Regime Dependent Determinants of Credit Default Swap Spreads." *Journal of Banking and Finance* 32(6), pp. 1008–1021.

ALTER, A. AND Y. S. SCHÜLER (2012): "Credit Spread Interdependencies of European States and Banks During the Financial Crisis." *Journal of Banking and Finance* 36(12), pp. 3444–3468.

AMMER, J. AND F. CAI (2011): "Sovereign CDS and Bond Pricing Dynamics in Emerging Markets: Does the Cheapest-to-Deliver Option Matter?" *Journal of International Financial Markets, Institutions and Money* 21(3), pp. 369–387.

ANG, A. AND F. A. LONGSTAFF (2013): "Systemic Sovereign Credit Risk: Lessons from the U.S. and Europe." *Journal of Monetary Economics* 60(5), pp. 493–510.

ANG, A. AND M. PIAZZESI (2003): "A No-Arbitrage Vector Autoregression of Term Structure Dynamics with Macroeconomic and Latent Variables." *Journal of Monetary Economics* 50, pp. 745–787.

BALDACCI, E., S. GUPTA, AND A. MATI (2008): "Is it (Still) Mostly Fiscal? Determinants of Sovereign Spreads in Emerging Markets." IMF Working Paper No. 259.

BALKE, N. S. (2000): "Credit and Economic Activity: Credit Regimes and Nonlinear Propagation of Shocks." *Review of Economics and Statistics* 82(2), pp. 344–349.

BERNDT, A. AND I. OBREJA (2010): "Decomposing European CDS Returns." *Review of Finance* 14(2), pp. 189–233.

BLANCO, R., S. BRENNAN, AND I. W. MARSH (2005): "An Empirical Analysis of the Dynamic Relationship Between Investment-Grade Bonds and Credit Default Swaps." *Journal of Finance* 60, pp. 2255–2281.

BYSTRÖM, H. (2006): "CreditGrades and the iTraxx CDS Index Market." *Financial Analysts Journal* 62, pp. 65–76.

CAMPBELL, J. Y. AND G. B. TAKSLER (2003): "Equity Volatility and Corporate Bond Yields." *Journal of Finance* 58, pp. 2321–2349.

CALICE, G., C. IOANNIDIS, AND R. H. MIAO (2012): "A Markov Switching Unobserved Component Analysis of the CDX Index Term Premium." Mimeo, University of Southampton.

CALICE, G., J. CHEN, AND J. WILLIAMS (2013): "Liquidity Spillovers in Sovereign Bond and CDS Markets: An Analysis of the Eurozone Sovereign Debt Crisis." *Journal of Economic Behavior and Organization* 85, pp. 122–143.

CECCHETTI, S. G., M. MOHANTY, AND F. ZAMPOLLI (2010): "The Future of Public Debt." In *Challenges to Central Banking in the Context of the Financial Crisis*, proceedings of the Reserve Bank of India conference.

CHEN, L., D. A. LESMOND, AND J. WEI. (2007): "Corporate Yield Spreads and Bond Liquidity." *Journal of Finance* 62, pp. 119–149.

CLARK, P. K. (1987): "The Cyclical Component of U.S. Economic Activity." *Quarterly Journal of Economics* 102, pp. 797–814.

COLLIN-DUFRESNE, P., R. S. GOLDSTEIN, AND J. S. MARTIN (2001): "The Determinants of Credit Spread Changes." *Journal of Finance* 56(6), pp. 2177–2207.

DE GRAUWE, P. AND Y. JI (2012): "Self-Fulfilling Crises in the Eurozone: An Empirical Test." *Journal of International Money and Finance* 34, pp. 15–36.

DELATTE, A. L., M. GEX, AND A. LÓPEZ-VILLAVICENCIO (2012): "Has the CDS Market Influenced the Borrowing Cost of European Countries During the Sovereign Crisis." *Journal of International Money and Finance* 31, pp. 481–497.

DIECKMANN, S. AND T. PLANK (2012): "Default Risk of Advanced Economies: An Empirical Analysis of Credit Default Swaps During the Financial Crisis." *Review of Finance* 16(4), pp. 903–934.

EDWARDS, S. (1984): "LDC's Foreign Borrowing and Default Risk: An Empirical Investigation." *The American Economic Review* 74(4), pp. 726–734.

ENGLE, R. AND M. WATSON (1981): "A One-Factor Multivariate Time Series Model of Metropolitan Wage Rates." *Journal of the American Statistical Association* 76, pp. 774–781.

EOM, Y. H., J. HELWEGE, AND J. Z. HUANG (2004): "Structural Models of Corporate Bond Pricing: An Empirical Analysis." *Review of Financial Studies* 17, pp. 499–544.

FAMA, E. F. AND K. R. FRENCH (1988): "Permanent and Temporary Components of Stock Prices." *Journal of Political Economy* 96, pp. 246–273.

FAVERO, C., M. PAGANO, AND E. L. VON THADDEN (2008): "How Does Liquidity Affect Government Bond Yields?" CEPR Discussion Paper No. 6649.

FONTANA, A. AND M. SCHEICHER (2010): "An Analysis of Euro Area Sovereign CDS and Their Relation with Government Bonds." ECB Working Paper No. 1271.

GARCIA, R. (1998): "Asymptotic Null Distribution of the Likelihood Ratio Test in Markov Switching Models." *International Economic Review* 39, pp. 763–788.

GARRATT, A., K. LEE, M. H. PESARAN, AND Y. SHIN (2006): *Global and National Macroeconometric Modelling: A Long Run Structural Approach*. Oxford University Press.

HAMILTON, J. D. (1994): "State Space Models." In R. Engle and D. McFadden (eds.): *Handbook of Econometrics*. Elsevier.

HAMILTON, J. D. (1989): "A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle." *Econometrica* 57, pp. 357–384.

HANSEN, B. E. (1992): "The Likelihood Ratio Test under Nonstandard Conditions: Testing the Markov Switching Model of GNP." *Journal of Applied Econometrics* 7, S61–S82.

HARRISON, J. M. AND D. M. KREPS (1979): "Martingales and Arbitrage in Multiperiod Securities Markets." *Journal of Economic Theory* 20(3), pp. 381–408.

HARVEY, A. C. (1985): "Trends and Cycles in Macroeconomic Time Series." *Journal of Business & Economic Statistics* 3, pp. 216–227.

HARVEY, A. C. (1989): *Forecasting Structural Time Series Models and the Kalman Filter*. Cambridge University Press: Cambridge.

HARVEY, A. C. AND N. SHEPHARD (1993): "Structural Time Series Models." In G. S. Maddala et al. (eds.): *Handbook of Statistics*. Elsevier Science Publisher.

IMF (2013): "A New Look at the Role of Sovereign Credit Default Swaps." In *Global Financial Stability Report*, April 2013.

KIM, C. J. (1994): "Dynamic Linear Models with Markov-Switching." *Journal of Econometrics* 60, pp. 1–22.

KIM, C. J. AND C. R. NELSON (1989): "The Time-Varying-Parameter Model for Modeling Changing Conditional Variance: The Case of the Lucas Hypothesis." *Journal of Business & Economic Statistics* 7, pp. 433–440.

LAMOUREUX, C. G. AND W. D. LASTRAPES (1990): "Persistence in Variance, Structural Change, and the GARCH Model." *Journal of Business & Economic Statistics* 8, pp. 225–234.

LO, A. W. AND A. C. MACKINLAY (1990): "Data-Snooping Biases in Tests of Financial Asset Pricing Models." *Review of Financial Studies* 3, pp. 431–467.

LONGSTAFF, F. A., S. MITHAL, AND E. NEIS (2005): "Corporate Yield Spreads: Default Risk or Liquidity? New Evidence from the Credit-Default Swap Market." *Journal of Finance* 60, pp. 2213–2253.

LONGSTAFF, F. A. AND A. RAJAN (2010): "An Empirical Analysis of the Pricing of Collateralized Debt Obligations." *Journal of Finance* 63(2), pp. 529–563.

LONGSTAFF, F. A., J. PAN, L. H. PEDERSEN, AND K. J. SINGLETON (2011): "How Sovereign is Sovereign Credit Risk?" *American Economic Journal: Macroeconomics* 3(2), pp. 75–103.

MERTON, R. C. (1974): "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates." *Journal of Finance* 29, pp. 449–470.

MODY, A. (2009): "From Bear Stearns to Anglo Irish: How Eurozone Sovereign Spreads Related to Financial Sector Vulnerability." IMF Working Paper No. 108.

NERLOVE, M., D. M. GRETHER, AND J. L. CARVALHO (1979): *Analysis of Economic Time Series: A Synthesis*. Academic Press: New York.

PALLADINI, G. AND R. PORTES (2011): "Sovereign CDS and Bond Pricing Dynamics in the Euro-Area." NBER Working Paper No. 17586.

PAN, J. AND J. K. SINGLETON (2008): "Default and Recovery Implicit in the Term Structure of Sovereign CDS Spreads." *Journal of Finance* LXIII(5), pp. 2345–2384.

POTERBA, J. M. AND L. H. SUMMERS (1988): "Mean Reversion in Stock Prices – Evidence and Implications." *Journal of Financial Economics* 22, pp. 27–59.

SHUKNECHT, L., J. VON HAGEN, AND G. WOLSWIJK (2010): "Government Bond Risk Premiums in the EU Revisited: The Impact of the Financial Crisis." ECB Working Paper No. 1152.

STULZ, R. M. (2010): "Credit Default Swaps and the Credit Crisis." *Journal of Economic Perspectives* 24(1), pp. 73–92

URIBE, M. AND V. Z. ZUE (2006): "Country Spreads and Emerging Countries: Who Drives Whom?" *Journal of International Economics* 69(1), pp. 6–36.

WATSON, M. W. (1986): "Univariate Detrending Methods with Stochastic Trends." *Journal of Monetary Economics* 18, pp. 49–75.

ZHU, H. (2006): "An Empirical Comparison of Credit Spreads Between the Bond Market and the Credit Default Swap Market." *Journal of Financial Services Research* 29, pp. 211–235.

Appendix A: Model Selection Results

Table A.1: Model Selection Results – Example for Spain

Model Specifications	No. of parameters	AIC	BIC	$\ln L$
Model 0: $\rho_{12} = \rho_{21} = 0$ (single regime)	4	-3.4635	-3.45036	2852.728071
Model 1: $\rho_{12} = \rho_{21} = 0$	10	-4.70752	-4.67466	3881.933
Model 2: $\rho_{12} = \rho_{21} = 0$ (k is regime dependent)	11	-4.75027	-4.71412	3918.095
Model 3: $\rho_{12} = \rho_{21} = 0$ (δ is regime dependent)	11	-3.84006	-3.80391	3169.448
Model 4: $\rho_{12} = \rho_{21} = 0$ (both k and δ are regime dependent)	12	-4.59205	-4.55262	3788.96
Model 5: $\rho_{12} = \rho_{21} \neq 0$	14	-4.78739	-4.74138	3951.626
Model 6: $\rho_{12} = \rho_{21} \neq 0$ (k is regime dependent)	15	-4.73602	-4.68673	3910.38
Model 7: $\rho_{12} = \rho_{21} \neq 0$ (δ is regime dependent)	15	-4.78735	-4.73806	3952.595
Model 8: $\rho_{12} = \rho_{21} \neq 0$ (both k and δ are regime dependent)	16	-4.8003	-4.7477	3964.2264672 3565

Note: Model 0 refers to the system of Equations 10–12 assuming $\rho_{12} = \rho_{21} = 0$. Model 1 builds on Model 0 with Markov-switching variances defined in Equations 10–12; Model 2 builds on Model 1 but allows k to switch regimes; Model 3 builds on Model 1 but allows δ to switch regimes; Model 4 builds on Model 1 but allows both k and δ to switch regimes; Models 5–8 differ from 1–4 in allowing correlations between the two components' disturbance terms. $\ln L$ denotes the natural logarithm of the likelihood value. AIC denotes the Akaike Information Criterion and BIC denotes the Schwarz Bayesian Information Criterion. A smaller AIC or BIC statistic corresponds to a smaller estimated Kullback-Leibler distance from the true model.

Table A.2: Likelihood Ratio Tests on Constraint $\rho_{12} = \rho_{21} = 0$ – Example for Spain

Constraint: $\rho_{12} = \rho_{21} = 0$	Likelihood ratio	p-value
Model 1 to Model 5	139.3857	3.82E-29
Model 2 to Model 6	15.42959	0.003888
Model 3 to Model 7	1566.294	0
Model 4 to Model 8	350.5322	1.35E-74

Table A.3: Likelihood Ratio Tests Within Groups – Example for Spain

Constraints	Likelihood ratio	p-value
Group of models applies $\rho_{12} = \rho_{21} = 0$		
Model 1 to Model 2 (k is regime dependent)	72.32416	0.0000
Model 1 to Model 3 (δ is regime dependent)	1424.97	0
Model 1 to Model 4 (both k and δ are regime dependent)	185.945	4.19E-41
Group of models applies $\rho_{12} = \rho_{21} \neq 0$		
Model 5 to Model 6 (k is regime dependent)	82.49119	1.06E-19
Model 5 to Model 7 (δ is regime dependent)	1.937544	0.163935
Model 5 to Model 8 (both k and δ are regime dependent)	25.20147	3.37E-06

Table A.4: Residual Diagnostic Test – Example for Spain

	Autocorrelation		ARCH		Autocorrelation		ARCH	
Lags	Q-stats	p-value	Q-stats	p-value	Q-stats	p-value	Q-stats	p-value
		Model 1				Model 5		
1	39.11062	4.00E-10	0.347891	0.555309	31.97669	1.56E-08	0.001199	0.972383
5	55.81159	8.89E-11	1.280766	0.936898	41.9961	5.90E-08	1.045453	0.958817
10	64.04229	6.18E-10	19.17475	0.038098	48.4867	5.06E-07	1.509542	0.998905
20	78.80701	6.25E-09	27.75592	0.115304	57.48877	1.73E-05	4.957407	0.999741
		Model 2				Model 6		
1	39.34167	3.56E-10	0.329926	0.565703	30.42881	3.46E-08	0.019129	0.889999
5	52.8439	3.62E-10	1.203485	0.944543	37.41972	4.93E-07	0.262627	0.998287
10	62.00618	1.51E-09	4.878999	0.899113	45.34582	1.88E-06	0.591742	0.999985
20	68.46635	3.24E-07	14.79917	0.787781	55.97484	2.93E-05	4.521084	0.999875
		Model 3				Model 7		
1	14.66075	0.000129	0.440285	0.506985	34.80504	3.64E-09	4.02E-05	0.994943
5	34.2106	2.16E-06	1.710293	0.887601	44.6695	1.69E-08	0.995786	0.962905
10	40.68729	1.28E-05	46.6557	1.09E-06	49.42069	3.41E-07	1.311012	0.999413
20	67.19603	5.19E-07	64.93531	1.20E-06	57.00196	2.05E-05	4.23871	0.999925
		Model 4				Model 8		
1	32.58264	1.14E-08	0.000779	0.977734	16.16636	5.80E-05	0.046774	0.828774
5	41.80586	6.45E-08	0.394182	0.995488	25.56847	1.08E-04	0.670731	0.984536
10	47.00404	9.43E-07	0.702961	0.999967	29.24316	1.14E-03	1.357448	0.999315
20	56.3584	2.57E-05	3.081561	0.999995	39.77742	5.33E-03	1.842954	1

Appendix B: Estimation Procedure

To estimate the state space Markov-switching model described in previous subsections, we use Kim's filter (Kim, 1994), which is a numerical algorithm that combines the Kalman filter for estimating state space models and the Hamilton filter (Hamilton, 1989a) for estimating Markov-switching models. In the conventional derivation of the Kalman filter for an invariant parameter state space model, the goal is to make predictions of the unobserved state variables based on the current information set, denoted $X_{t|t-1} = \mathrm{E}\left(X_t \mid I_{t-1}\right)$, where I_{t-1} represents all observed variables available at time $t-1$. The mean squared error of the prediction, denoted $P_{t|t-1}$, is $P_{t|t-1} = \mathrm{E}\left(\left(X_t - X_{t|t-1}\right)\left(X_t - X_{t|t-1}\right)' \mid I_{t-1}\right)$. The Kalman filter algorithm then implements a sequence of Bayesian updating on the unobserved variable X_t and the mean squared error P_t when observing a new data entry. The updated unobserved variable $X_{t|t}$, given the observation of the information set at time t, is formed as a weighted average of $X_{t|t-1}$ and the new information contained in the prediction error, where the weight assigned to this new information is called the Kalman gain. This prediction and updating process evolves over time and is conditional on the parameters of the model being correctly estimated. As a result, the Kalman filter will need to be initialized in first place. Specifically, some carefully chosen initial values need to be assigned to X_t and its mean squared error at time 0 conditional on the information up to time 0 ($X_{0|0}$ and $P_{0|0}$). For stationary X_t, $X_{0|0}$ and $P_{0|0}$ can be assigned with the unconditional mean and covariance matrix of X_t. For nonstationary X_t (or partially nonstationary X_t as in our case), however, the unconditional mean and covariance matrix of X_t do not exist. In this case, we follow Kim (1994), Kim, Piger, and Startz (2007), and Morley and Piger (2008) to arbitrarily set $X_{0|0}$ at some values based on wild guessing, and subsequently to tackle this very large uncertainty due to the wild guesses by assigning some very large values to the diagonal of $P_{0|0}$. Finally, the prediction errors and their variances, as by-products of the prediction process, will be used to construct the log-likelihood function

$$L\left(\theta\right) = -\frac{1}{2}\sum_{t=\tau+1}^{T} \ln\left[\left(2\pi\right)^n \left|\omega_{t|t-1}\right|\right] - \frac{1}{2}\sum_{t=\tau+1}^{T} \psi'_{t|t-1}\omega^{-1}_{t|t-1}\psi_{t|t-1}$$

where $\psi_{t|t-1}$ is the prediction error and $\omega_{t|t-1}$ is its conditional variance. As noted, this log-likelihood value function will be evaluated from $t = \tau + 1$, where τ is set to be large enough (in our case, we set $\tau = 10$) in order to minimize the effect of the arbitrary initial value $X_{0|0}$ on the log-likelihood value.

In addition to the nonstandard problem of initializing nonstationary state factors in a state space system, inferential procedures on the Markov-switching variables ($S_{1,t}$ and $S_{2,t}$) would undoubtedly complicate the estimation procedures. The prediction and updating processes of the unobserved variable X_t now depend on both the previous and current values of the Markov variables. Since we have two independent Markov chain processes in our model, for given realizations of the two Markov variables at times t and $t-1$ ($S_{1,t-1} = i$, $S_{1,t} = j$, $S_{2,t-1} = i$ and $S_{2,t} = j$, where $i = \{0,1\}$, $j = \{0,1\}$), the Kalman filter equations can then be represented as follows

$$
\begin{aligned}
X_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}} &= C + FX_{t-1|t-1}^{S_{1,t-1}S_{2,t-1}} \\
P_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}} &= FX_{t-1|t-1}^{S_{1,t-1}S_{2,t-1}}F' + \Sigma^{S_{1,t}S_{2,t}} \\
\eta_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}} &= Y_t - HX_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}} \\
f_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}} &= HP_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}}H' \\
X_{t|t}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}} &= X_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}} + \frac{P_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}}H'}{f_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}}}\eta_{t|t-1}^{S_{1,t-1}S_{2,t-1}} \\
P_{t|t}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}} &= P_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}} - \frac{P_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}}H'HP_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}}}{f_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}}}
\end{aligned}
$$

where $X_{t-1|t-1}^{S_{1,t-1}S_{2,t-1}}$ is the value of X_{t-1} based on the information up to time $t-1$, given that $S_{1,t-1} = i$ and $S_{2,t-1} = i$; $X_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}}$ is the updated value of X_t based on the information up to time $t-1$, given that $S_{1,t-1} = i$, $S_{1,t} = j$, $S_{2,t-1} = i$, and $S_{2,t} = j$; $P_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}}$ is the mean squared error of the unobserved $X_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}}$ given $S_{1,t-1} = i$, $S_{1,t} = j$, $S_{2,t-1} = i$, and $S_{2,t} = j$; $\eta_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}}$ is the prediction error of Y_t in the measurement equation, given the updated forecast of X_t as $X_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}}$ conditional on $S_{1,t-1} = i$, $S_{1,t} = j$, $S_{2,t-1} = i$, and $S_{2,t} = j$ based on the information up to time $t-1$; $f_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}}$ is the conditional variance of the forecast error $\eta_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}}$; $X_{t|t}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}}$ and $P_{t|t}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}}$ are the updated X_t and P_t based on the information up to time t, given that $S_{1,t-1} = i$, $S_{1,t} = j$, $S_{2,t-1} = i$, and $S_{2,t} = j$.

Since each iteration of the Kalman filter produces a four-fold increase in the number of cases to consider,[25] we reduce the 16 one-period posteriors $X_{t|t}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}}$ and $P_{t|t}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}}$ to four by taking appropriate approximations at the end of each iteration. This is computed using Kim's approximation procedures

$$X_{t|t}^{S_{1,t}S_{2,t}} = \frac{\sum_{S_{1,t-1}=0}^{1}\sum_{S_{2,t-1}=0}^{1} \Pr\left(S_{1,t-1}=i, S_{1,t}=j, S_{2,t-1}=i, S_{2,t}=j \mid I_t\right) X_{t|t}^{S_{1,t}S_{1,t-1}S_{2,t}S_{2,t-1}}}{\Pr\left(S_{1,t}=j, S_{2,t}=j \mid I_t\right)} \tag{B1}$$

$$\begin{aligned} P_{t|t}^{S_{1,t}S_{2,t}} &= \sum_{S_{1,t}=0}^{1}\sum_{S_{2,t}=0}^{1}\left(P_{t|t}^{S_{1,t}S_{1,t-1}S_{2,t}S_{2,t-1}} + \left(X_{t|t}^{S_{1,t}S_{2,t}} - X_{t|t}^{S_{1,t}S_{1,t-1}S_{2,t}S_{2,t-1}}\right)\left(X_{t|t}^{S_{1,t}S_{2,t}} - X_{t|t}^{S_{1,t}S_{1,t-1}S_{2,t}S_{2,t-1}}\right)'\right) \\ &\times \frac{\Pr\left(S_{1,t-1}=i, S_{1,t}=j, S_{2,t-1}=i, S_{2,t}=j \mid I_t\right)}{\Pr\left(S_{1,t}=j, S_{2,t}=j \mid I_t\right)} \end{aligned} \tag{B2}$$

where the probability terms in the above two equations are obtained from Hamilton's filter as

$$\begin{aligned} &\Pr\left(S_{1,t-1}=i, S_{1,t}=j, S_{2,t-1}=i, S_{2,t}=j \mid I_t\right) \\ &= \frac{\Pr\left(Y_t \mid S_{1,t-1}=i, S_{1,t}=j, S_{2,t-1}=i, S_{2,t}=j, I_{t-1}\right)\Pr\left(S_{1,t-1}=i, S_{1,t}=j, S_{2,t-1}=i, S_{2,t}=j \mid I_{t-1}\right)}{\Pr\left(Y_t \mid I_{t-1}\right)} \end{aligned}$$

with

$$\begin{aligned} &\Pr\left(Y_t \mid S_{1,t-1}=i, S_{1,t}=j, S_{2,t-1}=i, S_{2,t}=j, I_{t-1}\right) \\ &= \frac{1}{\sqrt{(2\pi)^N \left| f_{t|t-1}^{S_{1,t}S_{1,t-1},S_{2,t}S_{2,t-1}} \right|}} \exp\left(-\frac{1}{2}\frac{\left(\eta_{t|t-1}^{S_{1,t-1}S_{2,t-1}}\right)'\left(\eta_{t|t-1}^{S_{1,t-1}S_{2,t-1}}\right)}{f_{t|t-1}^{S_{1,t}S_{1,t-1},S_{2,t}S_{2,t-1}}}\right), \end{aligned}$$

$$\Pr\left(Y_t \mid I_{t-1}\right) = \sum_{S_{1,t-1}=0}^{1}\sum_{S_{1,t}=0}^{1}\sum_{S_{2,t-1}=0}^{1}\sum_{S_{2,t}=0}^{1} \Pr\left(Y_t, S_{1,t-1}=i, S_{1,t}=j, S_{2,t-1}=i, S_{2,t}=j \mid I_{t-1}\right)$$

[25] We have four cases to consider in each iteration of the Kalman filter: (1) both the stationary and random walk components are in the high-volatility regime; (2) the stationary component is in the high-volatility regime while the random walk component is in the low-volatility regime; (3) the stationary component is in the low-volatility regime while the random walk component is in the high-volatility regime; (4) both the stationary and random walk components are in the low-volatility regime. Therefore, in every new iteration, the first-order dependence of the current Markov chain variable on its previous value leads to a four-fold increase in the number of cases to consider.

and

$$\begin{aligned}&\Pr\left(S_{1,t-1}=i,S_{1,t}=j,S_{2,t-1}=i,S_{2,t}=j\mid I_{t-1}\right)\\&=\Pr\left(S_{1,t}=j\mid S_{1,t-1}=i\right)\Pr\left(S_{2,t}=j\mid S_{2,t-1}=i\right)\Pr\left(S_{1,t-1}=i,S_{2,t-1}=i\mid I_{t-1}\right)\end{aligned}$$

with

$$\Pr\left(S_{1,t-1}=i,S_{2,t-1}=i\mid I_{t-1}\right)=\sum_{S_{1,t-2}=0}^{1}\sum_{S_{2,t-2}=0}^{1}\Pr\left(S_{1,t-2}=i,S_{1,t-1}=i,S_{2,t-2}=i,S_{2,t-1}=i\mid I_{t-1}\right)$$

At the end of each iteration, Eq. (B1) and Eq. (B2) are used to collapse the 16 one-period posteriors ($X_{t|t}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}}$ and $P_{t|t}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}}$) into four ($X_{t|t}^{S_{1,t}S_{2,t}}$ and $P_{t|t}^{S_{1,t}S_{2,t}}$). As a by-product of the Hamilton filter, the approximate log likelihood function is given by

$$L(\theta)=\sum_{t=\tau+1}^{T}\ln f\left(Y_t\mid I_{t-1}\right)$$

which will be maximized with respect to the parameter vector space $\Theta=\left\{p_{1,00},p_{1,11},p_{2,00},p_{2,11},\delta,k,a,\sigma_{1,H},\sigma_{1,L},\sigma_{2,H},\sigma_{2,L},\rho_{12}\right\}$.