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RISK MANAGEMENT IN ACTION

ROBUST MONETARY POLICY RULES UNDER STRUCTURED UNCERTAINTY

by Paul Levine, Peter McAdam, Joseph Pearlman and Richard Pierse

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by Paul Levine[2],
Peter McAdam[3],
Joseph Pearlman[4]
and Richard Pierse[5]

In 2008 all ECB publications feature a motif taken from the €10 banknote.

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CONTENTS

Abstract 4

Non-technical summary 5

1 Introduction 8

2 The model 12
- 2.1 The Smets-Wouters model 12
- 2.2 Households 12
- 2.3 Firms 14
- 2.4 Price and wage-setting 15
- 2.5 Equilibrium and Interest rate rule 16
- 2.6 Zero-inflation steady state 17
- 2.7 The inefficiency of the zero-Inflation steady state 18

3 LQ Approximation and the ZLB Constraint 21
- 3.1 Representation of price-wage Dynamics as difference equations 21
- 3.2 The utility function in terms of wage and price dispersions 22
- 3.3 Large distortions quadratic approximation of utility 22
- 3.4 Linearization about the zero-inflation steady state 23
- 3.5 The LQ Problem 25
- 3.6 Imposing the nominal interest-rate zero lower bound 25

4 Estimation 26

5 Optimal monetary policy without model uncertainty 28
- 5.1 Optimal Policy 28
- 5.2 Optimized IFB rules 30
- 5.3 Calvo-type interest-rate rules 34
- 5.4 A Labour market based rule 36
- 5.5 Summary of performance of commitment rules 37

6 Optimal monetary policy with Model Uncertainty 37
- 6.1 Robust rules with model-consistent expectations 39
- 6.2 Robust rules with model-inconsistent expectations 44

7 Conclusions 49

A The Hamiltonian quadratic approximation of welfare 50

References 52

Tables and figures 56

European Central Bank Working Paper Series 58

Abstract

Recent interest in 'Risk Management' has highlighted the relevance of Bayesian analysis for robust monetary-policy making. This paper sets out a comprehensive methodology for designing policy rules inspired by such considerations. We design rules that are robust with respect to model uncertainty facing both the policymaker and private sector. We apply our methodology to three simple interest-rate rules: inflation-forecast-based (IFB) rules with a discrete forward horizon, one targeting a discounted sum of forward inflation, and a current wage inflation rule. We use an estimated DSGE model of the euro area and estimated measures of structured exogenous and parameter uncertainty for the exercise. We find that IFB rules with a long horizon perform poorly with or without robust design. Our discounted future targeting rule performs much better, indicating that policy can be highly forward-looking without compromising stabilization. The wage inflation rule dominates whether it is designed to have good robust properties or not.
JEL Classification: E52, E37, E58

Keywords: interest-rate rules, robustness, structured uncertainty

Non-Technical Summary

The design of robust policy rules is a fundamental concern for monetary authorities. This reflects the awareness that, when evaluating alternative monetary policies, uncertainties about the structure of the economy must be given due account. Robust policy is therefore policy which 'performs well' across a number of different states of the world. Different states of the world, however, carry different probabilities, some highly likely, some not. Moreover, not all realizations are symmetric: some events, even if inherently unlikely, may turn out to be highly malevolent. Accordingly, robust policy design might seek not only to empirically assess the consequences of different outcomes but condition policy on the probabilities of their being realized. This we consider the essence of the Bayesian and, in turn, 'risk management' approaches to robust policy design.

Risk management encompasses two broad arguments. First, the formalization of economic uncertainty (i.e., where uncertainty is captured by some known probability distribution, as opposed to Knightian uncertainty). Second, tailoring policy to insure against severe, adverse outcomes. Consider some examples. During the 1998 Russian debt default, although the Federal Reserve was understood to believe the US economy could weather the crisis, monetary policy was nonetheless loosened to partially insure against such an outcome. Largely speaking, the same events played out in 2003, when concern was expressed that declining inflation could escalate into deflation. Despite these specific examples, the risk management paradigm has tended to be viewed more as a conceptual framework than a practical recipe for policy making (Feldstein, 2004). However, some clear-cut points do emerge. Firstly, reflecting the real-world examples and Greenspan's quote, such a paradigm is fundamentally Bayesian; policy making involves attaching empirically-relevant probabilities to different states of the world. Secondly, it is different from the 'minimax' (or worst-case) approach since policy would then have completely accommodated such scenarios irrespective of their likelihood.

In this paper, we provide a formalization of robust policy design inspired by the Bayesian/risk management approaches. To our knowledge this has not so far been undertaken in the literature. Having decided upon and (Bayesian) estimated a number of rival empirical, micro-founded macro models, we use the estimated model probabilities and posterior parameter densities to design appropriate robust interest-rate rules. In so doing, we define risk-management in three dimensions: First, model-robust or 'M-robust' rules have stable and unique equilibria by design and, in addition, use the probabilities to minimize an expected loss function of the central bank subject to this model uncertainty across central estimates of the models. A typical outcome of Bayesian selection criteria

is degenerate odds: one model or scenario absorbs the mass of posterior probability. Accordingly, we consider a meaningful examination of risk management policies to be the identification of plausible scenarios with reasonably balanced Bayesian odds.

We then adopt a second, more stringent robustness requirement that minimizes the expected loss across all possible parameter values drawn from a large sample within the models constructed using the estimated posterior parameter distributions as well as the model probabilities. We refer to such rules as parameter-robust or 'P-robust'. The final dimension of risk management is as follows. One downside of robust Bayesian approaches is the heightened scope for expectation differences between the private and public sector. We therefore add a third, final dimension to Risk Management rules: that they be expectationally robust. The central bank must therefore consider scenarios over the distribution of parameter values where the private sector may believe in an incorrect state of the world. We refer to such rules as robust with respect to model-inconsistent rules and where perceptions coincide we use the term model-consistent rules.

Our approach differs from existing work on the design of robust policy rules in a number of important respects. First, a recent literature has assumed that uncertainty is unstructured, with malign Nature 'choosing' exogenous disturbances to minimize the welfare criterion that the policymaker is maximizing. However, the worst-case outcome is likely to represent a very low probability event and, from the Bayesian perspective, it is inappropriate to design policy heavily conditioned by it. Consequently, mini-max is not compatible with risk management robust rules. By contrast, we adopt a structured uncertainty approach.

Second, our paper differs from studies in this latter category that design robust rules across competing models, but arbitrarily calibrate the relative probabilities of alternative models being true representations of the economy. In keeping with our theme, we instead utilize estimated measures of model uncertainty for the design of robust rules. Third, the rival-model approach typically confines itself to the first (and weaker) robustness criteria that we consider: M-robustness. We impose an additional and more demanding robustness criterion, P-robustness, which may possibly (in our most stringent criterion) involve model-inconsistent expectations as defined previously. Fourth, we examine robust policy in a unified framework that compares different simple rules with each other, and with their optimal commitment counterparts.

Fifth, we design rules that are *implementable* in that they satisfy the zero lower bound constraint on the nominal interest rate. This turns out to be critical when analyzing robust policy rules. Finally, our analysis of optimal rules is welfare-optimal, based on a 'large distortions' quadratic approximation of the representative household's utility in a structural micro-founded model.

In particular, we apply our methodology to three simple interest-rate rules: inflation-forecast-based (IFB) rules with a discrete forward horizon, one targeting a discounted sum of forward inflation, and a current wage inflation rule. We use an estimated DSGE model of the euro area and estimated measures of structured exogenous and parameter uncertainty for the exercise. We find that IFB rules with a long horizon perform poorly with or without robust design. Our discounted future targeting rule performs much better, indicating that policy can be highly forward-looking without compromising stabilization. The wage inflation rule dominates whether it is designed to have good robust properties or not.

1 Introduction

... the conduct of monetary policy ... has come to involve, at its core, crucial elements of risk management. This conceptual framework emphasizes understanding as much as possible the many sources of risk and uncertainty that policymakers faces, quantifying those risks when possible, and assessing the costs associated with each of the risks. In essence, the risk management approach to monetary policymaking is an application of Bayesian decision-making. Alan Greenspan[1]

... the Governing Council of the ECB has no intention of being the prisoner of a single system ... We highly praise robustness. There is no substitute for a comprehensive analysis of the risks to price stability. Jean-Claude Trichet[2]

The design of robust policy rules is a fundamental concern for monetary authorities. This reflects the awareness that, when evaluating alternative monetary policies, uncertainties about the structure of the economy must be given due account. Robust policy is therefore policy which 'performs well' across a number of different states of the world. Different states of the world, however, carry different probabilities, some highly likely, some not. Moreover, not all realizations are symmetric: some events, even if inherently unlikely, may turn out to be highly malevolent. Accordingly, robust policy design might seek not only to empirically assess the consequences of different outcomes but condition policy on the probabilities of their being realized. This we consider the essence of the Bayesian and, in turn, 'risk management' approaches to robust policy design.

Risk management encompasses two broad arguments. First, the formalization of economic uncertainty (i.e., where uncertainty is captured by some known probability distribution, as opposed to Knightian uncertainty). Second, tailoring policy to insure against severe, adverse outcomes. Consider some examples. During the 1998 Russian debt default, although the Federal Reserve was understood to believe the US economy could weather the crisis, monetary policy was nonetheless loosened to partially insure against such an outcome. Largely speaking, the same events played out in 2003, when concern was expressed that declining inflation could escalate into deflation. Despite these specific examples, the Risk Management paradigm has tended to be viewed more as a conceptual framework than a practical recipe for policy making (Feldstein, 2004). However, some clear-cut points do emerge. Firstly, reflecting the real-world examples and Greenspan's quote, such a paradigm is fundamentally Bayesian; policy making involves attaching empirically-relevant

[1] Greenspan (2004)
[2] Trichet (2005)

probabilities to different states of the world. Secondly, it is different from the 'minimax' (or worst-case) approach since policy would then have completely accommodated such scenarios irrespective of their likelihood.

We assume the central bank commits to some policy rule (simple or complex) defining targets and a welfare function reflecting preferences. Implemented policy then reflects the interplay of shocks, the rule and the welfare criterion controlling for various forms of uncertainty. The key aspect being that these sources of uncertainty are weighted by their appropriate probabilities in the policy maker's expected welfare function. These probabilities can be wholly subjective reflecting the policy maker's views on the likelihood of different outcomes or their perceived impact. Alternatively, they may reflect actual posterior odds of various models and transmission mechanisms representing the economy. We prefer this latter definition since it captures *estimated* measures of uncertainty. Weighting events by (perceived) impact evokes mini-max control which has not described risk management in practice. In our case, the specification of a simple policy rule is the means by which the authorities asses the impact of different states of the world. Arguably, much of the existing robustness literature has failed to exploit the richness of the Bayesian methodology (e.g., Sims (2005)).[3] For instance, the rival models approach (e.g., Adalid *et al.* (2005); Levin *et al.* (2003)) defines a robust rule as one that "works well" across a number of models. In practice, these "number of models" turn out to be few and relatively similar (Svensson (2003)). However, a more important criticism is that aggregating rival models typically maps weakly to actual posterior odds; consequently, it is difficult to appreciate the gain from implementing a rule which performs well in n-1 models but fails in the nth most data-compatible one. Furthermore, as Cogley and Sargent (2005) demonstrate, monetary policy making has historically reflected judgment about the evolving probability odds of certain models representing the economy.

In this paper, we provide further formalization of robust policy design inspired by Bayesian/risk management approaches. To our knowledge this has not so far been undertaken in the literature. Having decided upon and (Bayesian) estimated a number of rival empirical, micro-founded macro models, we use the estimated model probabilities and posterior parameter densities to design appropriate robust interest-rate rules. In so doing, we define risk-management in three dimensions: First, model-robust or '*M-robust' rules* have stable and unique equilibria by design and, in addition, use the probabilities to minimize an expected loss function of the central bank subject to this model uncer-

[3] This is particularly ironic since the new breed of Bayesian micro-founded models increasingly employed by central banks (e.g., Smets and Wouters, 2003) precisely embody estimated measures of structural model uncertainty as by-products of their estimation.

tainty across central estimates of the models. On M-Robustness, one remark is relevant. A typical outcome of Bayesian selection criteria is degenerate odds: one model or scenario absorbs the mass of posterior probability (e.g., Gelman *et al.* (2003)). This undermines the entire approach for policy makers and, in so far as degenerate odds reflects weak model identification, masks the required degree of robustness required for policy. Accordingly, we consider a meaningful examination of risk management policies to be the identification of plausible interesting scenarios with reasonably balanced Bayesian odds.

We then adopt a second, more stringent robustness requirement that minimizes the expected loss across *all* possible parameter values drawn from a large sample within the models constructed using the estimated posterior parameter distributions as well as the model probabilities. We refer to such rules as parameter-robust or *'P-robust'*.

The final dimension of risk management is as follows. If the central bank optimizes over posterior probabilities spaces spanning both M- and P-robust, this may ensure favourable outcomes but the information environment becomes extremely rich. Consequently, one downside of robust Bayesian approaches (widely commented upon at the time of the Greenspan speech by 'Fed Watchers'[4]) is the heightened scope for expectation differences between the private and public sector. We therefore add a third, final dimension to Risk Management rules: that they be expectationally robust. In designing expectationally robust rules, the central bank must therefore consider scenarios over the distribution of parameter values where the private sector may believe in an incorrect state of the world. We refer to such rules as robust with respect to *model-inconsistent* rules and where perceptions coincide we use the term *model-consistent* rules.[5]

Our approach differs from existing work on the design of robust policy rules in a number of important respects. First, a recent literature draws on Hansen and Sargent (2003), Hansen and Sargent (2007) in assuming that uncertainty is unstructured, with malign Nature 'choosing' exogenous disturbances to minimize the welfare criterion that the policymaker is maximizing.[6] However, as Svensson (2000) comments, the worst-case outcome is likely to represent a very low probability event and, from the Bayesian perspective, it is inappropriate to design policy heavily conditioned by it. Consequently, mini-max is not compatible with risk management robust rules. By contrast, we adopt a structured

[4]See for example Miller (2003).

[5]Frankel and Rockett (1988) and Holtham and Hughes Hallett (1992) study model-inconsistent expectations in a different sense: in a world of interdependent economies each with their own central banks, the latter may each believe in different models.

[6]Walsh (2003) provides a effective overview of robust policy design including the Hansen-Sargent minimax approach. Tetlow and von zur Muehlen (2001) and Levine and Pearlman (2007) provides comparisons of robust design with structured and unstructured uncertainty.

uncertainty approach.

Second, our paper differs from studies in this latter category that design robust rules across competing models, but arbitrarily calibrate the relative probabilities of alternative models being true representations of the economy (e.g., Angeloni *et al.* (2003), Levin *et al.* (2003), Adalid *et al.* (2005), Cogley and Sargent (2005), Coenen (2007)). In keeping with our theme, we instead utilize *estimated* measures of model uncertainty for the design of robust rules.[7]

Third, the rival-model approach typically confines itself to the first (and weaker) robustness criteria that we consider: M-robustness. We impose an additional and more demanding robustness criterion, P-robustness, which may possibly (in our most stringent criterion) involve model-inconsistent expectations as defined previously. Fourth, we examine robust policy in a unified framework that compares different simple rules with each other, and with their optimal commitment counterparts.

Fifth, we design rules that are *implementable* in that they satisfy the zero lower bound constraint on the nominal interest rate. This turns out to be critical when analyzing robust policy rules. For instance, in contrast with the familiar Brainard (1967) result, mini-max robust control advocates a strong degree of policy activism. But strong policy activism risks running aground of the lower bound constraint (Levine and Pearlman (2007)). Indeed, in many robustness exercises the probability of optimal polices violating the lower bound constraint is left unreported and unknown. Feasible policy, however, must respect the lower bound. Finally, our analysis of optimal rules is welfare-optimal, based on a 'large distortions' quadratic approximation of the representative household's utility in a structural micro-founded model.

The paper proceeds as follows. Section 2 sets out the model. Section 3 describes the procedure for approximating the optimization problem in a LQ form. Section 4 provides the results for the Bayesian maximum-likelihood estimation of our core model and several variants. Section 5 first focuses on optimized simple interest-rate rules and the fully optimal rule without model uncertainty before we turn to the robust policy problem in section 6. Section 7 concludes.

[7]The Bayesian model averaging method of Brock *et al.* (2007) also estimate the relative probabilities, but is confined to M-robustness. Our extension to P-Robustness is in the spirit of the comments by Sims (2005) on that paper.

2 The Model

2.1 The Smets-Wouters Model

We employ the influential Smets-Wouters model of the Euro area (Smets and Wouters (2003), henceforth SW). The SW model is an extended version of the standard New-Keynesian DSGE closed-economy model with sticky prices and wages estimated by Bayesian techniques. It features three agents: households, firms and the monetary policy authority. Households maximize a utility function with two arguments (consumption and leisure) over an infinite horizon. Consumption appears in the utility function relative to a time-varying external habit-formation variable. Labour is differentiated over households, so that there is some monopoly power over wages, which results in an explicit wage equation and allows for the introduction of sticky nominal Calvo-type wages contracts. Households also rent capital services to firms and decide how much capital to accumulate given adjustment costs. Firms produce differentiated goods, decide on factor inputs, and set Calvo-type price contracts. Wage and price setting is augmented by the assumption that those prices and wages that can not be freely set are partially indexed to past inflation. Prices are therefore set as a function of current and expected real marginal cost, but are also influenced by past inflation. Real marginal cost depends on wages and the rental rate of capital. The short-term nominal interest rate is the instrument of monetary policy. The stochastic behavior of the model is driven by ten exogenous shocks: five shocks arising from technology and preferences, three cost-push shocks and two monetary-policy shocks. Consistent with the DSGE set up, potential output is defined as the level of output that would prevail under flexible prices and wages in the absence of cost-push shocks.

We incorporate one important modification to the SW model: the addition of distortionary taxes at the steady state. As we will see this has a bearing on the inefficiency at the steady state, the quadratic approximation of the utility function used for the welfare analysis and the existence of an inflationary bias.

2.2 Households

In a cashless economy version of the model, a representative household r maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t U_{C,t} \left[\frac{(C_t(r) - H_{C,t})^{1-\sigma}}{1-\sigma} - U_{L,t} \frac{L_t(r)^{1+\phi}}{1+\phi} \right] \tag{1}$$

where β is the household's discount factor, $U_{C,t}$, and $U_{L,t}$ are preference shocks common to all households, $C_t(r)$ is an index of consumption, $L_t(r)$ are hours worked, $H_{C,t}$ represents 'external habit' in consumption, or the desire not to differ too much from other households, and we choose $H_{C,t} = hC_{t-1}$, where $C_t = \frac{1}{\nu}\sum_{r=1}^{\nu} C_t(r)$ is the average consumption index, $h \in [0, 1)$. When $h = 0$, $\sigma > 1$ is the risk aversion parameter (or the inverse of the inter-temporal elasticity of substitution). We normalize the household number to unity.

The representative household r must obey a budget constraint:

$$(1 + T_{C,t})P_t(C_t(r) + I(r)) + E_t[D_{t+1}B_{t+1}(r)] = (1 - T_{Y,t})P_tY_t(r) + B_t(r) + TR_t \qquad (2)$$

where P_t is the GDP price index and $I_t(r)$ is investment. Assuming complete financial markets, $B_{t+1}(r)$ is a random variable denoting the payoff of the portfolio $B_t(r)$, purchased at time t, and D_{t+1} is the stochastic discount factor over the interval $[t, t+1]$ that pays one unit of currency in a particular state of period $t+1$ divided by the probability of an occurrence of that state given information available in period t. The nominal rate of return on bonds (the nominal interest rate), R_t, is then given by the relation $E_t[D_{t+1}] = \frac{1}{1+R_t}$. The tax structure is as follows: TR_t are lump-sum transfers to households by the government net of lump-sum taxes, $T_{C,t}$ and $T_{Y,t}$ are consumption and income tax rates respectively. The income tax rate is paid on total income, $P_tY_t(r)$, given by

$$P_tY_t(r) = W_t(r)L_t(r) + (R_{K,t}Z_t(r) - \Psi(Z_t(r))P_tK_{t-1}(r) + \Gamma_t(r) \qquad (3)$$

where $W_t(r)$ is the nominal wage rate, $R_{K,t}$ is the real return on beginning-of period t capital stock, K_{t-1}, owned by households, $Z_t(r) \in [0, 1]$ is the degree of capital utilization with costs $P_t\Psi(Z_t(r))K_{t-1}(r)$ where Ψ', $\Psi'' > 0$, and $\Gamma_t(r)$ is income from dividends derived from the imperfectly competitive intermediate firms plus the net cash inflow from state-contingent securities. We first consider the case of flexible wages and introduce wage stickiness later.

Capital accumulation is given by

$$K_t(r) = (1 - \delta)K_{t-1}(r) + (1 - S\left(X_t(r)\right)) I_t(r) \qquad (4)$$

where δ is the depreciation rate, $X_t(r) = \frac{U_{I,t}I_t(r)}{I_{t-1}(r)}$, $U_{I,t}$ is a shock to investment costs and the investment adjustment cost function, $S(\cdot)$, has the properties $S(1) = S'(1) = 0$. As seen below, intermediate firms employ differentiated labour with a CES aggregator with elasticity of substitution η. Then the demand for each consumer's labour is given by

$$L_t(r) = \left(\frac{W_t(r)}{W_t}\right)^{-\eta} L_t \qquad (5)$$

where $W_t = \left[\int_0^1 W_t(r)^{1-\eta}dr\right]^{\frac{1}{1-\eta}}$ is an average wage index and $L_t = \left[\int_0^1 L_t(r)^{\frac{\eta-1}{\eta}}dr\right]^{\frac{\eta}{\eta-1}}$ is average employment.

Household r chooses $\{C_t(r)\}$, $\{M_t(r)\}$, $\{K_t(r)\}$, $\{Z(r)\}$ and $\{W_t(r)\}$ to maximize (1) subject to (2)–(5), taking external habit $H_{C,t}$, interest rates and prices and as given.

The insurance provided by state-contingent securities (the complete financial markets assumption) enables us to impose symmetry on households (so that $C_t(r) = C_t$, etc). Then we have the first-order necessary conditions:

$$1 = \beta(1+R_t)E_t\left[\frac{MU^C_{t+1}}{MU^C_t}\frac{P_t}{P_{t+1}}\right] \quad (6)$$

$$Q_t = E_t\left[\beta\left(\frac{MU^C_{t+1}}{MU^C_t}\right)(Q_{t+1}(1-\delta)+R_{K,t+1}Z_t-\Psi(Z_{t+1}))\right] \quad (7)$$

$$\begin{aligned} 1 &= Q_t[1-(1-S(X_t)-S'(X_t)X_t)] \\ &+ \beta E_t Q_{t+1}\left(\frac{(C_{t+1}-H_{C,t+1})}{(C_t-H_{C,t})}\right)^{-\sigma} S'(X_t)\frac{U_{I,t+1}I^2_{t+1}}{I^2_t} \quad (8) \\ R_{K,t} &= \Psi'(Z_t) \quad (9) \\ \frac{W_t(1-T_{Y,t})}{(1+T_{C,t})P_t} &= -\frac{1}{(1-\frac{1}{\eta})}\frac{MU^L_t}{MU^C_t} \equiv \frac{1}{(1-\frac{1}{\eta})}MRS_t = \frac{U_{L,t}}{(1-\frac{1}{\eta})}L^{\phi}_t(C_t-H_{C,t})^{\sigma} \quad (10) \end{aligned}$$

where $MU^C_t = U_{C,t}(C_t - H_{C,t})^{-\sigma}$ and $MU^L_t = -U_{L,t}L^{\phi}_t$ are the marginal utilities of consumption and work respectively. Condition (6) is the familiar Keynes-Ramsey rule adapted to incorporate habit in consumption. In (7) and (8), Q_t is the real value of capital (Tobin's Q) and these conditions describe optimal investment behavior. (9) describes optimal capacity utilization and (10) equates the real disposable wage with the marginal rate of substitution (MRS_t) between consumption and leisure and reflects the monopolistic market power of households supplying a differentiated factor input with elasticity η.

2.3 Firms

Competitive final goods firms use a continuum of intermediate goods according to a constant returns CES technology to produce aggregate output

$$Y_t = \left(\int_0^1 Y_t(f)^{(\zeta-1)/\zeta}df\right)^{\zeta/(\zeta-1)} \quad (11)$$

where ζ is the elasticity of substitution and the firm number is normalized to unity. This implies a set of demand equations for each intermediate good f with price $P_t(f)$ of the form $Y_t(f) = \left(\frac{P_t(f)}{P_t}\right)^{-\zeta} Y_t$ where $P_t = \left[\int_0^1 P_t(f)^{1-\zeta}df\right]^{\frac{1}{1-\zeta}}$ is an aggregate price index.

In the intermediate goods sector each good f is produced by a single firm f using differentiated labour and capital with a Cobb-Douglas technology:

$$Y_t(f) = A_t(Z_t(f)K_{t-1}(f))^{\alpha}L_t(f)^{1-\alpha} - F \quad (12)$$

where $Z_t(f)$ denotes capacity utilization, F are fixed costs of production and

$$L_t(f) = \left(\int_0^1 L_t(r,f)^{(\eta-1)/\eta} dr \right)^{\eta/(\eta-1)} \tag{13}$$

is an index of differentiated labour types used by the firm, where $L_t(r,f)$ is the labour input of type r by firm f. A_t is an exogenous shock capturing shifts to trend total factor productivity in this sector. The cost of labour is $(1+T_{L,t})W_t$ where $T_{L,t}$ is a payroll tax paid by the firm. Minimizing costs $P_t R_{K,t} Z_t(f) K_{t-1}(f) + (1+T_{L,t})W_t L_t(f)$ and aggregating over firms leads to the demand for labour as in (5), where $\int_0^1 L_t(r,f)df = L_t(r)$, and to

$$\frac{(1+T_{L,t})W_t L_t(f)}{Z_t P_t R_{K,t} K_{t-1}(f)} = \frac{1-\alpha}{\alpha} \tag{14}$$

In an equilibrium of equal households and firms, all wages adjust to the same level W_t and it follows that $Y_t = A_t(Z_t K_{t-1})^\alpha L_t^{1-\alpha} - F$. The firm's cost-minimizing real marginal cost is therefore given by

$$MC_t = \frac{1}{A_t} \left(\frac{(1+T_{L,t})W_t}{P_t} \right)^{1-\alpha} R_{K,t}^\alpha \alpha^{-\alpha} (1-\alpha)^{-(1-\alpha)} \tag{15}$$

2.4 Price and Wage-Setting

Turning to price and wage-setting, we follow the standard Calvo framework supplemented with indexation. Thus, at each period there is a probability of $1-\xi_p$ and $1-\xi_w$ that the price and wage is set optimally. The optimal price derives from maximizing discounted profits whilst wages are set such as to maximize discounted the utility from labour consumption minus the disutility of labour effort. For those firms and workers unable to reset, prices and wages are indexed to last period's aggregate inflation, with indexation parameters indicated by γ_p and γ_w respectively. It can be shown that this leads to the following first-order conditions:

$$E_t \sum_{k=0}^{\infty} \xi_p^k D_{t+k} Y_{t+k}(f) \left[P_t^0(f) \left(\frac{P_{t+k-1}}{P_{t-1}} \right)^{\gamma_p} - \frac{\zeta}{(\zeta-1)} P_{t+k} \mathrm{MC}_{t+k} \right] = 0 \tag{16}$$

$$E_t \sum_{k=0}^{\infty} (\xi_w \beta)^k \ W_{t+k}^\eta \left(\frac{P_{t+k-1}}{P_{t-1}} \right)^{-\gamma_w \eta} L_{t+k} \frac{MU_{t+k}^C(r)}{P_{t+k}} \Bigg[W_t^0(r)(1-T_{Y,t+k}) \left(\frac{P_{t+k-1}}{P_{t-1}} \right)^{\gamma_w} - \frac{1}{(1-\frac{1}{\eta})} P_{t+k} MRS_{t+k}(r) \Bigg] = 0 \tag{17}$$

Then by the law of large numbers the evolution of the price and wage indices are given

$$P_{t+1}^{1-\zeta} = \xi_p \left(P_t \left(\frac{P_t}{P_{t-1}} \right)^{\gamma_p} \right)^{1-\zeta} + (1-\xi_p)(P_{t+1}^0(f))^{1-\zeta} \tag{18}$$

$$W_{t+1}^{1-\eta} = \xi_w \left(W_t \left(\frac{P_t}{P_{t-1}} \right)^{\gamma_w} \right)^{1-\eta} + (1-\xi_w)(W_{t+1}^0(r))^{1-\eta} \tag{19}$$

2.5 Equilibrium and Interest Rate Rule

In equilibrium, goods markets, money markets and the bond market all clear. Equating the supply and demand of the consumer good we obtain

$$Y_t = A_t(Z_t K_{t-1})^\alpha L_t^{1-\alpha} - F = C_t + G_t + I_t + \Psi(Z_t)K_{t-1} \tag{20}$$

We examine the dynamic behaviour in the vicinity of a steady state in which the government budget constraint is in balance; i.e.,

$$TR_t + P_t G_t = (T_{Y,t} + T_{C,t})P_t Y_t + T_{L,t} W_t L_t \tag{21}$$

As in Coenen *et al.* (2007) we further assume that changes in government spending are financed exclusively by changes in lump-sum taxes with tax rates $T_{Y,t}$, $T_{C,t}$ and $T_{L,t}$ held constant at their steady-state values.

Given the path of the interest rate, $\{R_t\}$ (expressed later in terms of an optimal or IFB rule) the money supply is fixed by the central banks to accommodate money demand. By Walras' Law we can dispense with the bond market equilibrium condition and therefore the household constraint. Then the equilibrium is defined at $t = 0$ by stochastic processes C_t, B_t, I_t, P_t, L_t, K_t, Z_t, $R_{K,t}$, W_t, Y_t, given past price indices and exogenous shocks and government spending processes.

The model is estimated in linearized form about a zero-inflation steady state, both set out below. For estimation purposes only, the model is closed with a linear 'empirical' Taylor rule of the form

$$\begin{aligned} r_t &= \rho r_{t-1} + (1-\rho)[\bar{\pi}_t + \theta_\pi E_t(\pi_{t+j} - \bar{\pi}_{t+j}) + \theta_y(y_t - \hat{y}_t)] + \theta_{\Delta\pi}(\pi_t - \pi_{t-1}) \\ &+ \theta_{\Delta y}(\Delta y_t - \Delta \hat{y}_t) + \epsilon_{R,t} \end{aligned} \tag{22}$$

where $\bar{\pi}_{t+1} = \rho_\pi \bar{\pi}_t + \epsilon_{\pi,t+1}$ is an inflation target shock process, and $\epsilon_{R,t}$ is an i.i.d. nominal interest-rate shock. In the policy exercises, this rule is replaced with optimal counterparts (fully optimal or optimized 'simple rules').

2.6 Zero-Inflation Steady State

For the cashless economy, deterministic zero-inflation steady state, denoted by variables without the time subscripts, $E_{t-1}(U_{C,t}) = 1$ and $E_{t-1}(U_{L,t}) = \kappa$ is given by

$$1 = \beta(1+R) \tag{23}$$

$$Q = \beta(Q(1-\delta) + R_K Z - \Psi(Z)) \tag{24}$$

$$R_K = \Psi'(Z) \tag{25}$$

$$Q = 1 \tag{26}$$

$$\frac{W(1-T_Y)}{P(1+T_C)} = \frac{\kappa(1-h)^\sigma}{1-\frac{1}{\eta}} L^\phi C^\sigma \tag{27}$$

$$\tag{28}$$

$$Y = A(KZ)^\alpha L^{1-\alpha} - F \tag{29}$$

$$\frac{W(1+T_L)L}{PZR_K K} = \frac{1-\alpha}{\alpha} \tag{30}$$

$$1 = \frac{P^0}{P} = \frac{\text{MC}}{\left(1-\frac{1}{\zeta}\right)} \tag{31}$$

$$\text{MC} = \frac{\left(\frac{W(1+T_L)}{P}\right)^{1-\alpha} R_K^\alpha}{A} \tag{32}$$

$$Y = C + (\delta + \Psi(Z))K + G \tag{33}$$

$$TR + PG = (T_Y + T_C)PY + T_L WL \tag{34}$$

determining R, Z, Q, $\frac{W}{P}$, L, K, R_K, MC, C, Y and possible tax structures, (TR, T_Y, T_C), given G. In our cashless economy the price level is indeterminate.

The solution for steady state values decomposes into a number of independent calculations. First from (23) the natural rate of interest is given by

$$R = \frac{1}{\beta} - 1 \tag{35}$$

which is thus pinned down by the household's discount factor. Equations (24) to (26) give

$$1 = \beta[1 - \delta + Z\Psi'(Z) - \Psi(Z)] \tag{36}$$

which determines steady state capacity utilization. As in SW we assume that $Z = 1$ and $\Psi(1) = 0$ so that (36) and (25) imply that $R_K = \Psi'(Z) = \frac{1}{\beta} - 1 + \delta = R + \delta$ meaning that perfect capital market conditions apply in the deterministic steady state.[8]

[8]As we shall see later Z is socially efficient thus justifying the assumption $Z = 1$.

From (30) to (32) a little algebra yields the capital-labour ratio and the real wage:

$$\frac{K}{L} = \left[A\left(1-\frac{1}{\zeta}\right)\frac{\alpha}{R_K}\right]^{\frac{1}{1-\alpha}} \tag{37}$$

$$\frac{W}{P} = \frac{(1-\alpha)R_K}{(1+T_L)\alpha}\frac{K}{L} \tag{38}$$

Denote the *total tax wedge* by T between the real effective wage income of households (the purchasing power of the post-tax wage) and the real effective labour cost of firms. Then

$$T \equiv 1 - \frac{1-T_Y}{(1+T_C)(1+T_L)} \simeq T_Y + T_C + T_L \tag{39}$$

Then combining (27), (29) and (33) and substituting for R_K from (37) we arrive at

$$\left(1+\frac{F}{Y}\right)^{\phi} Y^{\phi+\sigma}\left(1-\frac{\delta}{A}\left(\frac{K}{L}\right)^{1-\alpha} - \frac{G+\frac{\delta}{R_K}F}{Y}\right)^{\sigma} = \frac{(1-\alpha)(1-T)\left(1-\frac{1}{\eta}\right)\left(1-\frac{1}{\zeta}\right)A^{1+\phi}\left(\frac{K}{L}\right)^{\alpha(1+\phi)}}{\alpha\kappa(1-h)^{\sigma}} \tag{40}$$

Equations (40), with $\frac{K}{L}$ defined by (37), and $R_K = \frac{1}{\beta} - 1 + \delta$, together define the natural rate of output in terms of underlying parameters and the tax wedge T. Thus given government spending as a proportion of GDP, the natural rate of output falls as market power in output and labour markets increases (with decreases in ζ and η respectively) and the tax wedge T increases. However external habit in consumption causes households to supply more labour thus increasing the natural rate of output. Market power, taxes and external habit are all sources of inefficiency, but as we shall now see in section 2.7, they do not impact on efficiency in the same direction.

2.7 The Inefficiency of the Zero-Inflation Steady State

To examine the inefficiency of the steady state we consider the social planner's problem for the deterministic case obtained by maximizing

$$\Omega_0 = \sum_{t=0}^{\infty}\beta^t\left[\frac{(C_t-hC_{t-1})^{1-\sigma}}{1-\sigma} - \kappa\frac{L_t^{1+\phi}}{1+\phi}\right] \tag{41}$$

with respect to $\{C_t\}$, $\{K_t\}$, $\{L_t\}$ and $\{Z_t\}$, subject to the resource constraint

$$Y_t = A_t(Z_tK_{t-1})^{\alpha}L_t^{1-\alpha} - F = C_t + G_t + K_t - (1-\delta)K_{t-1} + \Psi(Z_t)K_{t-1} \tag{42}$$

To solve this optimization problem define the Lagrangian

$$\mathcal{L} = \Omega_0 + \sum_{t=0}^{\infty}\beta^t\mu_t\left[A_t(Z_tK_{t-1})^{\alpha}L_t^{1-\alpha} - C_t - G_t - K_t + (1-\delta)K_{t-1} - \Psi(Z_t)K_{t-1}\right] \tag{43}$$

First order conditions are:

$$C_t: \quad (C_t - hC_{t-1})^{-\sigma} - \beta h(C_{t+1} - hC_t)^{-\sigma} - \mu_t = 0 \tag{44}$$

$$K_t: \quad -\mu_t + \left[(1-\delta)\beta + \alpha\beta A_t Z_{t+1}\left(\frac{L_{t+1}}{Z_{t+1}K_t}\right)^{1-\alpha} - \beta\Psi(Z_{t+1})\right]\mu_{t+1} = 0 \tag{45}$$

$$L_t \quad : \quad -\kappa L_t^{\phi} + (1-\alpha)A_t\left(\frac{Z_t K_{t-1}}{L_t}\right)^{\alpha}\mu_t = 0 \tag{46}$$

$$Z_t: \quad \Psi'(Z_t) - \alpha A_t\left(\frac{L_t}{Z_t K_{t-1}}\right)^{1-\alpha} = 0 \tag{47}$$

The efficient steady-state levels of output $Y_{t+1} = Y_t = Y_{t-1} = Y^*$, is therefore found by solving the system:

$$[(1-h)C]^{-\sigma}(1-\beta h) - \mu = 0 \tag{48}$$

$$-1 + (1-\delta)\beta + \alpha\beta A Z\left(\frac{L}{ZK}\right)^{1-\alpha} - \beta\Psi(Z) = 0 \tag{49}$$

$$-\kappa L^{\phi} + (1-\alpha)A\left(\frac{ZK}{L}\right)^{\alpha}\mu = 0 \tag{50}$$

$$\Psi'(Z) - \alpha A\left(\frac{L}{ZK}\right)^{1-\alpha} = 0 \tag{51}$$

Solving as we did for the natural rate and denoting the social optimum by Z^*, Y^* etc we arrive at

$$1 = \beta[1 - \delta + Z^*\Psi'(Z^*) - \Psi(Z^*)] \tag{52}$$

Hence comparing (52) and (36) it can be seen that $Z^* = Z = 1$. Thus the *natural rate of capacity utilization is efficient*. However since

$$\frac{K}{L} = \left[\frac{A\left(1-\frac{1}{\zeta}\right)\alpha}{\Psi'(Z)}\right]^{\frac{1}{1-\alpha}} < \left[\frac{A\alpha}{\Psi'(Z^*)}\right]^{\frac{1}{1-\alpha}} = \frac{K^*}{L^*} \tag{53}$$

it follows that the *natural capital-labour ratio is below the social optimum*. The socially optimal level of output is now found from

$$\left(1+\frac{F^*}{Y^*}\right)^{\phi}(Y^*)^{\phi+\sigma}\left(1-\frac{\delta}{A}\left(\frac{K^*}{L^*}\right)^{1-\alpha} - \frac{G^* + \frac{\delta}{R_K}F^*}{Y^*}\right)^{\sigma} = \frac{(1-\alpha)A^{1+\phi}\left(\frac{K^*}{L^*}\right)^{\alpha(1+\phi)}(1-h\beta)}{\alpha\kappa(1-h)^{\sigma}} \tag{54}$$

The inefficiency of the natural rate of output can now be found by comparing (40) with (54). Since $Y^{\phi+\delta}$ is an increasing function of Y, we arrive at:[9]

[9]This generalizes the result in Choudhary and Levine (2006) which considered the same model, but without capital.

Proposition

The natural level of output, Y, is below the efficient level, Y^*, if and only if the following conditions are satisfied:

$$(1-T)\left(1-\frac{1}{\eta}\right)\left(1-\frac{1}{\zeta}\right)^{1+\alpha\phi} < (1-h\beta)\,\Theta \tag{55}$$

where

$$\Theta = \frac{\left(1-\delta\left[\left(1-\frac{1}{\zeta}\right)\frac{\alpha\beta}{1-\beta+\beta\delta}\right]-\frac{\left(G+\frac{\delta\alpha\beta}{1-\beta+\beta\delta}F\right)}{Y}\right)^{\sigma}\left(1+\frac{F}{Y}\right)^{\phi}}{\left(1-\delta\left[\frac{\alpha\beta}{1-\beta+\beta\delta}\right]-\frac{\left(G^*+\frac{\delta\alpha\beta}{1-\beta+\beta\delta}F^*\right)}{Y^*}\right)^{\sigma}\left(1+\frac{F^*}{Y^*}\right)^{\phi}}$$

Thus the term

$$\Phi_y \equiv 1-\frac{(1-T)\left(1-\frac{1}{\zeta}\right)\left(1-\frac{1}{\eta}\right)^{1+\alpha\phi}}{(1-h\beta)\Theta} \geq \text{or} \leq 0 \tag{56}$$

summarizes the overall distortion in the steady state natural level of output as a result of four distortions: taxes, market power in the output and labour markets and external habit.[10] Assume both government spending and fixed costs are efficient so that $\frac{G}{Y}=\frac{G^*}{Y^*}$ and $\frac{F}{Y}=\frac{F^*}{Y^*}$. It then follows that $\Theta > 1$. In the case where there is no habit persistence ($h=0$), then $\Phi_y > 0$ and (55) always holds. Then tax distortions and market power in the output and labour markets, captured by the elasticities $\eta \in (0,\infty)$ and $\zeta \in (0,\infty)$ respectively, drive the natural rate of output below the efficient level. If $h=T=0$ and $\eta=\zeta=\infty$, tax distortions and market power both disappear, $\Phi_y=0$ and the natural rate is efficient. But if $h>0$, this leads to the possibility that $\Phi_y<0$ and then the natural rate of output is actually *above* the efficient level (see Choudhary and Levine (2006)).

How big is the steady state distortion in the SW model? For parameter values estimated in section 4, for the core SW version and $\zeta = 7.67$, corresponding to a 15% mark-up, figure 1 shows the value for Φ_y corresponding to an interval $T \in [0, 0.5]$ for the tax wedge. In the original SW model taxes are assumed to be non-distortionary so $T=0$. Then the distortion is negative, (55) does not hold and the social optimum level of output in the steady state is *below* the natural rate. 'Corrective taxes' as in Layard (2006) may then be necessary to encourage people to work less. However, from Figure 1 we see that the appropriate level of such a tax is far less than the average tax wedge in the euro area reported in Coenen *et al.* (2007) of about 50%, at which level the distortion is positive and large. The quadratic approximation therefore requires the 'large distortions' procedure to which we now turn.

[10] This generalizes Woodford (2003), page 394 to include capital, labour-market power and habit.

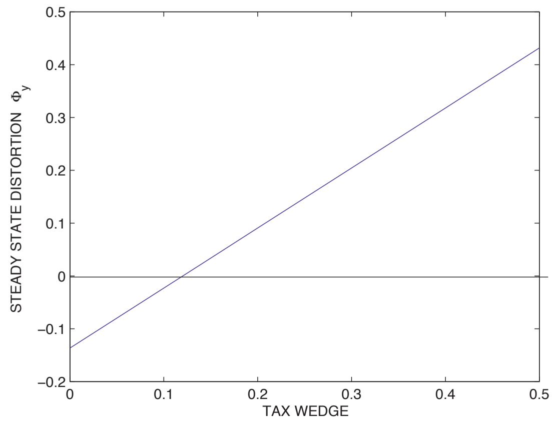

Figure 1: **Steady State Distortion and the Tax Wedge**

3 LQ Approximation and the ZLB Constraint

This section implements the procedure set out in Levine *et al.* (2007b) to obtain a quadratic approximation of the utility function and a linear state-space representation of the model dynamics in the region of the zero-inflation steady state. As we have shown that with euro-area levels of taxation, we cannot assume that distortions are small in this steady state. To implement the large distortions quadratic approximation, we first need to express the price and wage-setting first order conditions as stochastic non-linear difference equations.

3.1 Representation of Price-Wage Dynamics as Difference Equations

Define

$$\Pi_t \equiv \frac{P_t}{P_{t-1}} = \pi_t + 1 \tag{57}$$

$$\Upsilon_t \equiv P_t^0 / P_t \tag{58}$$

$$\tilde{\Pi}_t \equiv \frac{\Pi_t}{\Pi_{t-1}^{\gamma}} \tag{59}$$

and use $D_{t+k} = \beta^k \frac{MU_t^C}{P_{t+k}}$ where $MU_t^C = (C_t - hC_{t-1})^{-\sigma}$ is the marginal utility of consumption. Recalling that $\Lambda_t = \frac{MU_t^C}{P_t}$, aggregate price dynamics are then given by

$$H_t - \xi\beta E_t[\tilde{\Pi}_{t+1}^{\zeta-1} H_{t+1}] = Y_t MU_t^C \tag{60}$$

$$J_t - \xi\beta E_t[\tilde{\Pi}_{t+1}^{\zeta} J_{t+1}] = \frac{U_{L,t} L_t^{1+\phi}}{(1-1/\zeta)(1-1/\eta)(1-T_t)} \tag{61}$$

$$\Upsilon_t H_t = J_t \tag{62}$$

$$1 = \xi\tilde{\Pi}_t^{\zeta-1} + (1-\xi)\Phi_t^{1-\zeta} \tag{63}$$

For staggered wage-setting, the following difference equations apply:

$$\left(\frac{W_t^0}{P_t}\right)^{1+\eta\phi} N_t = O_t \tag{64}$$

$$N_t - \xi_W \beta E_t[\tilde{\Pi}_{t+1}^{\eta-1} N_{t+1}] = \left(\frac{W_t}{P_t}\right)^{\eta} (1-T_t) L_t MU_t^C \tag{65}$$

$$O_t - \xi_W \beta E_t[\tilde{\Pi}_{t+1}^{\eta(1+\phi)} O_{t+1}] = \left(\frac{W_t}{P_t}\right)^{\eta(1+\phi)} \frac{U_{L,t} L_t^{1+\phi}}{(1-1/\eta)} \tag{66}$$

$$\left(\frac{W_{t+1}}{P_{t+1}}\right)^{1-\eta} = \xi_W \left(\frac{W_t}{P_t}\right)^{1-\eta} \tilde{\Pi}_{t+1}^{\eta-1} + (1-\xi_W)\left(\frac{W_{t+1}^0}{P_{t+1}}\right)^{1-\eta} \tag{67}$$

3.2 The Utility Function in Terms of Wage and Price Dispersions

Ignoring the welfare implications of monetary frictions, the utility of household r is

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_t(r) - hC_{t-1})^{1-\sigma}}{1-\sigma} - \kappa \frac{L_t(r)^{1+\phi}}{1+\phi}\right] \tag{68}$$

Since we assume complete risk-sharing within each bloc, each consumer's consumption is identical; i.e., $C_t(r) = C_t$. However households differ in their labour supply because in each period some can re-optimize their wage contracts. To obtain the social welfare we then require the average value of $L_t(r)^{1+\phi}$ across households. Let $L_t(f,r)$ be the labour supplied to firm f by household r. Then from (5) we have

$$L_t(r) = \int_0^1 L_t(r,f)df = \left(\frac{W_t(r)}{W_t}\right)^{-\eta} \int_0^1 L_t(f)df$$

where $L_t(f)$ the index of differentiated labour employed by firm f. Then following Levine *et al.* (2007c), henceforth LMP, we eventually arrive at

$$\int_0^1 L_t(r)^{1+\phi} dr = L_t^{1+\phi}\left(1 + \frac{Y_t}{F+Y_t}\frac{1}{2}\zeta(1+\phi)D_t^P\right)(1 + \frac{1}{2}\eta(1+\eta\phi)(1+\phi)D_t^W) \tag{69}$$

where D_t^P and D_t^W are price and wage dispersion. Up to second order terms these are given respectively by

$$D_t^P = \xi_p D_{t-1}^P + \frac{\xi_p}{1-\xi_p}(\pi_t - \gamma_p \pi_{t-1})^2 \tag{70}$$

$$D_t^W = \xi_w D_{t-1}^W + \frac{\xi_w}{1-\xi_w}(\Delta wr_t + \pi_t - \gamma_w(\Delta wr_{t-1} + \pi_{t-1}))^2 \tag{71}$$

3.3 Large Distortions Quadratic Approximation of Utility

We have now expressed utility in terms of price and wage variances. Together with the household's first-order conditions, the capital accumulation equation, price and wage setting expressed as difference equations, we can write down the deterministic non-linear

Ramsey optimization problem. The quadratic approximation of the single-period utility derived from (68), (70) and (71) can then be obtained in a conceptually straightforward fashion as the second-order Taylor series approximation about the steady state of the Ramsey problem of the associated Hamiltonian. For our model and parameter values this turns out to be the zero-inflation steady state. Details are given in Levine *et al.* (2007b) and Appendix A provides a brief summary.

In the limit as the zero-inflation steady state becomes efficient then it is possible to obtain an analytical form of the quadratic approximation of the single-period welfare loss, U_t. As shown in LMP, this takes the form

$$\begin{aligned} U_t &= w_c(c_t - hc_{t-1})^2 + w_l l_t^2 + w_\pi(\pi_t - \gamma_p \pi_{t-1})^2 + w_{\Delta w}(\Delta w_t - \gamma_w \Delta w_{t-1})^2 \\ &+ w_{lk}(l_t - k_{t-1} - z_t - \frac{1}{1-\alpha} a_t)^2 + w_z(z_t + \psi a_t)^2 - w_{al} a_t l_t - w_i(i_t - i_{t-1})^2 \end{aligned} \quad (72)$$

where positive weights w_c etc are defined in LMP. All variables are in log-deviation form about the steady state as in the linearization. The first four terms in (72) give the welfare loss from consumption, employment, price inflation and wage inflation variability respectively. The remaining terms are contributions that arise from the resource constraint in our quadratic approximation. In a flexi-price and flexi-wage economy $w_{\Delta w}$ and w_π are zero and the first two terms in (72) dominate. For the high levels of price and wage stickiness estimated in the SW model however, it is these coefficients on price and wage inflation variability that dominate. Although we use a large-distortions quadratic approximation without the convenient analytical form of (72), this feature of the small-distortions approximation is indicative of the importance of price and wage inflation variability in the choice of policy rules.

3.4 Linearization about the Zero-Inflation Steady State

We finally linearize about the deterministic zero-inflation steady state. Define all lower case variables as proportional deviations from this baseline steady state except for rates of change which are absolute deviations.[11] Then the linearization takes the form:

[11] That is, for a typical variable X_t, $x_t = \frac{X_t - X}{X} \simeq \log\left(\frac{X_t}{X}\right)$ where X is the baseline steady state. For variables expressing a rate of change over time such as r_t and π_t, $x_t = X_t - X$.

$$
\begin{aligned}
c_t &= \frac{h}{1+h}c_{t-1} + \frac{1}{1+h}E_t c_{t+1} \\
&- \frac{1-h}{(1+h)\sigma}(r_t - E_t\pi_{t+1} + E_t u_{C,t+1} - u_{C,t}) \qquad (73)\\
q_t &= \beta(1-\delta)E_t q_{t+1} - (r_t - E_t\pi_{t+1}) + \beta Z E_t r_{K,t+1} + \epsilon_{Q,t} \qquad (74)\\
z_t &= \frac{r_{K,t}}{Z\Psi''(Z)} = \frac{\psi}{R_K}r_{K,t} \quad \text{where } \psi = \frac{\Psi'(Z)}{Z\Psi''(Z)} \qquad (75)\\
i_t &= \frac{1}{1+\beta}i_{t-1} + \frac{\beta}{1+\beta}E_t i_{t+1} + \frac{1}{S''(1)(1+\beta)}q_t + u_{I,t} \qquad (76)\\
\pi_t &= \frac{\beta}{1+\beta\gamma_p}E_t\pi_{t+1} + \frac{\gamma_p}{1+\beta\gamma_p}\pi_{t-1} + \frac{(1-\beta\xi_p)(1-\xi_p)}{(1+\beta\gamma_p)\xi_p}mc_t + \epsilon_{P,t} \qquad (77)\\
k_t &= (1-\delta)k_{t-1} + \delta i_t \qquad (78)\\
mc_t &= (1-\alpha)wr_t + \frac{\alpha}{R_K}r_{K,t} - a_t \qquad (79)
\end{aligned}
$$

$$
\begin{aligned}
wr_t &= \frac{\beta}{1+\beta}E_t wr_{t+1} + \frac{1}{1+\beta}wr_{t-1} + \frac{\beta}{1+\beta}E_t\pi_{t+1} - \frac{1+\beta\gamma_w}{1+\beta}\pi_t + \frac{\gamma_w}{1+\beta}\pi_{t-1} \\
&+ \frac{(1-\beta\xi_w)(1-\xi_w)}{(1+\beta)\xi_w(1+\eta\phi)}(mrs_t - wr_t) + \epsilon_{W,t} \qquad (80)\\
mrs_t &= \frac{\sigma}{1-h}(c_t - hc_{t-1}) + \phi l_t + u_{L,t} \qquad (81)\\
l_t &= k_{t-1} + \frac{1}{R_K}(1+\psi)r_{K,t} - wr_t \qquad (82)\\
y_t &= c_y c_t + g_y g_t + i_y i_t + k_y \psi r_{K,t} \qquad (83)\\
y_t &= \phi_F[a_t + \alpha(\frac{\psi}{R_K}r_{K,t} + k_{t-1}) + (1-\alpha)l_t] \quad \text{where } \phi_F = 1 + \frac{F}{Y} \qquad (84)\\
u_{C,t+1} &= \rho_C u_{C,t} + \epsilon_{C,t+1} \qquad (85)\\
u_{L,t+1} &= \rho_L u_{L,t} + \epsilon_{L,t+1} \qquad (86)\\
u_{I,t+1} &= \rho_I u_{I,t} + \epsilon_{I,t+1} \qquad (87)\\
g_{t+1} &= \rho_g g_t + \epsilon_{g,t+1} \qquad (88)\\
a_{t+1} &= \rho_a a_t + \epsilon_{a,t+1} \qquad (89)
\end{aligned}
$$

where “inefficient cost-push” shocks $\epsilon_{Q,t+1}$, $\epsilon_{P,t+1}$ and $\epsilon_{W,t+1}$ have been added to value of capital, the marginal cost and real wage equations respectively. Variables y_t, c_t, mc_t, $u_{C,t}$, $u_{L,t}$, a_t, g_t are proportional deviations about the steady state. $[\epsilon_{C,t}, \epsilon_{l,t}, \epsilon_{g,t}, \epsilon_{a,t}]$ are i.i.d. disturbances. π_t, $r_{K,t}$ and r_t are absolute deviations about the steady state.[12] To obtain the output gap, the difference between output for the sticky price model obtained above, and output when prices and wages are flexible, $\hat{y}_t$ say. Following SW we also eliminate

[12]Note that in the SW model $\hat{r}_{K,t}$ is defined as $\frac{r_{K,t}}{R_K}$. Then $z_t = \frac{\Psi'(Z)}{Z\Psi''(Z)}\hat{r}_{K,t} = \psi\hat{r}_{K,t}$. In our set-up $z_t = \frac{\psi}{R_K}r_{K,t}$ has been eliminated.

the inefficient shocks from this target level of output. The latter is obtained by setting $\xi_p = \xi_w = \epsilon_{Q,t+1} = \epsilon_{P,t+1} = \epsilon_{W,t+1} = 0$ in (77) to (83).[13]

Table 1 provides a summary of our notation.

3.5 The LQ Problem

We can express the linearized model in state-space form as:

$$\begin{bmatrix} \mathsf{z}_{t+1} \\ \mathsf{x}^e_{t+1,t} \end{bmatrix} = A \begin{bmatrix} \mathsf{z}_t \\ \mathsf{x}_t \end{bmatrix} + Br_t + C\epsilon_t \tag{90}$$

where z_t is an $(n-m) \times 1$ vector of predetermined variables including non-stationary processed, z_0 is given, x_t is an $m \times 1$ vector of non-predetermined variables and $\mathsf{x}^e_{t+1,t}$ denotes rational (model consistent) expectations of x_{t+1} formed at time t. Let $\mathsf{y}'_t = [\mathsf{z}'_t\ \mathsf{x}'_t]$.

Following the procedure set out above, the inter-temporal welfare loss of the representative household at time $t = 0$ can be approximated by

$$\Omega_0 = \frac{1}{2} E_t \sum_{t=0}^{\infty} \beta^t [\mathsf{y}'_t Q \mathsf{y}_t] \tag{91}$$

where Q is symmetric and non-negative definite. This completes the LQ approximation of the original non-linear, non-quadratic optimization problems considered in the rest of the paper. The procedures for evaluating the optimal policy rules are outlined in Levine *et al.* (2007c) (or Currie and Levine (1993) for a more detailed treatment).[14]

3.6 Imposing the Nominal Interest-Rate Zero Lower Bound Constraint

Now our optimization problem is expressed in LQ form we can impose an interest-rate zero lower bound (ZLB) constraint in a straightforward way. As in Woodford (2003), chapter 6, this is implemented by modifying the welfare loss function to

$$\Omega_0 = \frac{1}{2} E_t \sum_{t=0}^{\infty} \beta^t [\mathsf{y}'_t Q \mathsf{y}_t + w_r r_t^2] \tag{92}$$

[13] Note that the zero-inflation steady states of the sticky and flexi-price steady states are the same. In fact given the rules we actually examine in this paper, the output gap is only required in the estimation.

[14] The standard DSGE model assumes no growth, so when estimating one has to de-trend the data. When investigating optimal policy, there is again little problem, since all quadratic weights for the linear-quadratic approximation are weighted by the same term $C^{1-\sigma}$. If we assume a balanced growth steady state path however, the utility function of the household has to be modified. A simple modification that leaves the linearization, estimation and policy rules unchanged is of the form

$$\Omega_0 = E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_t(r)/\bar{C}_t - hC_{t-1}/\bar{C}_{t-1})^{1-\sigma}}{1-\sigma} - \frac{\kappa}{1+\phi} L_t(r)^{1+\phi} \right]$$

where $\bar{C}_t$ is the steady state consumption balanced growth path.

Then following Levine *et al.* (2007c), the policymaker's optimization problem is to choose w_r such that the probability, p, of the interest rate hitting the lower bound is very low. This is implemented by calibrating the weight w_r *for each of our policy rules* so that $z_0(p)\sigma_r < R$ where $z_0(p)$ is the critical value of a standard normally distributed variable Z such that prob $(Z \leq z_0) = p$.[15]

4 Estimation

The Bayesian Maximum-Likelihood approach taken in this paper follows work by DeJong *et al.* (2000b,a), Otrok (2001), and SW. There are by now numerous applications of the approach which can be seen as a combination of likelihood methods and the calibration methodology. Bayesian analysis allows formally incorporating uncertainty and prior information regarding the parametrization of the model by combining the likelihood with a prior density for the parameters of interest based on results from earlier microeconometric or macroeconometric studies.

Let us assume that the model outlined in section 2 represents the central bank's fundamental view of the economy. Nevertheless, the policy maker will always harbour uncertainty about the strength of various frictions within that framework. In this particular model, such frictions include habit persistence, factor adjustment costs, autoregressive shock processes, nominal stickiness etc; uncertainty about the strength of *any* of these frictions have implications for the transmission and conduct of monetary policy. Accordingly, in this paper, we examine robust monetary policy whereby the likelihood of various states of the world conditions the decision strategies of the policy maker. Two concerns were upper most in our minds: to identify frictions directly relevant for monetary policy but which also generate a reasonably even spread of "Bayesian Odds" in order to generate

[15]The ZLB constraint can be further eased by shifting the interest rate distribution to the right. Then steady state inflation rate in the optimal policy is positive. Let $\pi^* > 0$ be this rate. Then $R = \frac{1}{\beta} - 1 + \pi^*$ is the steady state nominal interest rate. Given σ_r the steady state positive inflation rate that will ensure $r_t \geq 0$ with probability $1 - p$ is given by $\pi^* = \max[z_0(p)\sigma_r - \left(\frac{1}{\beta} - 1\right) \times 100, 0]$. Furthermore if π^* is chosen in a optimal fashion, it is a credible new steady state inflation rate. (See Levine *et al.* (2007c)). In this paper however we retain zero inflation as a steady state feature of the policy rules. Note that in our LQ framework, the zero interest-rate bound is very occasionally hit. Then the interest rate is allowed to become negative, possibly using a scheme proposed by Gesell (1934) and Keynes (1936). Our approach to the ZLB constraint (following Woodford, 2003) in effect replaces it with a nominal interest-rate variability constraint which ensures the ZLB is hardly ever hit. By contrast the work of a number of authors including Adam and Billi (2007), Coenen and Wieland (2003), Eggertsson and Woodford (2003) and Eggertsson (2006) study optimal monetary policy with commitment in the face of a non-linear constraint $r_t \geq 0$ which allows for frequent episodes of liquidity traps in the form of $r_t = 0$.

a meaningful exercise.

Naturally, a key concern for monetary policy relates to how much nominal inertia exists in the economy. The degree of nominal stickiness in the economy has implications, for example, for the speed with which central banks react to shocks. Accordingly, we take our core estimated model and (re-)estimate another four variants with key nominal-persistence mechanisms altered: namely, where indexation in prices and wages is eliminated jointly or individually or treated symmetrically. The results are shown in **Table 2** at the end of the paper.[16]

We see that the baseline model - where both wage and price indexation is allowed - performs relatively well with the second highest odds. However, it would appear that the data prefer no indexation in the real wage to inflation (with a model probability 0.85), whilst it strongly rejects zero price indexation. Given this distribution of odds, the policy maker may conclude there are two highly likely states of the world: the core model and one where real wages are driven by their own dynamics, expected inflation, an optimality condition but without indexation to inflation histories. However, two other states of the world appear non negligible: whereby indexation is treated jointly zero or symmetrically (adding up to odds of 6%). Accordingly, in what follows, we condition our robust monetary policy exercises on the these four cases. Then given equal prior weights, one can determine the *posterior odds ratio* of each model:

$$\frac{p_j}{p_i} \equiv \frac{p(Model\ j|data)}{p(Model\ i|data)} = \frac{p(data|Model\ j)}{p(data|Model\ i)} = \frac{\exp^{LL_j}}{\exp^{LL_i}} \tag{93}$$

To summarize, it turns out that the probability for the $\gamma_P = 0$ variant is negligible, so we retain only the following four variants for our subsequent robustness analysis:

Model 1: $\gamma_P > 0$, $\gamma_W > 0$, probability=0.09
Model 2: $\gamma_P = \gamma_W > 0$, probability=0.05
Model 3: $\gamma_P = \gamma_W = 0$, probability=0.01
Model 4: $\gamma_W = 0$, probability=0.85.

As discussed in the introduction for the design of M-robust rules to be useful we require odds which do not massively support one variant. In fact we have a spread of odds which are all reasonably significant and so satisfy this requirement.

[16] As in SW the following parameters are imposed with calibrated values: $\beta = 0.99$, $\alpha = 0.3$, $\eta = 3$, $\delta = 0.025$, $c_y = 0.60$, $i_y = 0.22$ and $g_y = 0.18$. Furthermore, we put $T = 0.4$.

5 Optimal Monetary Policy without Model Uncertainty

5.1 Optimal Policy

In the absence of model uncertainty we first examine the four estimate variants of the model under the optimal commitment policy. Parameter values are fixed at the mean of the posterior distributions. Each model in turn is considered as the true model believed by the private sector and central bank alike. First consider optimal policy without a nominal interest-rate ZLB constraint. Table 3a sets out the outcomes in terms of steady state variances of key variables. Throughout the paper, we adopt a *conditional* welfare loss measure, starting at the zero-inflation steady state.[17]

	Baseline (Model 1)	$\gamma_W = 0$ (Model 4)	$\gamma_W = \gamma_P = 0$ (Model 3)	$\gamma_P = \gamma_W$ (Model 2)
var(y_t)	20.4	21.3	21.1	20.8
var(c_t)	21.0	21.8	22.5	21.4
var(π_t)	0.0654	0.0626	0.0521	0.0611
var(q_t)	64.3	62.3	60.7	62.4
var(i_t)	116	120	121	119
var(l_t)	7.45	7.78	7.49	7.47
var(wr_t)	9.00	10.3	9.79	9.35
var(r_t)	3.00	2.80	3.11	3.10
Prob ZLB	0.281	0.274	0.284	0.284
$\Omega_0(w_r)$	152.9	145.1	142.7	155.5
$c_e^{MODEL}(\%)$	0	-0.17	-0.23	0.06

Table 3a: Variances in $\%^2$ and Expected Welfare Loss: $w_r = 0.001$.

In this table, $c_e^{MODEL} = \frac{\Omega_0(w_r)^i - \Omega_0(w_r)^1}{1-h^a} \times 10^{-2}$ across alternative model variants is the welfare difference relative to model 1 in % permanent consumption equivalent units relative to the steady state, where $h^a = 0.55$ is the average of h across model variants.[18]

[17] An *unconditional* welfare loss measure averages over all possible initial states using the distribution of states calculated under the optimal commitment policy (see, for example Schmitt-Grohe and Uribe (2006)). However, for a discount factor close to unity, the differences between the measures are second order.

[18] To work out the welfare in terms of a consumption equivalent percentage increase, expanding $U(C) = \frac{C^{1-\sigma}(1-h)^{1-\sigma}}{1-\sigma}$ as a Taylor series, a 1% permanent increase in consumption of 1 per cent yields a first-order welfare increase $(1-h)^{1-\sigma}C^{-\sigma}\Delta C = C^{1-\sigma}(1-h)^{1-\sigma} \times 0.01$. Losses X reported in the Tables have been scaled such that utility loss is $C^{1-\sigma}(1-h)^{-\sigma}X \times 10^{-4}$. Hence $c_e = \frac{X}{(1-h)} \times 0.01$.

In terms of welfare outcomes we see that the four variants differ significantly. Eliminating wage indexation yields a welfare improvement (reduction in welfare loss) of $c_e = 0.17\%$ permanent increase in consumption. The further elimination of price indexation increases this improvement to $c_e = 0.23\%$. Imposing $\gamma_P = \gamma_W > 0$ on the other hand sees a welfare reduction (increase in welfare loss) of $c_e = 0.06\%$. The difference between the best and worst welfare outcomes is $c_e = 0.29\%$, a substantial welfare gain. It should be noted that in our LQ approximation our procedure includes all components of welfare and does not drop and terms independent of policy as is sometimes the case in the literature. It follows that the welfare for different model variants in Table 3a can be compared and that these values are measures of the minimum costs of fluctuations driven by the exogenous processes. In consumption equivalent terms, these are over 3% of steady state consumption, an order of magnitude of 100 times those in Lucas (1987).[19]

	Baseline (Model 1)	$\gamma_W = 0$ (Model 4)	$\gamma_W = \gamma_P = 0$ (Model 3)	$\gamma_P = \gamma_P$ (Model 2)
var(y_t)	20.0	20.9	20.9	20.4
var(c_t)	19.9	20.8	21.4	20.4
var(π_t)	0.065	0.062	0.052	0.061
var(q_t)	62.6	61.3	59.3	60.6
var(i_t)	120	123	124	121
var(l_t)	7.21	7.57	7.23	7.24
var(wr_t)	9.16	10.4	9.89	9.51
var(r_t)	0.243	0.246	0.243	0.249
Prob ZLB)	0.021	0.022	0.021	0.023
$\Omega_0(w_r)$	160.4	151.7	149.8	163.3
$\Omega_0(0)$	156.8	148.4	146.4	159.6
c_e^{MODEL}	0	-0.19	-0.23	0.062
c_e^{ZLB}	0.087	0.073	0.082	0.091
w_r	30	27	28	30

Table 3b: Variances in $\%^2$ and Expected Welfare Loss: $w_r = 30$. [20]

[19] The reason for these contrasting results lies in the contribution of wage and price variability to that of labour supply in household utility (see also Ball and Romer (1990)). Our welfare costs of fluctuations compare with estimates around 2% found in Levin *et al.* (2006) for a similar model, but with internal habit and estimated using US data.

[20] c_e^{ZLB} is the cost of the ZLB constraint in consumption equivalent units defined as: $c_e^{ZLB} = \frac{\Omega_0(0)^{NO\ CONSTRAINT} - \Omega_0(0)^{CONSTRAINT}}{1-h} \times 10^{-2}$.

In a sense these results are misleading since the probability of hitting the interest-rate lower bound is high in all cases. Almost one in three quarters would see this event happen on average. To impose the ZLB constraint we increase the weight penalizing the variability of the interest rate, w_r, in (92) until this probability reduces to less than 0.025. Then the true welfare is evaluated under this rule by subtracting the contribution of interest-rate variability to the modified loss function. Table 3b shows the results of this exercise. The ZLB constraint with a probability of less than 0.025 of hitting a zero interest rate is achieved by choosing w_r between 27 and 30, depending on the model variant. The cost in terms of a reduced stochastic welfare in percentage consumption equivalent terms is in the region $[0.073, 0.091]$.

5.2 Optimized IFB Rules

We now turn to optimized IFB rules feeding back on either current inflation alone or on j-period ahead, $j \geq 1$ expected inflation of the general form:

$$r_t = \rho r_{t-1} + \theta_\pi E_t \pi_{t+j} \tag{94}$$

where $\rho \in [0, 1]$, $\theta_\pi > 0, j \geq 0$. In what follows we denote such a rule by IFBj. For $\rho < 1$, (94) can be written as $\Delta r_t = \frac{1-\rho}{\rho}\left[\theta_\pi(1-\rho)^{-1}E_t\pi_{t+j} - r_t\right]$ which is a partial adjustment to a static IFBj rule $r_t = \theta_\pi(1-\rho)^{-1}E_t\pi_{t+j}$. If $\rho = 1$ we have an *integral rule* that is equivalent to the interest rate responding to a *price level target*.[21]

w_r	$[\rho, \theta_\pi]$	σ_r^2	Prob ZLB	$\Omega(w_r)$	$\Omega(0)$
0	[0, 5.765]	2.17	0.251	182.08	182.08
10	[0.588, 1.830]	0.49	0.076	186.53	184.08
20	[0.600, 1.603]	0.42	0.062	188.83	184.66
30	[0.625, 1.484]	0.38	0.053	190.52	184.87
40	[0.719, 1.008]	0.24	0.021	190.87	186.88

Table 4a: Imposing the ZLB: Baseline Model and $j = 0$ Price Inflation Rule.

Table 4a sets out the procedure for imposing the ZLB for the contemporaneous inflation rule (IFB0) and the baseline model 1. As we increase w_r, we reduce the equilibrium variance and the probability of hitting the ZLB. Rules become more inertial (ρ rises) and

[21] Unlike its non-integral counterpart, an integral rule responding to inflation does not require observations of the steady state (natural) rate of interest, about which i_t is expressed, to implement. The merits of price level versus inflation targeting are examined in Vestin (2006).

the instantaneous feedback from current inflation (θ_π) falls. At $w_r = 40$ the probability of hitting the lower bound is below 0.025, the target we impose in all our subsequent exercises.

Table 4b sets out analogous results for IFBj rules, $j \in [1, 8]$ for the baseline model and includes the result from table 4a for comparison. In these tables we add a column that calculates the consumption equivalent loss compared with the Ramsey policy in table 3a defined as $c_e = \frac{(\Omega_0^{IFBj} - \Omega_0^{OP}) \times 10^{-2}}{(1-h)}$. Tables 4c - 4e repeat this exercise for model variants 2-4.

j	w_r	$[\rho, \theta_\pi]$	σ_r^2	Prob ZLB	$\Omega(w_r)$	$\Omega(0)$	c_e
0	40	$[0.719, 1.008]$	0.24	0.021	190.87	186.88	0.77
1	20	$[0.719, 1.008]$	0.24	0.021	190.87	186.88	0.77
2	10	$[0.545, 6.172]$	0.25	0.023	179.57	178.32	0.58
3	0	$[0.591, 9.271]$	0.22	0.017	178.80	178.80	0.59
4	0	$[0.758, 11.54]$	0.16	0.006	180.23	180.23	0.62
5	0	$[0.916, 16.74]$	0.14	0.004	181.49	181.49	0.65
6	0	$[1.000, 11.629]$	0.085	0.000	182.25	182.25	0.67
7	0	$[1.000, 3.926]$	0.044	0.000	185.88	185.88	0.75
8	0	$[1.000, 1.551]$	0.026	0.000	194.97	194.97	0.96

Table 4b: Baseline Model: Optimized IFBj rules with ZLB Constraint.

j	w_r	$[\rho, \theta_\pi]$	σ_r^2	Prob ZLB	$\Omega(w_r)$	$\Omega(0)$	c_e
0	40	$[0.691, 1.153]$	0.25	0.023	194.22	189.65	0.78
1	25	$[0.627, 3.222]$	0.24	0.021	185.52	182.52	0.61
2	5	$[0.515, 7.567]$	0.24	0.021	181.92	181.32	0.57
3	0	$[0.607, 8.563]$	0.18	0.009	181.98	181.98	0.57
4	0	$[0.772, 10.06]$	0.14	0.004	183.26	183.26	0.60
5	0	$[0.956, 23.45]$	0.14	0.004	184.56	184.56	0.65
6	0	$[1.000, 10.19]$	0.075	0.000	185.14	185.14	0.67
7	0	$[1.000, 3.489]$	0.039	0.000	189.81	189.81	0.78
8	0	$[1.000, 1.454]$	0.024	0.000	199.82	199.82	1.01

Table 4c: Model 2: Optimized IFBj rules with ZLB Constraint.

j	w_r	$[\rho, \theta_\pi]$	σ_r^2	Prob ZLB	$\Omega(w_r)$	$\Omega(0)$	c_e
0	50	$[0.750, 1.550]$	0.25	0.023	179.05	172.74	0.65
1	10	$[0.038, 12.55]$	0.23	0.019	169.02	167.87	0.57
2	0	$[0.513, 8.738]$	0.16	0.006	168.24	168.24	0.58
3	0	$[0.713, 10.73]$	0.14	0.004	168.96	168.96	0.60
4	0	$[0.813, 10.50]$	0.099	0.000	169.52	169.52	0.61
5	0	$[1.000, 21.18]$	0.096	0.000	170.11	170.11	0.62
6	0	$[1.000, 7.113]$	0.048	0.000	171.96	171.96	0.67
7	0	$[1.000, 2.727]$	0.028	0.000	178.32	178.32	0.81
8	0	$[1.000, 1.190]$	0.018	0.000	190.41	190.41	1.08

Table 4d: Model 3: Optimized IFBj rules with ZLB Constraint.

j	w_r	$[\rho, \theta_\pi]$	σ_r^2	Prob ZLB	$\Omega(w_r)$	$\Omega(0)$	c_e
0	40	$[0.716, 1.086]$	0.25	0.023	182.89	177.75	0.75
1	0.75	$[0.800, 2.155]$	0.18	0.009	172.65	172.58	0.60
2	5	$[0.505, 6.795]$	0.25	0.023	172.26	172.03	0.59
3	0	$[0.601, 7.338]$	0.17	0.008	172.73	172.73	0.60
4	0	$[0.772, 8.203]$	0.13	0.003	174.44	174.44	0.64
5	0	$[0.912, 14.075]$	0.12	0.002	175.82	175.82	0.67
6	0	$[1.000, 10.330]$	0.078	0.000	176.35	176.35	0.68
7	0	$[1.000, 3.582]$	0.039	0.000	179.90	179.90	0.76
8	0	$[1.000, 1.475]$	0.024	0.000	188.82	188.82	1.19

Table 4e: Model 4: Optimized IFBj rules with ZLB Constraint.

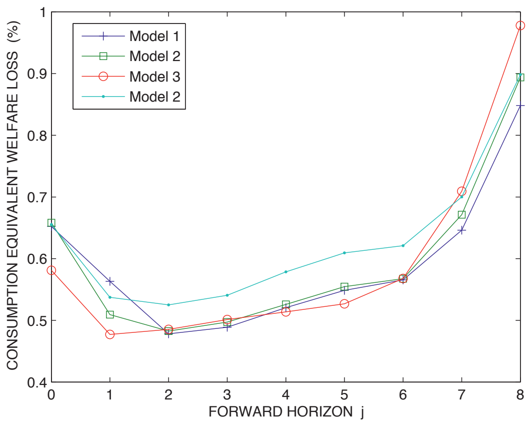

Figure 2: **Optimal Forward Horizon in IFBj Rules for Baseline Model**

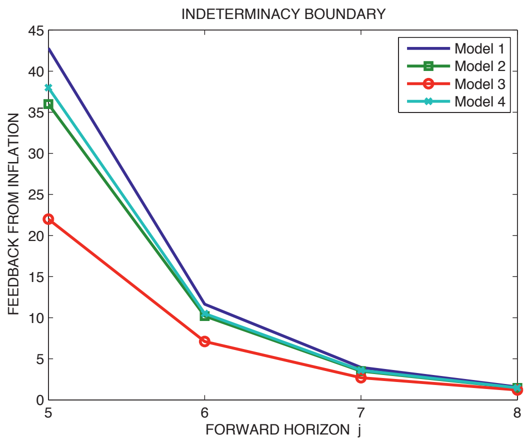

Figure 3: **Indeterminacy Boundary for all Models.** $\rho = 1$**.**

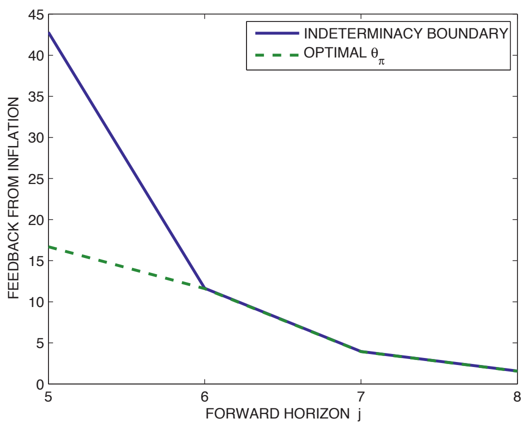

Figure 4: **Indeterminacy Boundary for Baseline Model and Optimized Rules:** $\rho = 1$**.**

A number of points emerge. First optimized IFBj rules improve stabilization performance in terms of welfare as the forward horizon j increases from $j = 0$ (a current inflation rule) to $j = 1$ for model 3 and $j = 2$ for the other three models. But as j increases further the rules deteriorate and sharply so for $j \geq 6$. Figure 2 shows that the optimal horizon is reached at $j = 1$ or $j = 2$ for the four model variants. The reason for this deterioration is the existence of a downward-sloping *indeterminacy boundary*: as j increases the upper bound on θ_π for which the rules deliver determinacy falls. This indeterminacy boundary is shown in Figure 3 for all three model variants. The existence of this downward-sloping boundary acts as a constraint on the optimal choice of θ_π, as illustrated in Figure 4 for the baseline model.[22] This is the "too much, too soon" result first shown by Batini and Pearlman (2002).

Second, as rules become more forward-looking the interest rate variance falls and at $j = 2$ or $j = 3$, depending on the model, the ZLB constraint ceases to bind.

Third, as the horizon j increases the optimized IFBj rules become more inertial (i.e., ρ increases). For $j \geq 6$ in all models they become integral rules (so an IFB rule becomes a price level forecast rule). Finally, compared with the optimal rule there is a significant welfare loss from the restriction implied by pursuing IFBj rules. At optimal horizons with $j = 1$ or $j = 2$ this loss is equivalent to a permanent consumption loss of $c_e = 0.57 - 0.59\%$ depending on the model. For $j = 8$ this loss rises to around 1% for models 1 - 2, 1.08% for model 3 and 1.19% for model 4.

5.3 Calvo-Type Interest-Rate Rules

An alternative way of thinking about IFB rules, first raised by Levine *et al.* (2007a), is in terms of *Calvo-type interest-rate rules*.[23] To formulate this first define the discounted sum of future expected inflation rates as

$$\Theta_t = (1 - \varphi)E_t(\pi_t + \varphi\pi_{t+1} + \varphi^2\pi_{t+2} + \cdots)\,;\ \varphi \in (0, 1) \tag{95}$$

Then

$$\varphi E_t\Theta_{t+1} - \Theta_t \quad = \quad -(1 - \varphi)\pi_t \tag{96}$$

[22] Figures 3 and 4 start at $j = 5$, because below this horizon the indeterminacy boundary occurs at very high values of θ_π and there is no real indeterminacy problem.

[23] We use this terminology since they have the same structure as Calvo-type contracts. (Calvo (1983)). One can think of the rule as a feedback from expected future inflation which continues in any one period with probability φ and is switched off with probability $1 - \varphi$. The probability of the rule lasting for just j periods is then $(1 - \varphi)\varphi^j$ and the mean lead horizon is therefore $(1 - \varphi)\sum_{j=1}^{\infty} j\varphi^j = \frac{\varphi}{1-\varphi}$.

With this definition, a rule of the form

$$r_t \quad = \quad \rho r_{t-1} + \theta_\pi \Theta_t \tag{97}$$

emerges which describes feedback on forward-looking inflation with a mean lead horizon of $\frac{\varphi}{1-\varphi}$. Thus with $\varphi = 0.5$, for example, we have a Calvo-type rule that compares with (94) with a horizon $j = 1$. It is of interest to note that for $\rho \in [0, 1)$, this rule can also be expressed as

$$r_t = (1 + \varphi\rho)^{-1}[\rho r_{t-1} + \varphi E_t r_{t+1} + \theta_\pi(1 - \rho)\pi_t] \tag{98}$$

Whether the rule is expressed in this way or as (97), it is evident that current variables and one-step ahead forecasts are sufficient statistics for the decisions of the policymaker.

Table 5a shows the stabilization performance of the Calvo interest-rate rule as φ increases from very close to zero (corresponding to a current inflation rule) to $\varphi = 0.95$ corresponding to a mean horizon of 19 quarters. A striking result emerges: interest-rate targeting can be very forward-looking with a Calvo rule without the sharp deterioration in stabilization performance typically seen with IFBj rules. The basic reason for this result is shown in Levine *et al.* (2007a) for a simple New Keynesian model: for integral rules Calvo rules can be shown never to be indeterminate. Our model is more complicated and not amenable to the same kind of analysis. However we can confirm numerically that in the whole region of (j, θ_π) space of Figure 3 (associating j with $\frac{\varphi}{1-\varphi}$), integral Calvo rules are determinate.

φ	$[\rho, \theta_\pi]$	σ_r^2	Prob ZLB	$\Omega(w_r) = \Omega(0)$
0.001	$[0, 5.83]$	2.21	0.25	182.15
0.1	$[0, 6.074]$	2.14	0.25	181.76
0.2	$[0, 6.440]$	2.06	0.24	181.33
0.3	$[0, 6.847]$	1.96	0.24	180.87
0.4	$[0, 7.403]$	1.85	0.23	180.37
0.5	$[0, 8.179]$	1.73	0.22	179.83
0.6	$[0, 9.274]$	1.59	0.21	179.24
0.7	$[0.3441, 6.779]$	0.80	0.13	178.87
0.8	$[0.6218, 5.052]$	0.37	0.051	178.86
0.9	$[0.8075, 5.331]$	0.19	0.011	179.44
0.95	[0.8698, 10.229]	0.16	0.006	179.71

Table 5a: Baseline Model 1: $w_r = 0$, Calvo Price Inflation Rule.

Table 5b imposes a ZLB constraint on the Calvo rule for each model in turn as in table 4a. Here and in the rest of the paper we focus on the case $\varphi = 4/5$ and $\varphi = 8/9$, corresponding to a mean forward horizons of 4 and 8 quarters respectively. By analogy with IFBj rules, we subsequently refer to these rules using the notation Calvoj, $j = 4, 8$. The consumption equivalent calculations in the last column confirm the important difference between IFBj and Calvoj with a ZLB constraint imposed: as j increases beyond $j = 4$. the former deteriorate markedly in terms of stabilization performance, whilst the performance of the latter hardly changes.

Model	j (φ)	w_r	$[\rho, \theta_\pi]$	σ_r^2	Prob ZLB	$\Omega_0(w_r)$	$\Omega_0(0)$	c_e
Model 1	4 ($\frac{4}{5}$)	14	$[0.656, 3.791]$	0.25	0.023	180.98	179.29	0.60
Model 2	4 ($\frac{4}{5}$)	14	$[0.662, 4.193]$	0.25	0.023	183.93	182.28	0.61
Model 3	4 ($\frac{4}{5}$)	6	$[0.695, 7.312]$	0.25	0.023	168.11	167.38	0.54
Model 4	4 ($\frac{4}{5}$)	12	$[0.678, 3.989]$	0.25	0.023	172.72	171.28	0.58
Model 1	8 ($\frac{8}{9}$)	0	$[0.789, 5.511]$	0.21	0.015	179.34	179.34	0.60
Model 2	8 ($\frac{8}{9}$)	0	$[0.786, 6.349]$	0.21	0.015	182.37	182.37	0.61
Model 3	8 ($\frac{8}{9}$)	0	$[0.823, 11.33]$	0.20	0.013	167.88	167.88	0.55
Model 4	8 ($\frac{8}{9}$)	0	$[0.786, 6.180]$	0.22	0.017	171.10	171.10	0.58

Table 5b Optimized Calvoj Rules with ZLB Imposed

5.4 A Labour Market Based Rule

We now examine a current wage inflation rule, found by Levin *et al.* (2006) to have good stabilization properties

$$r_t = \rho r_{t-1} + \theta_{\Delta w} \Delta w_t \tag{99}$$

where $\rho \in [0, 1], \theta_{\Delta w} > 0$. As with the price level rule, if $\rho = 1$, (99) reduces to a *wage level* target rule. Table 6 is analogous to table 5b.

Model	w_r	$[\rho, \theta_{\Delta w}]$	σ_r^2	Prob ZLB	$\Omega_0(w_r)$	$\Omega_0(0)$	c_e
Model 1	45	[1.000, 0.873]	0.25	0.023	167.48	161.99	0.12
Model 2	45	[1.000, 0.877]	0.25	0.023	170.77	165.23	0.13
Model 3	45	[1.000, 0.916]	0.25	0.023	159.69	154.15	0.17
Model 4	45	[1.000, 0.919]	0.25	0.023	159.51	154.05	0.13

Table 6 Optimized Wage Inflation Rule with ZLB Imposed

From table 6 we see that the optimized form of the wage inflation rule of the integral type for all models. Such rules are a substantially better than their IFBj or Calvoj counterparts amounting to an improvement of around 0.4% in terms of a permanent percentage consumption equivalent. The reason for this result can be seen by recalling that the loss function penalizes heavily both wage and price inflation. A rule that responds to either will therefore help to reduce expected welfare; but since $\Delta w_t = \Delta wr_t + \pi_t$ a wage inflation rule implicitly responds to both real wage and inflation and has a direct role in stabilizing employment as well.[24]

5.5 Summary of Performance of Commitment Rules

To summarize, we have examined the optimal commitment (Ramsey) interest-rate rule and three forms of simple Taylor-type rules that respond only to expected future or current inflation at some specified horizon j (IFBj rules); a discounted future sum of inflation rates (a Calvo rule), and to wage inflation. We have found that the stabilization performance of the IFBj rules deteriorates sharply as j rises above $j = 5$ quarters owing to a determinacy constraint. By contrast the Calvo rule can be very forward-looking without a losing its ability to stabilize. The current wage inflation rule outperforms either of these rules.

6 Optimal Monetary Policy with Model Uncertainty

In this section we consider model uncertainty in the form of uncertain estimates of the non-policy parameters of the model, Γ. Suppose the state of the world s is described by a model with $\Gamma = \Gamma^s$ expressed in state-space form as

$$\begin{bmatrix} \mathsf{z}_{t+1}^s \\ E_t \mathsf{x}_{t+1}^s \end{bmatrix} = A^s \begin{bmatrix} \mathsf{z}_t^s \\ \mathsf{x}_t^s \end{bmatrix} + B^s r_t^s + C^s \epsilon_t \tag{100}$$

where z_t^s is a vector of predetermined variables at time t and x_t are non-predetermined variables in state s of the world. In (100) it is important to stress that variables are in deviation form about a zero-inflation steady state of the model in state s. For example output in deviation form is given by $y_t^s = \frac{Y_t^s - Y^s}{Y_s}$ where Y^s is the steady state of the model in state s defined by parameters Γ^s and $r_t^s = R_t - R^s$ where the natural rate of interest in model s, $R^s = \frac{1}{\beta^s} - 1$. In our estimation however we imposed $\beta^s = \beta = 0.99$ so $R^s = R = \frac{1}{\beta} - 1$, which simplifies robust policy design somewhat.

[24]Interestingly such a rule is implicitly suggested by a current member of the monetary policy committee of the Bank of England, David Blanchflower (see Blanchflower and Shadforth (2007)). Analogous to IFBj rules, wage inflation forecast based rules of the form $r_t = \rho r_{t-1} + \theta_{\Delta w} E_t \Delta w_{t+j}$ are also of interest, but are not pursued in this paper.

For P-robustness (92) is replaced with the average expected utility loss across a large number of draws, n, from all models constructed using both the posterior model probabilities and the posterior parameter distributions for each model.

$$\Omega_0 = \frac{1}{2}\sum_{s=1}^{n} E_t \sum_{t=0}^{\infty} \beta^t [\mathsf{y}_t^{s\prime} Q^s \mathsf{y}_t^s + w_r r_t^2] \tag{101}$$

We denote the policy rule that results from this procedure, **Robustness with Model-Consistent Expectations**. We use the draws from the Markov Chain Monte Carlo (MCMC) Bayesian estimation as a representation of the ex post probability distribution of the parameters of the system. The results that follow are based on $n = 5000$ such draws of which proportions 0.09, 0.05, 0.01 and 0.85, corresponding to the estimated probabilities in table 2, taken from models 1 - 4 respectively.[25]

However there is one further consideration first raised by Levine (1986) that is usually ignored in the literature. Up to this point we have assumed that in each state of the world, s, private sector expectations $E_t\mathsf{x}_{t+1}^s$ are state s model-consistent expectations. In other words, in each state of the world the private sector knows the state and faces no model uncertainty. In a more general formulation of the problem we can relax this assumption and assume that *both* the policymaker and the private sector face model uncertainty. Suppose that in state s of the world the latter believes model u is the correct one. Then $E_t\mathsf{x}_{t+1}^s$ must be replaced by the expectation $E_t\mathsf{x}_{t+1}^u$ where the expectational operator at time t is now conditional on model u.

Consider simple rules of the general form

$$r_t = Dy_t = D\begin{bmatrix} z_t \\ x_t \end{bmatrix} \tag{102}$$

where D is constrained to be sparse in some specified way as in the IFBj, Calvoj and wage inflation rules of section 5. In state of the world s with the private sector believing state of the world r, the system under control (100), with the interest-rate rules (believed by the private sector) given by (102), has a rational expectations solution with $x_t^{su} = -N^u z_t^{su}$ where $N^u = N^u(D)$ is calculated on the basis of model u. Hence

$$z_{t+1}^{su} = (G_{11}^s - G_{12}^s N^u) z_t^{su} \tag{103}$$

[25] We do not incorporate learning about the environment as in Cogley and Sargent (2005), for example. This is not straightforward when considering our optimal commitment rules owing to their time inconsistency. Learning about time-inconsistent rules must address the question of how a reputation for commitment can be established when the private sector may be unable to distinguish legitimate re-optimization, arising from new information, from an opportunistic re-optimization that can occur without such new information. How a central bank can establish a reputation for commitment rules is considered in Levine *et al.* (2007c).

where $G^s = \begin{bmatrix} G^s_{11} & G^s_{12} \\ G^s_{21} & G^s_{22} \end{bmatrix} = A^s + B^s D$ is partitioned conformably with $\begin{bmatrix} z^s_t \\ x^s_t \end{bmatrix}$. For P-robustness the counterpart of (101) is the minimand

$$\Omega_0 \;=\; \frac{1}{2}\sum_{s=1}^{n}\sum_{u=1}^{n} E_t \sum_{t=0}^{\infty} \beta^t [\mathrm{y}_t^{su\prime} Q^s \mathrm{y}_t^{su} + w_r r_t^2] \qquad (104)$$

We denote this by **Robustness with Model-Inconsistent Expectations**. However this pairwise optimization with model-inconsistent expectations is computationally very time-consuming. With $n = 5000$, and up to 100 calculations of the welfare loss for each optimized rule over feedback parameters, this involves of the order 2.5 billion optimizations! We therefore choose a smaller sample of $n = 200$ MCMC draws. To ensure a reasonably representative sample from the least probable variants we can no longer choose draws from each in proportion to the model probabilities. For instance this would lead to only 2 draws for model 3. We therefore choose draws $n_1 = 50$, $n_2 = n_3 = 25$ and $n_4 = 100$ for models $1 - 4$. Then define an *adjusted probability per pair of draws* $\bar{p}_{us} = \frac{p_u p_s}{n_u n_s}$ where p_u and p_s are the original model probabilities associated with the models from which the draws originate. The minimand then becomes

$$\Omega_0 \;=\; \frac{1}{2}\sum_{s=1}^{n}\sum_{u=1}^{n} \bar{p}_{us} E_t \sum_{t=0}^{\infty} \beta^t [\mathrm{y}_t^{su\prime} Q^s \mathrm{y}_t^{su} + w_r r_t^2] \qquad (105)$$

6.1 Robust Rules with Model-Consistent Expectations

In this subsection we calculate optimal simple interest-rate rules IFBj, Calvoj and wage inflation rules. As above, we have done this for various weights on the interest rate in the welfare loss function. This enables us to calculate the probability of hitting the zero lower bound for the nominal interest rate. For each of the $n = 5000$ draws, we calculate the equilibrium steady state variance of the interest rate. Then for each draw we use the variance of the interest rate to calculate the probability of hitting the zero lower bound; once again the average of these appears as Prob ZLB in the tables and the average variance of these is included in the table as σ_r^2. Thus with an equilibrium interest rate of 1% per quarter (4% per annum), the latter are given by

$$\sigma_r^2 \;=\; \frac{1}{n}\sum_{j=1}^{n} \sigma_r^2(j) \qquad (106)$$

$$\mathit{Prob\ ZLB} \;=\; \frac{1}{n}\sum_{j=1}^{n} Z\left(-\frac{1}{\sigma_r(j)}\right) \qquad (107)$$

where $Z(x)$ is the probability that a standard normal random variable has a value less than x.

Table 7a represents the results of averaging the welfare loss over 5000 draws, 450 from model 1, 250 from model 2, 50 from model 3 and the rest from model 4.

j	w_r	$[\rho, \theta_\pi]$	σ_r^2	Prob ZLB	$\Omega_0(w_r)$	$\Omega_0(0)$
0	45	[0.759, 1.009]	0.25	0.023	235.55	229.89
4	0	[0.87, 8.490]	0.15	0.007	225.6	225.64
8	0	[1.000, 0.650]	0.02	0.000	262.66	262.66

Table 7a Model Consistent-Robust IFBj Rules

How do the robust rules of Table 7a compare with their non-robust counterparts in Tables 4b - 4e? A careful comparison reveals first, that the degree of interest-rate smoothing, ρ, is higher for robust rules up to the long horizon $j = 8$ where rules are of the integral type in both cases. Notice that all the rules satisfy the *modified Taylor principle* for rules with inertia, $\rho + \theta_\pi > 1$, which for integral rules simply becomes $\theta_\pi > 0$ (see Woodford (2003), page 255). Second, robust rules have a lower response to current or expected inflation (θ_π is lower). Robust rules, in other words, respond less quickly and less aggressively to deviations of current or forward-looking inflation from the zero-inflation target.[26]

Table 7b provides outcomes for IFBj rules designed for each model, then implemented in a model economy described by all four model variants with parameter values taking central values. These outcomes are compared with those for the robust rules of Table 7a. Now when rules designed for Model i, with central values, are applied to model $j \neq i$ we see two noteworthy results. First, IFB0 rules without robust design are in fact reasonable robust: Off-diagonal entries are very close to the diagonal values except for the rule designed for model 3, without any indexation. Even in that case the drop in welfare is only of the order $c_e = 0.03$. As well as a drop in welfare the other consequence or applying the rule for model 3 in the other models is that the ZLB constraint is violated. However the robust rules avoids the latter, by design, and achieves an outcome very close to the optimal non-robust rule for each of the models, except model 3 where the cost of robustness is of the order $c_e = 0.08$.

[26]The result that model uncertainty calls for a more cautious policy goes back to Brainard (1967), but contrasts with the robust policy rules that arise from the Hansen-Sargent minmax approach that see robust policy as being faster and more aggressive.

Rule IFBj(i)	Model 1	Model 2	Model 3	Model 4
IFB0(1)	186.88 (0.023)	190.20 (0.017)	177.70 (0.004)	178.38 (0.019)
IFB0(2)	186.18 (0.030)	189.65 (0.023)	176.90 (0.008)	177.69 (0.027)
IFB0(3)	188.07 (0.078)	190.02 (0.064)	172.74 (0.025)	177.73 (0.071)
IFB0(4)	186.47 (0.030)	190.06 (0.025)	175.82 (0.006)	177.75 (0.025)
Robust	186.50 (0.023)	189.97 (0.017)	176.49 (0.004)	178.33 (0.021)
IFB4(1)	180.23 (0.006)	indet	indet	indet
IFB4(2)	180.30 (0.005)	183.26 (0.004)	indet	174.51 (0.004)
IFB4(3)	180.75 (0.004)	183.78 (0.003)	169.52 (0.003)	175.01 (0.004)
IFB4(4)	180.36 (0.003)	183.31 (0.003)	indet	174.44 (0.003)
Robust	181.11 (0.002)	184.20 (0.000)	170.07 (0.000)	175.29 (0.000)
IFB8(1)	194.97 (0.000)	indet	indet	indet
IFB8(2)	195.78 (0.000)	199.82 (0.000)	indet	189.07 (0.000)
IFB8(3)	197.93 (0.000)	202.26 (0.000)	190.41 (0.000)	192.82 (0.000)
IFB8(4)	185.71 (0.000)	indet	indet	188.82 (0.000)
Robust	205.92 (0.000)	213.74 (0.000)	204.74 (0.000)	200.96 (0.000)

Table 7b: Robust IFBj Rules Across 4 Models

Notation: IFBj(i) is IFB rule with j horizon designed for model i from tables 4b - 4e. In this section with model-consistent expectations, the true model is believed by the private sector. (Values in brackets are ZLB Probabilities)

For IFBj rules, $j \geq 4$ applying the non-robust rule designed for the wrong model has far more serous implications. A rule designed for model 1 leads to indeterminacy when implemented in the other three models. Model 3 exhibits indeterminacy if subjected to rules other than the correct one. Again this problem is avoided with the robust rules of Table 7a, but this comes at a cost of as much as $c_e = 0.3$ in an economy described by model 2 ($\gamma_P = \gamma_W > 0$).

j (φ)	w_r	$[\rho, \theta_\pi]$	σ_r^2	Prob ZLB	$\Omega_0(w_r)$	$\Omega_0(0)$
4 ($\varphi = \frac{4}{5}$)	7	[0.793,3.316]	0.23	0.019	223.29	222.50
8 ($\varphi = \frac{8}{9}$)	0	[0.878,5.511]	0.19	0.012	222.88	222.88

Table 8a Model-Consistent Robust Calvoj Rules

Rule Calvoj(i)	Model 1	Model 2	Model 3	Model 4
Calvo4(1)	179.29 (0.023)	indet	indet	indet
Calvo4(2)	179.13 (0.031)	182.28 (0.023)	indet	171.20 0.027
Calvo4(3)	181.41 (0.11)	183.42 (0.09)	167.38 (0.023)	172.51 (0.10)
Calvo4(4)	179.24 (0.027)	182.41 (0.019)	indet	171.28 (0.023)
Robust	180.23 (0.017)	183.35 (0.011)	169.66 (0.000)	171.84 (0.013)
Calvo8(1)	179.34 (0.015)	indet	indet	171.17 (0.014)
Calvo8(2)	179.40 (0.023)	182.37 (0.015)	indet	171.10 (0.019)
Calvo8(3)	183.26 (0.093)	184.94 (0.074)	167.88 (0.013)	173.79 (0.083)
Calvo8(4)	179.36 (0.021)	182.37 (0.013)	indet	171.10 (0.017)
Robust	180.68 (0.013)	183.61 (0.008)	169.43 (0.000)	172.00 (0.009)

Table 8b: Robust Calvoj Rules Across 4 Models

Notation: Calvoj(i) is IFB rule with expected $j = \frac{\varphi}{1-\varphi}$ horizon designed for model i. (Values in brackets are ZLB Probabilities)

Now consider the Calvo$j$ rules. From Table 8a it can be seen that the greater interest-rate smoothing and less aggressive response to expected inflation applies to this form of rule too. But from Table 8b, although the non-robust form of these rules are prone to indeterminacy as well, the robust form of the rules comes at a much lower cost of around $c_e = 0.02$ as compared to the worst case of $c_e = 0.3$ for the discrete IFB8 rule. Policymakers can be forward-looking in terms of inflation targets with robust rules of the Calvo type and the compromise to achieve robustness does not cost much in terms of welfare loss.

Finally the wage inflation rule that was found to mimic the optimal rule in the case of no model uncertainty, now can be seen from Table 9b to have remarkable robustness properties as well. Rules designed for model i perform well in model $j \neq i$. Implementing the wrong rule for the model does not lead to any serious violation of the ZLB constraint. Robust design, using the rule set out in Table 9a, avoids the latter altogether at a very small welfare costs indeed. The robust rule in table 9a is, in fact, the non-robust rule in Table 6 designed for model 2. All our optimized wage inflation rules are of the integral type, but the robust rule is less aggressive than that required for models 3 and 4, and almost the same as model 1. The Brainard result - that model uncertainty calls for a more cautious policy - applies to the wage inflation rule as well.

w_r	$[\rho, \theta_{\Delta w}]$	σ_r^2	Prob ZLB	$\Omega_0(w_r)$	$\Omega_0(0)$
65	[1, 0.877]	0.26	0.025	208.03	199.56

Table 9a: Model-Consistent Robust Current Wage Inflation Rule

Rule Wage(i)	Model 1	Model 2	Model 3	Model 4
Wage(1)	161.99	165.26	154.49	154.40
	(0.023)	(0.023)	(0.021)	(0.021)
Wage(2)	161.96	165.23	154.47	154.37
	(0.025)	(0.023)	(0.023)	(0.021)
Wage(3)	161.67	164.92	154.15	154.07
	(0.027)	(0.027)	(0.023)	(0.023)
Wage(4)	161.65	164.90	154.12	154.05
	(0.027)	(0.027)	(0.027)	(0.023)
Robust	161.96	165.23	154.46	154.37
	(0.023)	(0.025)	(0.021)	(0.021)

Table 9b: Robust and Wage Inflation Rule Across 4 Models (Values in brackets are ZLB Probabilities)

To summarize: we have designed robust rules across model variants and for each across parameter draws taken from the estimated posterior joint distribution, for simple interest-rate rules of the type IFBj, Calvoj and a feedback from wage inflation. A common result for all rules is that robust rules exhibit the Brainard property of more caution than those designed without model uncertainty. Where rules are of the non-integral type (IFBj and Calvoj) the robust ones show more interest-rate smoothing (a higher ρ) and a lower immediate response to changes in current or expected future inflation (a smaller θ_π).

By far the most robust rule, in the sense that its optimized form is not too sensitive to the model for which it is designed, is that feeding back on wage inflation. Robust design across models or the parameter distribution is not really essential for this rule. The next best performing rules, Calvoj, however do require robust design to avoid indeterminacy, but robustness comes at a low welfare cost. IFBj rules again require robust design, but now for $j \geq 4$, robustness comes at a high welfare cost.

6.2 Robust Rules with Model-Inconsistent Expectations

We finally turn to robust rules where the central bank and private sector have different perceptions of the state of the world. Rules are designed to be robust to outcomes where the true model is i but the private sector believes in model $j \neq i$ necessarily. As explained in section 6.1 we limit the possible states of the world to a smaller sample of 200 in total from all four models. Rules IFB0, the wage inflation rule, Calvo4 and Calvo8 are considered in turn.

First consider the form of the robust rules designed for model-inconsistent expectations in table 10a. To help comparison with the rules in section 6.1 designed assuming model-inconsistent expectations the latter are reassembled in a comparable table 10b.

Rule	w_r	$[\rho, \theta_\pi]$ or $[\rho, \theta_{\Delta w}]$	σ_r^2	Prob ZLB	$\Omega_0(w_r)$	$\Omega_0(0)$
IFB0	55	[0.775, 0.956]	0.24	0.023	242.11	235.48
Wage Inflation	75	[1, 0.810]	0.25	0.024	210.32	200.88
Calvo4	20	[0.759, 3.604]	0.26	0.025	227.79	225.22
Calvo8	0	[0.851, 5.962]	0.25	0.022	225.54	225.54

Table 10a: Robust Rules for Model-Inconsistent Expectations

Rule	w_r	$[\rho, \theta_\pi]$ or $[\rho, \theta_{\Delta w}]$	σ_r^2	Prob ZLB	$\Omega_0(w_r)$	$\Omega_0(0)$
IFB0	45	[0.759, 1.009]	0.25	0.024	235.55	229.89
Wage Inflation	65	[1, 0.877]	0.26	0.025	208.08	199.56
Calvo4	7	[0.793, 3.316]	0.23	0.019	223.29	222.50
Calvo8	0	[0.878, 5.511]	0.19	0.022	222.88	222.88

Table 10b: Comparison with Robust Rules for Model-Consistent Expectations

Two observations are first, the Brainard property that increasing model uncertainty (now to include private sector expectations) should induce policy caution extends to the IFB0 and wage inflation rules, but not to the Calvoj rules. For the latter adding more model uncertainty leads to less interest-rate smoothing and a more aggressive immediate response to an expected inflation rate change. However these rules are still more cautious than their counterparts designed without model uncertainty. Second, the welfare costs of robust design for this extra uncertainty obtained by comparing the final columns of tables 10a and 10b are $c_e = .12$, .03, 0.06% for the IFB0, wage inflation and Calvoj rules respectively. These are quite significant losses especially for IFB0 and the wage inflation rules emerges with the lowest robustness costs.

To assess the robustness qualities of all the rules up to now we consider the outcomes when the true model is variant $i = 1, 4$ with central parameter values, but the private sector believes in variant $j \neq i$ necessarily. Tables 11-14 set out these results for the four forms of rule in turn.

True Model	Model 1 Believed	Model 2 Believed	Model 3 Believed	Model 4 Believed
1	186.88 (0.020)	182.84 (0.013)	166.32 (0.001)	188.94 (0.017)
2	195.04 (0.033)	189.65 (0.024)	172.08 (0.002)	197.51 (0.029)
3	230.33 (0.14)	217.24 (0.12)	172.74 (0.026)	227.03 (0.13)
4	178.72 (0.027)	174.46 (0.019)	156.56 (0.001)	177.75 (0.023)

Table 11a. IFB0 Non-Robust Rules

Notation: In cell ij the IFB0 rule designed for model i from is implemented in model i with the private sector believing model j. (Values in brackets are ZLB Probabilities)

In Table 11a, first consider the non-robust current inflation rate rules, IFB0, designed for model variants $i = 1, 4$ given in the first rows of tables 4b-4c. The off-diagonal cells indicates two things that can go seriously wrong with these interest-rate rules when private and public sector perceptions differ. First the welfare loss rises substantially when model 3 (with no price or wage indexing) is the true model, but the private sector believes in variants 1, 2 or 4 with indexing and makes adjustments to their price and wage decisions accordingly. The second failure of these non-robust rules is that private sector miss-perceptions result in a serious violation of the ZLB constraint as can be seen by the large probabilities of hitting the ZLB in brackets when model 3 is the correct one.

Now consider, in Table 11b, the model-inconsistent robust IFB0 rule of Table 10a designed across different variants and parameter draws within each variant which take into account private-public expectation differences. Now a more cautious robust rule satisfies the ZLB constraint coupled with smaller off-diagonal welfare losses in 9 out of 13 cells. This comes at a cost in diagonal cells where perceptions coincide which are non-negligible for model 3.

True Model	Model 1 Believed	Model 2 Believed	Model 3 Believed	Model 4 Believed
1	187.30 (.019)	182.28 (0.012)	165.51 (0.000)	189.51 (0.016)
2	196.32 (0.022)	190.49 (0.014)	172.02 (0.001)	199.07 (0.018)
3	228.59 (0.057)	217.24 (0.041)	176.14 (0.003)	225.40 (0.050)
4	178.88 (0.020)	174.45 (0.013)	156.33 (0.001)	178.15 (0.017)

Table 11b. IFB0 Model-Inconsistent Robust Rule
Notation: In cell ij the single robust IFB0 rule from table 10a is implemented in model i with the private sector believing model j.

Tables 12–14 repeat this comparison for the current wage inflation rule and the two Calvo rules. For the latter forward-looking rules, their non-robust forms in Tables 13a and 14a can result in *indeterminacy* when perceptions of the private sector and central banks differ. Robust rules for model-inconsistent expectations address this problem at the expense of modest increases on the diagonals. Nevertheless, with Calvo rules, as with the IFB0 rule there remains some very large entries for a world described by model 3 which are not mitigated by robust design. This suggests that private sector misperceptions of the correct model are far more serious than those of the policymaker designing the rule.

By contrast, from Table 12 with robust or otherwise wage inflation rules, differences in perceptions do not create a ZLB problem and large off-diagonal entries. This rule is remarkably robust even when designed without model-uncertainty considerations.

Model	Model 1 Believed	Model 2 Believed	Model 3 Believed	Model 4 Believed
1	161.99 (0.023)	158.24 (0.022)	143.55 (0.020)	162.34 (0.022)
2	169.64 (0.024)	165.23 (0.024)	149.04 (0.020)	170.36 (0.023)
3	202.59 (0.026)	193.02 (0.026)	154.15 (0.021)	197.21 (0.025)
4	156.24 (0.021)	152.80 (0.020)	136.24 (0.017)	154.06 (0.020)

Table 12a Current Wage Inflation Non-Robust Rules
Notation: In cell ij the Current Wage Inflation rule designed for model i from table 6 is implemented in model i with the private sector believing model j. (Values in brackets are ZLB Probabilities)

Model	Model 1 Believed	Model 2 Believed	Model 3 Believed	Model 4 Believed
1	162.51 (0.018)	158.75 (0.018)	144.06 (0.015)	162.91 (0.018)
2	170.22 (0.019)	165.80 (0.019)	149.60 (0.015)	170.99 (0.018)
3	203.55 (0.021)	193.98 (0.021)	155.06 (0.016)	198.21 (0.020)
4	157.05 (0.016)	153.60 (0.016)	137.02 (0.013)	154.92 (0.016)

Table 12b Current Wage Inflation Model-Inconsistent Robust Rule
Notation: In cell ij the model-inconsistent robust Current Wage Inflation rule from table 11a is implemented in model i with the private sector believing model j.

Model	Model 1 Believed	Model 2 Believed	Model 3 Believed	Model 4 Believed
1	179.29 (0.025)	indet	indet	indet
2	187.18 (0.035)	182.28 (0.025)	indet	188.40 (0.028)
3	225.53 (0.18)	212.62 (0.14)	167.39 (0.025)	219.64 (0.16)
4	173.32 (0.017)	169.28 (0.010)	indet	171.28 (0.025)

Table 13a: Non-Robust Calvo Rules $\varphi = 0.8$

Notation: In cell ij the Calvo rules designed for model i from table 5b is implemented in model i with the private sector believing model j.(Values in brackets are ZLB Probabilities)

Model	Model 1 Believed	Model 2 Believed	Model 3 Believed	Model 4 Believed
1	179.89 (0.022)	175.54 (0.012)	159.72 (0.000)	180.89 (0.016)
2	188.02 (0.024)	182.95 (0.014)	165.65 (0.000)	189.44 (0.018)
3	222.51 (0.055)	211.60 (0.035)	169.37 (0.001)	217.19 (0.045)
4	173.42 (0.023)	169.36 (0.013)	151.56 (0.000)	171.57 (0.017)

Table 13b: Model-Inconsistent Robust Calvo Rule $\varphi = 0.8$

In cell ij the model-consistent robust Calvo rule from table 12a is implemented in model i with the private sector believing model j.

Model	Model 1 Believed	Model 2 Believed	Model 3 Believed	Model 4 Believed
1	179.34 (0.014)	indet	indet	180.27 (0.010)
2	187.46 (0.026)	182.37 (0.015)	indet	188.73 (0.020)
3	226.12 (0.16)	212.81 (0.12)	167.88 (0.013)	220.68 (0.14)
4	173.05 (0.022)	168.98 (0.013)	indet	171.10 (0.017)

Table 14a: Non-Robust Calvo Rules $\varphi = 8/9$

Model	Model 1 Believed	Model 2 Believed	Model 3 Believed	Model 4 Believed
1	180.33 (0.018)	175.80 (0.001)	159.69 (0.000)	181.31 (0.013)
2	188.50 (0.020)	183.22 (0.011)	165.60 (0.000)	189.90 (0.015)
3	221.65 (0.048)	210.72 (0.030)	169.02 (0.001)	216.64 (0.038)
4	173.53 (0.019)	169.39 (0.010)	151.19 (0.000)	171.72 (0.013)

Table 14b: Model-Inconsistent Robust Calvo Rule $\varphi = 8/9$

7 Conclusions

We examined robust policy design through the lens of Bayesian and risk management perspectives. The paper has made two principal contributions to the literature on interest-rate rules.

First, we have set out a comprehensive methodology for designing rules that are robust with respect to model uncertainty facing both the policymaker and the private sector. In a welfare-based study, we reduced the optimization problem to an LQ one using a large distortions quadratic approximation of the representative household's utility. In the steady-state analysis of the SW model with taxes we showed that distortions are indeed large, justifying this form of approximation. Within this LQ framework we imposed a zero-lower-bound constraint in the design of optimized interest-rate rules. Unlike previous literature, we assumed both the policymaker and the private sector faced model uncertainty in the form of an estimated joint distribution of parameters and the estimated probabilities of four model variants.

Our second contribution involved the application of this methodology to three particular intuitive and interesting rules: inflation-forecast-based (IFB) rules with a discrete forward horizon, a 'Calvo' price inflation rule, and a current wage inflation rule. We found that IFB rules with a long horizon perform badly with or without robust design. Our Calvo rule performed much better, indicating that central banks *can* (contrary to received wisdom) be highly forward-looking without compromising stabilization. But, in a result consistent with Levin *et al.* (2006), the current wage level rule outperformed these alternatives by far, whether the rule was designed to have good robust properties or not.

Results on the relative performance of these three rules are naturally dependent on the model and the variants chosen. As Lombardo and Vestin (2007) and others have shown, the welfare costs of price and wage inflation are sensitive to the modelling of nominal inertia and labour supply by households. The Bayesian methodology set out in this paper provides a framework for assessing these rival approaches and their implications for robust policy design.

A The Hamiltonian Quadratic Approximation of Welfare

Consider the following general deterministic optimization problem

$$\max \sum_{t=0}^{\infty} \beta^t U(X_{t-1}, W_t) \;\; s.t. \;\; X_t = f(X_{t-1}, W_t) \tag{A.1}$$

where X_{t-1} is vector of state variables and W_{t-1} a vector of instruments.[27] There are given initial and the usual tranversality conditions. For our purposes, we consider this as including models with forward-looking expectations, so that the optimal solution to the latter setup is the *pre-commitment* solution. Suppose the solution converges to a steady

[27] An alternative representation of the problem is $U(X_t, W_t)$ and $E_t[X_{t+1}] = f(X_t, W_t)$ where X_t includes forward-looking non-predetermined variables and $E_t[X_{t+1}] = X_{t+1}$ for the deterministic problem where perfect foresight applies. Whichever one uses, it is easy to switch from one to the other by a simple re-definition. Note that Magill (1977) adopted a continuous-time model without forward-looking variables. As we demonstrate in Levine *et al.* (2007c), although the inclusion of forward-looking variables significantly alters the nature of the optimization problem, these changes only affect the boundary conditions and the second-order conditions, but not the steady state of the optimum which is all we require for LQ approximation.

state X, W as $t \to \infty$ for the states X_t and the policies W_t. Define $x_t = X_t - X$ and $w_t = W_t - W$ as representing the first-order approximation to absolute deviations of states and policies from their steady states.[28]

The Lagrangian L for the problem is defined as

$$L = \sum_{t=0}^{\infty} \beta^t [U(X_{t-1}, W_t) - \lambda_t^T (X_t - f(X_{t-1}, W_t))] \tag{A.2}$$

so that a *necessary* condition for the solution to (A.1) is that the Lagrangian is stationary at all $\{X_s\}$, $\{W_s\}$ i.e.

$$U_W + \lambda_t^T f_W = 0 \qquad U_X - \frac{1}{\beta}\lambda_{t-1}^T + \lambda_t^T f_X = 0 \tag{A.3}$$

Assume a steady state λ for the Lagrange multipliers exists as well. Now define the Hamiltonian $H_t = U(X_{t-1}, W_t) + \lambda^T f(X_{t-1}, W_t)$. The following is the discrete time version of Magill (1977):

Theorem 1: If a steady state solution (X, W, λ) to the optimization problem (A.1) exists, then any perturbation (x_t, w_t) about this steady state can be expressed as the solution to

$$\max \frac{1}{2} \sum_{t=0}^{\infty} \beta^t \begin{bmatrix} x_{t-1} & w_t \end{bmatrix} \begin{bmatrix} H_{XX} & H_{XW} \\ H_{WX} & H_{WW} \end{bmatrix} \begin{bmatrix} x_{t-1} \\ w_t \end{bmatrix} \quad s.t. \;\; x_t = f_X x_{t-1} + f_W w_t \tag{A.4}$$

where H_{XX}, etc denote second-order derivatives evaluated at (X, W). This can be directly extended to the case incorporating disturbances. Thus our general procedure is as follows:

1. Set out the deterministic non-linear problem for the Ramsey Problem, to maximize the representative agents utility subject to non-linear dynamic constraints.

2. Write down the Lagrangian for the problem.

3. Calculate the first order conditions. We do not require the initial conditions for an optimum since we ultimately only need the steady-state of the Ramsey problem.

4. Calculate the steady state of the first-order conditions. The terminal condition implied by this procedure is that the system converges to this steady state.

5. Calculate a second-order Taylor series approximation, about the steady state, of the Hamiltonian associated with the Lagrangian in 2.

[28] Alternatively $x_t = (X_t - X)/X$ and $w_t = (W_t - W)/W$, depending on the nature of the economic variable (See footnote 9). Then Theorem 1 follows in a similar way with an appropriate adjustment to the Jacobian Matrix.

6. Calculate a first-order Taylor series approximation, about the steady state, of the first-order conditions and the original constraints.

7. Use 4. to eliminate the steady-state Lagrangian multipliers in 5. By appropriate elimination both the Hamiltonian and the constraints can be expressed in minimal form. This then gives us the accurate LQ approximation of the original non-linear optimization problem in the form of a minimal linear state-space representation of the constraints and a quadratic form of the utility expressed in terms of the states.

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π_t	producer price inflation over interval $[t-1,t]$
r_t	nominal interest rate over interval $[t,t+1]$
$wr_t = w_t - p_t$	real wage
mc_t	marginal cost
mrs	marginal rate of substitution between work and consumption
l_t	employment
z_t	capacity utilization
k_t	end-of-period t capital stock
i_t	investment
$r_{K,t}$	return on capital
q_t	Tobin's Q
c_t	consumption
y_t,	output
$u_{i,t+1} = \rho_a u_{i,t} + \epsilon_{i,t+1}$	AR(1) processes for utility preference shocks, $u_{i,t}$, $i = C,\ L,\ I$
$a_{t+1} = \rho_a a_t + \epsilon_{a,t+1}$	AR(1) process for factor productivity shock, a_t
$g_{t+1} = \rho_g g_t + \epsilon_{g,t+1}$	AR(1) process government spending shock, g_t
β	discount parameter
$\gamma_p,\ \gamma_w$	indexation parameters
h	habit parameter
$1-\xi_p,\ 1-\xi_w$	probability of a price, wage re-optimization
σ	risk-aversion parameter
ϕ	disutility of labour supply parameter
ϕ_F	$1 + \frac{F}{Y}$

Table 1. Summary of Notation (Variables in Deviation Form).

	Core	$\gamma_p = 0$	$\gamma_w = 0$	$\gamma_w = \gamma_p = 0$	$\gamma_w = \gamma_p$
ρ_a	0.97 [0.95:0.98]	0.97 [0.96:0.99]	0.97 [0.95:0.98]	0.97 [0.96:0.99]	0.97 [0.95:0.98]
$\rho_{\bar{\pi}}$	0.85 [0.70:0.99]	0.95 [0.87:1.00]	0.85 [0.67:0.99]	0.91 [0.80:1.00]	0.85 [0.70:0.99]
ρ_C	0.85 [0.80:0.91]	0.87 [0.81:0.92]	0.86 [0.80:0.91]	0.87 [0.82:0.91]	0.85 [0.80:0.91]
ρ_g	0.93 [0.89:0.97]	0.94 [0.90:0.98]	0.94 [0.90:0.97]	0.94 [0.90:0.97]	0.94 [0.90:0.98]
ρ_L	0.92 [0.89:0.96]	0.93 [0.90:0.97]	0.93 [0.89:0.96]	0.93 [0.90:0.97]	0.93 [0.89:0.96]
ρ_I	0.90 [0.84:0.97]	0.92 [0.87:0.97]	0.91 [0.86:0.97]	0.92 [0.86:0.97]	0.91 [0.85:0.97]
$S''(1)$	6.41 [4.71:8.11]	6.23 [4.56:7.92]	6.33 [4.63:8.01]	6.25 [4.56:7.89]	6.32 [4.69:8.07]
σ	1.58 [1.11:2.05]	1.63 [1.16:2.10]	1.61 [1.16:2.05]	1.66 [1.23:2.12]	1.62 [1.15:2.08]
h	0.56 [0.44:0.68]	0.55 [0.44:0.67]	0.55 [0.43:0.66]	0.54 [0.42:0.66]	0.56 [0.45:0.67]
ϕ	1.93 [0.90:2.84]	2.16 [1.10:3.18]	1.89 [0.86:2.85]	2.17 [1.18:3.13]	1.99 [0.97:2.94]
ϕ_F	1.50 [1.31:1.67]	1.47 [1.28:1.66]	1.49 [1.31:1.66]	1.48 [1.29:1.66]	1.50 [1.32:1.67]
ψ	1.96 [1.38:2.54]	1.98 [1.39:2.57]	1.96 [1.35:2.53]	1.96 [1.34:2.56]	1.94 [1.33:2.54]
ξ_W	0.74 [0.69:0.79]	0.73 [0.67:0.79]	0.73 [0.68:0.79]	0.74 [0.69:0.79]	0.74 [0.68:0.79]
ξ_P	0.92 [0.91:0.94]	0.91 [0.90:0.93]	0.92 [0.91:0.94]	0.91 [0.90:0.93]	0.92 [0.91:0.93]
ξ_e	0.76 [0.72:0.80]	0.75 [0.71:0.79]	0.77 [0.73:0.80]	0.75 [0.71:0.79]	0.76 [0.72:0.80]
γ_w	0.31 [0.14:0.48]	0.28 [0.12:0.44]	-	-	0.35 [0.23:0.48]
γ_p	0.42 [0.28:0.55]	-	0.40 [0.27:0.54]	-	0.35 [0.23:0.48]
θ_π	1.70 [1.54:1.86]	1.71 [1.55:1.87]	1.70 [1.54:1.86]	1.70 [1.54:1.87]	1.71 [1.54:1.88]
$\theta_{\Delta\pi}$	0.13 [0.06:0.21]	0.15 [0.08:0.22]	0.14 [0.08:0.22]	0.15 [0.07:0.22]	0.13 [0.07:0.20]
ρ	0.97 [0.95:0.98]	0.95 [0.91:0.99]	0.97 [0.95:0.99]	0.97 [0.95:0.99]	0.97 [0.95:0.98]
θ_y	0.13 [0.06:0.19]	0.12 [0.06:0.18]	0.13 [0.06:0.19]	0.12 [0.05:0.19]	0.13 [0.06:0.20]
$\theta_{\Delta y}$	0.20 [0.16:0.23]	0.19 [0.16:0.24]	0.20 [0.16:0.23]	0.20 [0.17:0.24]	0.19 [0.16:0.23]
$sd(\epsilon_a)$	0.56 [0.44:0.67]	0.55 [0.45:0.67]	0.58 [0.46:0.70]	0.55 [0.43:0.65]	0.56 [0.46:0.67]
$sd(\epsilon_{\bar{\pi}})$	0.02 [0.00:0.03]	0.08 [0.00:0.16]	0.02 [0.00:0.06]	0.03 [0.00:0.08]	0.01 [0.01:0.02]
$sd(\epsilon_C)$	2.28 [1.77:2.78]	2.36 [1.82:2.88]	2.27 [1.76:2.77]	2.32 [1.84:2.82]	2.32 [1.82:2.77]
$sd(\epsilon_g)$	1.67 [1.48:1.85]	1.67 [1.49:1.86]	1.67 [1.49:1.85]	1.66 [1.47:1.84]	1.67 [1.48:1.86]
$sd(\epsilon_L)$	3.17 [1.95:4.33]	3.19 [1.92:4.31]	3.12 [1.82:4.27]	3.19 [1.98:4.32]	3.19 [1.97:4.34]
$sd(\epsilon_I)$	0.07 [0.04:0.11]	0.08 [0.05:0.11]	0.07 [0.04:0.10]	0.08 [0.04:0.11]	0.07 [0.04:0.11]
$sd(\epsilon_R)$	0.07 [0.04:0.09]	0.07 [0.04:0.10]	0.07 [0.04:0.10]	0.06 [0.04:0.09]	0.07 [0.04:0.09]
$sd(\epsilon_Q)$	7.80 [5.41:10.04]	7.56 [5.33:9.71]	7.72 [5.53:10.00]	7.60 [5.34:9.77]	7.66 [5.58:9.82]
$sd(\epsilon_P)$	0.17 [0.14:0.19]	0.23 [0.19:0.26]	0.17 [0.15:0.20]	0.24 [0.20:0.27]	0.18 [0.15:0.20]
$sd(\epsilon_W)$	0.21 [0.18:0.23]	0.21 [0.18:0.23]	0.20 [0.17:0.22]	0.20 [0.17:0.23]	0.21 [0.18:0.23]
LL	-263.70	-269.82	-261.44	-265.84	-264.25
prob	0.09	0.00	0.85	0.01	0.05

Table 2. Bayesian Estimation of Parameters[29]

[29] 5th and 95th percentiles are given in squared brackets below the posterior mean estimates, "- " indicates not applicable, LL denotes Log Likelihood and prob denotes Bayesian Odds ratios.

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