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REAL EXCHANGE RATE FORECASTING

A CALIBRATED HALF-LIFE PPP MODEL CAN BEAT THE RANDOM WALK

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In 2013 all ECB publications feature a motif taken from the €5 banknote.

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Abstract

This paper brings three new insights into the Purchasing Power Parity (PPP) debate. First, we show that a half-life PPP model is able to forecast real exchange rates (RER) better than the random walk (RW) model at both short and long-term horizons. Secondly, we find that this result holds only if the speed of adjustment to the sample mean is calibrated at reasonable values rather than estimated. Finally, we find that it is also preferable to calibrate, rather than to elicit as a prior, the parameter determining the speed of adjustment to PPP.

Keywords: Exchange rate forecasting; purchasing power parity; half-life.

JEL classification: C32; F31; F37

Non-technical summary

There is broad agreement between policy makers and academics that beating the random walk in forecasting nominal exchange rates is extremely challenging. Views are however less unanimous on whether real exchange rates can be forecast.

Real exchange rates forecasting is crucial to gauge the evolution of price competitiveness and export performance. The task that we set ourselves is to investigate whether real exchange rate forecasting of major world currencies can be achieved more accurately with a simple economic-theory-based model or with a naive random walk benchmark. The standard theoretical reference on this issue is the PPP hypothesis, which suggests that the relative price of two identical, domestic and foreign, baskets of goods is constant when expressed in a common currency. Although PPP is one of the most prominent theories in economics, it remains highly controversial, as it is thought to fail in the short-run. As for the long run it is generally recognized that mean reversion to the PPP implied rate is one of the factors at play.

In this paper we suggest that a PPP-based model can perform better than is generally recognized from a forecasting perspective. We show that a simple version of the model, which assumes slow convergence of the real exchange rate to the sample mean, generally overwhelmingly beats the RW model in terms of real exchange rate forecasting. What is particularly remarkable, also with reference to previous studies, is that we find that it strongly outperforms the RW model even at short-term horizons.

Our article qualifies this result from an important perspective. We find that if the speed of convergence to the mean is estimated, the model is generally beaten by the RW model. Our main contribution relative to the previous literature is to overturn this result by imposing a speed of adjustment to PPP, which implies that half of the adjustment is completed in 3 or 5 years. Such calibration, consistent with the findings of the exchange rate literature on the "PPP puzzle", is sufficient to ensure that RER forecasts are much better than those derived with the RW model. The analysis is overall encouraging on the usefulness of exchange rates theory: by changing the battlefield, i.e. turning to real rather than nominal exchange rate forecasting, a theory based model outperforms sizably the RW. The rest of the paper is devoted to understanding this result better from a theoretical point of view and to assess its empirical robustness. We also explore to what extent Bayesian

techniques, which allow us to set prior beliefs on the speed of adjustment to PPP, could improve the forecast performance of the half-life PPP model. We find that a reasonably calibrated half-life model is, however, a difficult benchmark to beat even with a Bayesian autoregressive model.

1 Introduction

Following the seminal papers by Meese and Rogoff (1983a,b), a consensus view has emerged that the economic theory is of little help in forecasting exchange rates. Although in the mid-1990s Mark (1995) and Chinn and Meese (1995) claimed that the random walk (RW) model could be beaten at longer horizons, this more optimistic perspective was short-lived and vigorously contested (see in particular Faust et al., 2003 or Cheung et al., 2005). Looking forward, as stated by Rogoff (2009) the unpredictability of exchange rates is likely to remain the consensus view for the conceivable future. Some authors have still argued that (i) the exchange rate theory cannot be falsified only on the basis of its forecasting performance and (ii) there could still be some exchange rate predictability, especially over longer horizons, with panel data models (Mark and Sul, 2001; Engel et al., 2008; Lopez-Suarez and Rodriguez-Lopez, 2011). The aim of this paper is not to review the exchange rate puzzle but to signal that there might be other promising research avenues. While sharing the fascination of many economists and market analysts for attempting to understand what drives exchange rates, in this paper we propose to turn the attention to real exchange rate forecasting. Not only this task might be easier but it could also be more relevant from a macroeconomic perspective, considering that to assess a country's outlook the relevant concept is price competitiveness and not the level of the exchange rate. The strong longing to understand nominal exchange rates could help explain why only a handful of studies have so far investigated the predictability of RERs. The main exceptions are the papers by Meese and Rogoff (1988) and Mark and Choi (1997). These papers, however, reach opposite conclusions: the first rather skeptical and the second more supportive on the scope for RER forecasting.

The standard theoretical reference on RER is the PPP hypothesis, one of the most prominent and controversial theories in the history of economic thinking. In their review of the PPP debate Taylor and Taylor discuss how the consensus has shifted for and against PPP over time: in their assessment the common view is now back to what had prevailed before the 1970s, i.e. "that short run PPP does not hold, that long-run PPP may hold in the sense that there is significant mean reversion of the RER, although there may be factors impinging on the equilibrium RER through time" (Taylor and Taylor, 2004, p. 154). The empirical literature that conducts unit root tests to evaluate the mean-reversion of RERs usually finds that it is not possible to reject the null of RERs non-stationarity. The evidence, however, is not conclusive owing to the low power of the tests for persistent processes (Sarno and Taylor, 2002, p. 59-67). The "PPP puzzle" literature, which estimates the speed of mean-reversion of RERs, generally concludes that it takes between 3 and 5 years to halve PPP deviations (see the discussion in Rogoff, 1996, Murray and Papell (2002), Kilian and Zha, 2002 or Norman (2010)).

This paper adds to the above literature from three perspectives. First, we show that a calibrated half-life model, which postulates a gradual adjustment of RER to the PPP level, outperforms the RW model in forecasting RERs at both short and long-term horizons. We claim that this finding provides new evidence in the PPP debate, suggesting that a PPP based framework outperforms the RW. Second, we show that the calibrated model outperforms its estimated counterpart in light of the fundamental role played by estimation error in the presence of a very persistent process. Finally we show that it is in general preferable to calibrate the speed of adjustment parameter rather than setting it as a prior in a Bayesian autoregressive model.

The rest of the paper is structured as follows. Section 2 outlines the alternative models that we shall use in our exchange rate forecasting competition. In section 3 we provide empirical support for the PPP hypothesis using monthly data for real effective exchange rates of major currencies for the period between January 1975 and March 2012. In Section 4 we provide an analytical investigation of our empirical findings, which highlights the important role of estimation error. Section 5 shows that the results are robust to the choice of different estimation windows and other currencies. We also show that the Bayesian autoregressive model, in which the speed of adjustment is set as a prior, fails to outperform the corresponding calibrated half-life model.

2 The models

Let us define the log of the real exchange rate as $y_t \equiv s_t + p_t - p_t^*$, where s_t is the log of the nominal exchange rate expressed as the foreign currency price of a unit of domestic currency, and p_t and p_t^* are the logs of home and foreign price levels, respectively. Let us assume that the DGP for y_t is a simple autoregression (AR) of the form:

$$(y_t - \mu) = \rho(y_{t-1} - \mu) + \epsilon_t, \epsilon_t \sim \mathcal{N}(0, \sigma^2) \tag{1}$$

with $|\rho| < 1$ measuring the speed of reversion to μ, which we interpret as the level of PPP. As mentioned in the introduction, the consensus view is that the half-life of deviations from the PPP:

$$hl = \log(0.5)/\log(\rho) \tag{2}$$

is somewhere between 3 and 5 years. This view implicitly assumes that RERs are mean reverting and hence predictable. We show that this claim is generally justified by comparing the accuracy of RER forecasts derived from the following competing models, which are a specific form of (1).

The first model is a random walk, for which the h step ahead forecast is:

$$y_{T+h|T}^{RW} = y_T. \tag{3}$$

The next two models assume that the half-life amounts to 3 or 5 years (HL3 and HL5), thus RERs converge to their sample mean values at pace $\bar{\rho}$ consistent with the duration of the half-life in line with (2). The h step ahead forecast is:

$$y_{T+h|T}^{HL} = \bar{\mu} + \bar{\rho}^h(y_T - \bar{\mu}), \tag{4}$$

where $\bar{\mu}$ is the sample mean. The last competitor is the AR model of the form (1) for which:

$$y_{T+h|T}^{AR} = \hat{\mu} + \hat{\rho}^h(y_T - \hat{\mu}), \tag{5}$$

where $\hat{\mu}$ and $\hat{\rho}$ are OLS estimates.

3 Empirical evidence

To assess the predictability of RERs we gather monthly data for nine major currencies of the following countries: Australia (AUD), Canada (CAD), euro area (EUR), Japan (JPY), Mexico (MXN), New Zealand (NZD), Switzerland (CHF), the United Kingdom (GBP) and the United States (USD) for the period between 1975:1 and 2012:3. For all currencies we model narrow real effective exchange rates as calculated by the Bank for International Settlements (Klau and Fung, 2006). The values of the analyzed series are presented in Figure 1.

The out-of-sample forecast performance is analyzed for horizons ranging from one up to sixty months ahead, whereas the evaluation is based on data from the period 1990:1 to 2012:3. In our baseline the models are estimated using rolling samples of 15 years ($R = 180$ months). The first set of forecasts is elaborated with the rolling sample 1975:1-1989:12 for the period 1990:1-1994:12. This procedure is repeated with the rolling samples ending in each month from the period 1990:2-2012:2. Since the data available end in 2012:3, the 1-month-ahead forecasts are evaluated on the basis of 267 observations, 2-month-ahead forecasts on the basis of 266 observations, and 60-month-ahead forecasts on the basis of 208 observations.

The forecasting performance is measured with two standard statistics: the mean squared forecast errors (MSFEs) and the correlation coefficient between forecast and realized RER changes. Table 1 and Figure 2 present the values of MSFEs. As is generally done in the forecasting literature, we report the actual MSFEs values for the RW model, while for the remaining models the numbers are expressed as ratios, so that values below unity indicate that a given model dominates the RW. We also test the null of equal forecast accuracy with the two-sided Diebold and Mariano (1995) test.

In terms of the MFSE criterion the two HL model-based forecasts are much better than the RW for seven out of nine currencies (EUR, MXN, NZD, CHF, GBP, USD, JPY). For example in the case of HL5, we find that the MSFEs are on average 9% and 23% lower than that of the RW model at the two and five-year horizon, respectively. Both H3 and H5 model-based forecasts are also considerably more precise than those based on the AR model for the following five currencies (CAD, EUR, JPY, GBP and USD) while the outcomes are broadly comparable for the other four currencies. Particularly interesting is that HL models are generally able to outperform other models, including the RW, even at short-term horizons: forecasts from the HL models are more accurate than those from the RW model in the case of EUR, MXN and NZD and are broadly comparable for the other currencies. For example, for the HL5 model at the one-year horizon the MSFEs are on average lower by 3% and 12% than those from the RW and AR models, respectively. Finally, at short-term horizons the AR model performs much worse than the RW model.

Further evidence that the HL models beat the alternatives can be found using our second criterion, which consists in computing the correlation coefficient between the realized and forecast changes of RERs:

$$r_{M,h} = cor(y^M_{T+h|T} - y_T, y_{T+h} - y_T), \tag{6}$$

where M stands for the model name. Note that (3) and (4) imply that $r_{RW,h}$ is zero and $r_{HL,h}$ does not depend on the duration of the half-life: for that reason in Table 2 we report only the results for two models, a common HL and the AR model. The table shows that the correlation coefficients for the HL model are generally positive for all currencies at all horizons, except for the AUD. The average value of $r_{HL,h}$ also increases with the forecast horizon: from just 0.04 for the one-month ahead forecasts to 0.53 for the five-year ahead forecasts. In the case of the AR model the results are less supportive: MXN is the only currency with a positive $r_{AR,h}$ throughout the forecast horizon. Moreover, the average value of $r_{AR,h}$ is positive only for horizons above two years. Finally, at all horizons $r_{AR,h}$ is visibly lower than $r_{HL,h}$.

To sum up, the evidence suggests that RERs of major currencies are mean reverting and forecastable, as shown by the good performance of the HL models. What is puzzling is the poor performance of the estimated AR model. We provide an interpretation of this result in the next section.

4 Analytical interpretation of the results

Here we show analytically that the finite sample determines a sizable estimation error, which tends to distort the results in favor of the RW model even when the estimation windows cover several years of monthly data. Let us assume that the DGP for y_t is given by (1) so that the unbiased and efficient forecast is:

$$y_{T+h|T} = \mu + \rho^h (y_T - \mu), \tag{7}$$

and the variance of the forecast error:

$$E\{(y_{T+h} - y_{T+h|T})^2\} = \sigma^2 \frac{1 - \rho^{2h}}{1 - \rho^2}. \tag{8}$$

The only source of forecast errors is the existence of the random term, whose future values are unknown. In the case of forecasts given by equations (3)-(5) the variance of forecast errors is higher than that in (8) because the coefficients μ and ρ are unknown and have to be estimated or calibrated.

Let us decompose the variance of the forecast error from a generic model $M \in \{RW, HL, AR\}$ into three components:

$$\begin{aligned} E\{(y_{T+h} - y^M_{T+h|T})^2\} = E\{(y_{T+h} - y_{T+h|T})^2\} + E\{(y_{T+h|T} - y^M_{T+h|T})^2\} + \\ + 2E\{(y_{T+h} - y_{T+h|T})(y_{T+h|T} - y^M_{T+h|T})\}. \end{aligned} \tag{9}$$

The value of the first component, which is given by (8), represents the random error common for all models. The second component captures the error that originates from having estimated or calibrated the model. Finally, since we cannot forecast future shocks, the third component is zero and can be disregarded. In what follows we provide the analytical expressions for the second component, which determines the relative performance of our competing models.

In the case of the RW model the error equals:

$$y_{T+h|T} - y^{RW}_{T+h|T} = (\rho^h - 1)(y_T - \mu) \tag{10}$$

and thus:

$$E\{(y_{T+h|T} - y^{RW}_{T+h|T})^2\} = (\rho^h - 1)^2 \times E\{(y_T - \mu)^2\}, \tag{11}$$

where:

$$E\{(y_T-\mu)^2\}=\frac{\sigma^2}{1-\rho^2}.$$

For the HL model, the combination of (4) and (7) yields:

$$y_{T+h|T}-y^{HL}_{T+h|T}=(\rho^h-\bar{\rho}^h)(y_T-\mu)-(1-\bar{\rho}^h)(\bar{\mu}-\mu). \tag{12}$$

The first term describes the forecast error caused by the wrong calibration of parameter ρ and the second one is the forecast error related to the estimation of μ. The resulting variance is:

$$\begin{aligned} E\{(y_{T+h|T}-y^{HL}_{T+h|T})^2\}&=(\rho^h-\bar{\rho}^h)^2\times E\{(y_T-\mu)^2\}+(1-\bar{\rho}^h)^2\times\\ &\times E\{(\bar{\mu}-\mu)^2\}-2(\rho^h-\bar{\rho}^h)(1-\bar{\rho}^h)\times E\{(y_T-\mu)(\bar{\mu}-\mu)\}, \end{aligned} \tag{13}$$

where:

$$E\{(\bar{\mu}-\mu)^2\}=\frac{\sigma^2}{1-\rho^2}\times\frac{1}{R^2}\times(R+2\sum_{j=1}^{R-1}(R-j)\rho^j)$$

$$E\{(y_T-\mu)(\bar{\mu}-\mu)\}=\frac{\sigma^2}{1-\rho^2}\times\frac{1}{R}\times\frac{1-\rho^R}{1-\rho}.$$

Finally, as derived in Fuller and Hasza (1980), for the AR model the value of the second component is approximately:

$$E\{(y_{T+h|T}-y^{AR}_{T+h|T})^2\}\simeq\sigma^2\times\frac{1}{R}\times\left[h^2\rho^{2(h-1)}+\left(\frac{1-\rho^h}{1-\rho}\right)^2\right]. \tag{14}$$

Given (8)-(14), the assumptions for the DGP coefficients (μ, ρ and σ) and the sample size (R), one can calculate the theoretical value of MSFE for all competing models (RW, HL and AR) at different forecast horizons ($h=1,2,\dots,H$). It is worth noting that the theoretical MSFEs of all models do not depend on the value of μ and are proportional to the value of σ^2. Consequently, the relative MSFEs depend solely on the convergence coefficient ρ, the sample size R and forecast horizon h.

In what follows we consider values of ρ corresponding to a half-life varying from one to ten years, the sample size of 180 monthly observations and forecast horizons up to 60 months ahead. These values correspond to the empirical analysis described in Section 3. The results are presented in Figure 3, where the theoretical MSFEs of a given model are shown as a ratio of the MSFEs of the RW model.

The analytical results, which we cross-checked with Monte Carlo simulations, suggest that the HL3 and HL5 model-based forecasts are more accurate than those from RW models as long as the half-life of DGP is below 10 years. Moreover, they are also more accurate than those from the AR model if the DGP half-life is above one year. Finally in terms of point forecast accuracy, the AR model outperforms the RW model only for relatively low values of the DGP half-life, i.e. not exceeding three years. The estimation error of the AR model is especially severe for more persistent processes.

The above results tell the following story: even if the true DGP is a simple autoregression with the duration of the half-life between three and five years, an estimated AR model will not usually forecast RERs better than the RW model. This result is explained by the estimation forecast error of the AR model, which outweighs the accuracy loss of choosing the mis-specified RW model. A simple remedy is to employ a reasonably calibrated HL model, which assumes a gradual mean reversion to the sample mean. Our insight is therefore that the estimation error is so important that the HL model remains competitive even when the calibrated parameter is not chosen in a very precise way.

5 Sensitivity analysis

The baseline empirical results discussed in the previous sections refer to out-of-sample forecasts for nine currencies generated using rolling regressions with a window of 15 years. In this section we evaluate whether the baseline results are robust to changes in these forecasting settings. We begin by analyzing whether a change in the length of the rolling window has an impact on our findings. We then investigate how accurate the forecasts generated from an AR model would be when eliciting a prior on the half-life parameter ρ. Third, we check whether the results are valid for other currencies.

Rolling window length

A change in length of the rolling window might affect the accuracy of RER forecasts generated by the HL and AR models in two directions. If the RER is generated by a mean-reverting process of form (1), then the lengthening of the rolling window should increase the accuracy of the HL and AR models, as implied by (13) and (14). However, if we relax the assumption that the equilibrium value of the RER is time-invariant, the lengthening of the rolling window could imply that the sample average of the RER approximates poorly the equilibrium exchange rate level (see Rossi, 2006, for a discussion on the importance of parameter instability).

The results for 5, 10 and 20 year rolling windows are presented in tables 3, 4 and 5, respectively. With a 5 year rolling window we find that the RW becomes more competitive, even if it is still outperformed by the HL models for 5 out of 9 currencies. The AR model instead generates inaccurate forecasts, which are much worse than in the baseline. With a 10 year rolling window, the results are broadly similar to those of the baseline. Finally with a 20 year rolling window, the HL models outperform the RW model for 8 out of 9 currencies at most horizons. We also find that the accuracy of the AR model tends to increase with the length of the rolling window. These results confirm that the estimation error is the main source of the weak performance of the AR model.

Prior on the half-life parameter

Setting the half-life parameter ρ as prior information could potentially improve the accuracy of the forecasts relative to the calibrated version of the model. To this aim we consider a Bayesian autoregressive model (BAR) to forecast RERs, along the line suggested by Kilian and Zha (2002). As regards the prior, we use the standard Minnesota setting commonly for vector autoregressions. In particular, we write down the model (1) in the standard AR form:

$$y_t = \delta + \rho y_{t-1} + \epsilon_t, \tag{15}$$

where $\delta = (1 - \rho)\mu$. The prior for $\alpha = [\delta\ \rho]'$ is assumed to be $\mathcal{N}(\underline{\alpha}, \underline{V})$ with $\underline{\alpha} = [(1 - \bar{\rho})\bar{\mu}\ \bar{\rho}]'$ and $\underline{V} = diag(\lambda\sigma^2, \lambda)$, where σ is the residual standard error from the AR model, $\bar{\rho}$ is the mean-reversion parameter calibrated so that the half-life is five years and λ is the overall tightness hyperparameter. The expected value of the posterior is:

$$\overline{\alpha} = \left(\underline{V}^{-1}\underline{\alpha} + \sigma^{-2}X'X\hat{\alpha}\right),$$

where $\hat{\alpha}$ is the OLS estimate of (15), X is the observation matrix and $\overline{V} = (\underline{V}^{-1} + \sigma^{-2}X'X)$ (see Robertson and Tallman, 1999, for details). Let us note that for $\lambda = 0$ the value of $\overline{\alpha}$ equals to $\underline{\alpha}$ (HL5 model) and for $\lambda \to \infty$ to $\hat{\alpha}$ (AR model).

The ratios of the MSFEs from BAR models with λ equal to 0, 0.001, .1 and ∞ relative to those from the RW are reported in Table 6. The ratios for λ set to 0 and ∞ in Table 6 are exactly the same as the ratios for HL5 and AR models in Table 1, respectively. With values of λ between 0.001 and 0.1, the ratios of the MSFEs from the BAR model are within those of the HL5 and AR models for almost all currencies and horizons, suggesting that the relationship between MSFE and λ is monotonic. As a result, in most cases the corner solution of calibrating, rather than eliciting as prior, the half-life parameter ρ turns out to be the better solution from a forecasting perspective.

For the one-month horizon we also provide a graphical illustration of what we have just said. Figure 4 presents the relationship between the MSFE and λ , where the values of MSFE are normalized so that MSFE is equal to 100 for $\lambda = 0$. It can be seen that for six currencies (EUR, JPY, NZD, CHF, GBP, USD) the relationship is monotonic and for MXN 'bell-shaped'. Only for AUD and CAD the relationship is 'U-shaped', which means that there could be some gains from using the BAR model. Given the above results, we draw the general conclusion that the calibrated half-life model forecasts RERs more accurately than the BAR model. Only for a few currencies and specific ranges of λ there might be potential additional gains from using a Bayesian autoregressive model.

Other currencies

The last question that we address in this paper is whether the baseline results are also valid for other currencies. We therefore consider the full set of real effective exchange rates indices available in the Bank for International Settlements database. The additional sample consists of eighteen currencies for the following countries: Austria (ATS), Belgium (BEF), Taiwan (TWD), Denmark (DKK), Finland (FIM), France (FRF), Germany (DEM), Greece (GRD), Hong Kong (HKG), Ireland (IEP), Italy (ITL), the South Korea (KRW), the Netherlands (NLG), Norway (NOK), Portugal (PTE), Singapore (SGD), Spain (ESP) and Sweden (SEK). The results are reported in Table 7 and lead to similar conclusions to those reached earlier. In comparison to the RW model the forecasts based on the two HL models are more accurate for 9 of the 18 currencies, comparable for 6 and less accurate for 3. The HL models deliver more precise forecasts than the AR model for most currencies. To summarize, the implications from the above robustness exercise provide further support to our findings.

6 Conclusions

The consensus view in the literature is that exchange rates are not predictable. By choosing a different field of forecasting competition, i.e. real rather than nominal exchange rates, we show that a calibrated half-life model is remarkably successful in forecasting RERs. In particular, it overwhelmingly beats the random walk at both short and long horizons. We also find that it is generally preferable to calibrate, rather than to estimate or elicit as a prior, the parameter determining the speed of adjustment to PPP. Our results are robust with respect to the forecasting scheme setting and the choice of currency.

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Table 1: Mean Squared Forecast Errors (15Y rolling window)

h	RW	HL3	HL5	AR1	RW	HL3	HL5	AR1	RW	HL3	HL5	AR1
	AUD				CAD				EUR			
1	0.05	1.03	1.01	1.02*	0.02	1.04	1.02	1.03*	0.02	1.00	1.00	1.04**
6	0.44	1.07	1.03	1.06*	0.23	1.10	1.03	1.11**	0.19	0.96	0.96	1.12*
12	0.82	1.13	1.06	1.09*	0.46	1.17	1.06	1.20**	0.43	0.90	0.92*	1.14*
24	1.53	1.22**	1.10*	1.09	0.94	1.24*	1.10	1.19*	0.89	0.80*	0.83**	1.18**
36	2.10	1.25*	1.12	1.06	1.58	1.18	1.06	1.20*	1.28	0.72**	0.77**	1.13*
60	3.00	1.22	1.11	1.06	3.02	0.99	0.94	1.45**	2.06	0.58**	0.66**	0.91
	JPY				MXN				NZD			
1	0.06	1.01	1.00	1.01	0.12	0.99	0.99	0.99	0.03	1.00	1.00	1.03*
6	0.59	0.99	0.98	1.04	0.83	0.94	0.96*	0.92	0.32	0.96	0.96	1.06
12	1.00	1.00	0.97	1.10*	1.55	0.89*	0.92*	0.87*	0.72	0.90*	0.92*	1.03
24	2.34	0.92	0.91	1.17**	3.01	0.79**	0.84**	0.78*	1.59	0.77**	0.83**	0.89*
36	3.53	0.86	0.86	1.19**	3.66	0.72**	0.78**	0.74**	2.44	0.65**	0.74**	0.74**
60	3.12	0.97	0.89	1.19*	3.56	0.73*	0.74**	0.72*	3.01	0.55**	0.64**	0.62**
	CHF				GBP				USD			
1	0.02	1.00	1.00	1.06	0.03	1.00	1.00	1.02	0.02	1.00	1.00	1.03*
6	0.12	0.98	0.98	1.22*	0.24	0.97	0.97	1.04	0.19	0.96	0.96	1.10*
12	0.25	0.97	0.97	1.16	0.47	0.95	0.95	1.03	0.31	0.94	0.93	1.21**
24	0.50	0.84*	0.88*	0.99	1.06	0.85*	0.87*	0.98	0.55	0.84	0.84*	1.21*
36	0.72	0.73**	0.80**	0.78**	1.49	0.77**	0.82**	0.93	0.69	0.71*	0.72**	1.13
60	0.79	0.65**	0.72**	0.69**	1.98	0.59**	0.67**	0.70**	1.41	0.45**	0.53**	0.91

Notes: For the RW model MSFEs are reported in levels (multiplied by 100), whereas for the remaining methods they appear as the ratios to the corresponding MSFE from the RW model. Asterisks ** and * denote the rejection of the null of the Diebold and Mariano (1995) test, stating that the MSFE from RW are not significantly different from the MSFE of a given model, at 1%, 5% significance level, respectively.

Table 2: Correlation of forecast and realized changes of real exchange rates

h	AUD	CAD	EUR	JPY	MXN	NZD	CHF	GBP	USD	av.
	Half-life models									
1	-0.04	-0.03	0.06	0.04	0.11	0.06	0.08	0.06	0.07	0.04
6	-0.01	0.02	0.21	0.15	0.30	0.23	0.21	0.19	0.22	0.17
12	-0.05	0.01	0.32	0.18	0.42	0.36	0.25	0.26	0.29	0.23
24	-0.10	0.02	0.46	0.31	0.60	0.56	0.50	0.43	0.43	0.36
36	-0.12	0.11	0.55	0.38	0.66	0.73	0.66	0.53	0.56	0.45
60	-0.20	0.26	0.71	0.31	0.61	0.81	0.74	0.71	0.78	0.53
	AR model									
1	-0.04	-0.09	-0.19	-0.04	0.13	-0.07	-0.13	-0.03	-0.09	-0.06
6	-0.07	-0.22	-0.29	-0.10	0.33	-0.05	-0.21	0.02	-0.17	-0.08
12	-0.06	-0.31	-0.24	-0.22	0.44	0.05	-0.04	0.08	-0.24	-0.06
24	0.00	-0.14	-0.22	-0.33	0.60	0.35	0.22	0.19	-0.13	0.06
36	0.07	-0.09	-0.06	-0.32	0.64	0.61	0.52	0.26	0.05	0.19
60	0.10	-0.43	0.32	0.04	0.63	0.78	0.64	0.58	0.34	0.33

Table 3: Mean Squared Forecast Errors (5Y rolling window)

h	RW	HL3	HL5	AR1	RW	HL3	HL5	AR1	RW	HL3	HL5	AR1
	AUD				CAD				EUR			
1	0.05	1.01	1.00	1.05**	0.02	1.03*	1.02*	1.05	0.02	1.02	1.01	1.06**
6	0.44	1.00	0.99	1.15*	0.23	1.07	1.04	1.10	0.19	1.03	1.01	1.19*
12	0.82	0.99	0.98	1.10	0.46	1.13*	1.08	1.12	0.43	1.01	1.00	1.25*
24	1.53	0.97	0.96	1.09	0.94	1.22**	1.15**	1.53*	0.89	1.00	0.99	1.72*
36	2.10	0.96	0.95	1.20	1.58	1.23**	1.16**	2.25*	1.28	1.01	0.99	2.75*
60	3.00	1.06	1.02	1.63	3.02	1.22**	1.17*	6.03*	2.06	0.97	0.96	4.45
	JPY				MXN				NZD			
1	0.06	1.00	1.00	1.06**	0.12	1.00	1.00	1.09*	0.03	1.02	1.01	1.10**
6	0.59	0.97	0.97	1.16**	0.83	0.98	0.98	1.23*	0.32	1.02	1.01	1.35**
12	1.00	0.95	0.95	1.30**	1.55	0.96	0.96	1.42*	0.72	1.00	0.98	1.57**
24	2.34	0.84*	0.86**	1.59**	3.01	0.88	0.89	2.21	1.59	0.88	0.89*	2.32**
36	3.53	0.75**	0.78**	1.67*	3.66	0.79*	0.80**	4.78	2.44	0.73**	0.76**	3.47**
60	3.12	0.81	0.82*	2.78*	3.56	0.51**	0.54**	71.96	3.01	0.53**	0.58**	11.26
	CHF				GBP				USD			
1	0.02	1.01	1.00	1.07	0.03	1.01	1.01	1.12*	0.02	1.01	1.00	1.03
6	0.12	1.02	1.01	1.36	0.24	1.01	1.00	1.68	0.19	1.00	1.00	1.06
12	0.25	1.04	1.02	1.29**	0.47	1.00	0.99	3.06	0.31	1.04	1.02	1.16
24	0.50	0.95	0.95	1.44**	1.06	0.93	0.93	22.94	0.55	1.11	1.07	1.29
36	0.72	0.83**	0.85**	1.31**	1.49	0.89	0.90*	346.58	0.69	1.21	1.15	1.47*
60	0.79	0.69**	0.72**	1.37	1.98	0.80**	0.81**	4.12	1.41	1.07*	1.04*	3.21*

Notes: As in Table 1.

Table 4: Mean Squared Forecast Errors (10Y rolling window)

h	RW	HL3	HL5	AR1	RW	HL3	HL5	AR1	RW	HL3	HL5	AR1
	AUD				CAD				EUR			
1	0.05	1.01	1.00	1.05	0.02	1.05*	1.03	1.02	0.02	1.01	1.01	1.04*
6	0.44	1.00	0.99	1.15	0.23	1.12*	1.07	1.10*	0.19	1.01	0.99	1.14**
12	0.82	0.99	0.98	1.10	0.46	1.21*	1.14*	1.21	0.43	0.98	0.96	1.24**
24	1.53	0.97*	0.96*	1.09	0.94	1.34**	1.23**	1.24**	0.89	0.93	0.92	1.55**
36	2.10	0.96*	0.95	1.20	1.58	1.31**	1.21**	1.32*	1.28	0.89	0.88	1.96**
60	3.00	1.06	1.02	1.63	3.02	1.15	1.09	1.26*	2.06	0.76*	0.77*	2.34*
	JPY				MXN				NZD			
1	0.06	1.00	1.00	1.02	0.12	0.99	0.99	1.02	0.03	1.00	1.00	1.00
6	0.59	0.99	0.98	1.03	0.83	0.94	0.95	1.06	0.32	0.96	0.96	0.99
12	1.00	1.00	0.99	1.08*	1.55	0.89*	0.90**	1.15	0.72	0.91	0.92**	0.97
24	2.34	0.93	0.92	1.10	3.01	0.77*	0.81**	1.26*	1.59	0.81**	0.84**	0.92
36	3.53	0.87	0.87	1.08	3.66	0.69**	0.73**	1.10	2.44	0.71**	0.75**	0.96
60	3.12	0.98	0.95	1.14	3.56	0.67**	0.68**	0.89	3.01	0.60**	0.64**	1.26
	CHF				GBP				USD			
1	0.02	1.00	1.00	1.05	0.03	1.00	1.00	1.05	0.02	1.02	1.01	1.04**
6	0.12	0.99	0.99	1.24	0.24	0.98	0.98	1.25	0.19	1.03	1.01	1.17**
12	0.25	1.00	0.99	1.09	0.47	0.99	0.98	1.52	0.31	1.09	1.04	1.38**
24	0.50	0.91	0.92	1.02	1.06	0.93	0.93	3.35	0.55	1.12	1.05	1.55**
36	0.72	0.82**	0.84**	0.88**	1.49	0.91	0.91	13.45	0.69	1.12	1.03	1.78**
60	0.79	0.74**	0.76**	0.86	1.98	0.76**	0.78**	0.82*	1.41	0.76*	0.75*	1.87**

Notes: As in Table 1.

Table 5: Mean Squared Forecast Errors (20Y rolling window)

h	RW	HL3	HL5	AR1	RW	HL3	HL5	AR1	RW	HL3	HL5	AR1
	AUD				CAD				EUR			
1	0.05	1.01*	1.00	1.05	0.02	1.04*	1.02	1.01	0.02	1.00	1.00	1.00
6	0.44	1.00	0.99	1.15	0.23	1.09	1.05	1.05*	0.19	0.95	0.95	0.98
12	0.82	0.99*	0.98*	1.10*	0.46	1.16	1.09	1.11**	0.43	0.89*	0.90*	0.94
24	1.53	0.97**	0.96*	1.09**	0.94	1.24*	1.15*	1.17**	0.89	0.80*	0.82**	0.90*
36	2.10	0.96**	0.95*	1.20*	1.58	1.19	1.11	1.18**	1.28	0.74**	0.77**	0.86**
60	3.00	1.06*	1.02	1.63	3.02	1.02	0.98	1.35**	2.06	0.64**	0.67**	0.75**
	JPY				MXN				NZD			
1	0.06	1.01	1.00	1.01	0.12	0.99	0.99	1.00	0.03	1.00	1.00	1.01
6	0.59	1.00	0.99	1.05*	0.59	0.93*	0.94 *	0.94	0.32	0.96	0.97	0.99
12	1.00	1.04	1.00	1.12**	1.00	0.86**	0.89**	0.87	0.72	0.91	0.92*	0.94
24	2.34	0.97	0.94	1.17**	2.34	0.75**	0.79**	0.72**	1.59	0.79 **	0.82**	0.82**
36	3.53	0.91	0.89	1.16**	3.53	0.68**	0.72**	0.67**	2.44	0.67**	0.72**	0.69**
60	3.12	0.97	0.90	1.13	3.12	0.67 *	0.68 **	0.69*	3.01	0.54**	0.59**	0.55**
	CHF				GBP				USD			
1	0.02	1.00	1.00	1.02	0.03	1.00	0.99	1.00	0.02	1.00	1.00	0.99
6	0.12	0.98	0.98	1.06	0.24	0.95	0.96	0.96	0.19	0.96	0.96	0.96*
12	0.25	0.97	0.97	1.02	0.47	0.92	0.93	0.91	0.31	0.94	0.94	0.92**
24	0.50	0.88*	0.89*	0.87	1.06	0.82*	0.84**	0.85**	0.55	0.87	0.86*	0.87**
36	0.72	0.80**	0.82**	0.74**	1.49	0.76**	0.79**	0.77**	0.69	0.79	0.78*	0.86**
60	0.79	0.77 *	0.78*	0.66**	1.98	0.60**	0.65**	0.62**	1.41	0.56**	0.59**	0.82**

Notes: As in Table 1.

Table 6: Mean Squared Forecast Errors (MSFEs) – BAR model

h	HL5	BAR		AR1	HL5	BAR		AR1	HL5	BAR		AR1
λ	0	.001	.1	∞	0	.001	.1	∞	0	.001	.1	∞
	AUD				CAD				EUR			
1	1.01	1.01	1.02*	1.02*	1.02	1.02*	1.03*	1.03*	1.00	1.03**	1.04**	1.04**
6	1.03	1.04	1.05*	1.06*	1.03	1.09**	1.11**	1.11**	0.96**	1.10*	1.12*	1.12*
12	1.06	1.06	1.09*	1.09*	1.06	1.17**	1.19**	1.20**	0.92*	1.12*	1.14*	1.14*
24	1.10*	1.06	1.09	1.09	1.10	1.17*	1.19*	1.19*	0.83**	1.14	1.17*	1.18*
36	1.12	1.04	1.06	1.06	1.06	1.17*	1.20*	1.20*	0.77**	1.08	1.12*	1.13*
60	1.11	1.05	1.06	1.06	0.94	1.39**	1.44**	1.45**	0.66**	0.87*	0.91	0.91
	JPY				MXN				NZD			
1	1.00	1.01	1.01	1.01	0.99	0.99	0.99	0.99	1.00	1.02*	1.03*	1.03*
6	0.98	1.03	1.04	1.04	0.96*	0.94	0.92	0.92	0.96	1.04	1.05	1.06
12	0.97	1.09*	1.10*	1.10*	0.92*	0.89	0.87*	0.87*	0.92*	1.02	1.03	1.03
24	0.91	1.13*	1.16**	1.17**	0.84**	0.82*	0.78*	0.78*	0.83**	0.90**	0.89**	0.89**
36	0.86	1.14*	1.19**	1.19**	0.78**	0.79**	0.74**	0.74**	0.74**	0.76**	0.75**	0.74**
60	0.89	1.13	1.19*	1.19*	0.74**	0.75*	0.72*	0.72*	0.64**	0.64**	0.62**	0.62**
	CHF				GBP				USD			
1	1.00	1.04	1.05	1.06	1.00	1.02	1.02	1.02	1.00	1.02*	1.03*	1.03*
6	0.98	1.14*	1.21*	1.22*	0.97	1.03	1.04	1.04	0.96	1.09*	1.10*	1.10*
12	0.97	1.12	1.15	1.16	0.95	1.03	1.03	1.03	0.93	1.19*	1.21*	1.21*
24	0.88*	0.98	0.99	0.99	0.87*	0.98	0.98	0.98	0.84*	1.19*	1.21*	1.21*
36	0.80**	0.81**	0.79**	0.78**	0.82**	0.94	0.93	0.93	0.72**	1.10	1.12	1.13
60	0.72**	0.72**	0.70**	0.69**	0.67**	0.73**	0.70**	0.70**	0.53**	0.88*	0.91	0.91

Notes: For all models MSFEs are reported in the ratios to the RW model. Asterisks ** and * denote the rejection of the null of the Diebold and Mariano (1995) test, stating that the MSFE from RW are not significantly different from the MSFE of a given model, at 1%. 5% significance level, respectively.

Table 7: Mean Squared Forecast Errors for other currencies

h	RW	HL3	HL5	AR1	RW	HL3	HL5	AR1	RW	HL3	HL5	AR1
	ATS				BEF				TWD			
1	0.05	1.01	1.00	1.05	0.00	1.00	1.00	1.04	0.02	1.02	1.01	1.01
6	0.44	1.00	0.99	1.15	0.03	0.97	0.96	1.09	0.13	1.12*	1.07	1.06
12	0.82	0.99	0.98	1.10	0.07	0.92	0.92	1.07	0.25	1.25**	1.15*	1.13
24	1.53	0.97	0.96	1.09	0.14	0.81*	0.83*	1.00	0.43	1.58**	1.39**	1.34**
36	2.10	0.96	0.95	1.20	0.21	0.71**	0.74**	0.94	0.56	1.95**	1.69**	1.49**
60	3.00	1.06*	1.02*	1.63	0.38	0.55**	0.60**	0.64**	1.05	2.06**	1.83**	1.57**
	DKK				FIM				FRF			
1	0.00	1.00	1.00	1.02	0.01	1.02	1.01	1.01	0.00	1.00	1.00	1.02
6	0.04	0.98	0.97	1.07	0.15	1.01	0.99	1.03	0.03	0.97	0.97	1.06
12	0.08	0.95	0.94	1.10	0.40	0.96	0.94	1.06**	0.08	0.92	0.92	1.05
24	0.12	0.96	0.94	1.19*	1.07	0.84	0.85*	1.12**	0.15	0.81*	0.83**	1.10
36	0.14	1.12	1.06	1.26*	1.34	0.83	0.83	1.19**	0.21	0.73**	0.76**	1.11
60	0.18	1.32*	1.23*	1.43**	0.89	1.13	1.02	1.60**	0.33	0.65**	0.67**	0.89
	DEM				GRD				HKD			
1	0.01	0.99	0.99	1.03	0.02	1.01	1.01	1.03*	0.02	1.07	1.04	1.05*
6	0.06	0.95	0.95	1.12	0.05	1.26**	1.17**	1.35**	0.23	1.18	1.11	1.23**
12	0.13	0.87*	0.89*	1.11	0.08	1.62**	1.41**	1.76**	0.48	1.32*	1.20	1.50**
24	0.28	0.72**	0.76**	1.03	0.23	1.75**	1.52**	1.80**	1.23	1.30*	1.18	1.84**
36	0.41	0.60**	0.65**	0.91	0.41	1.75**	1.55**	1.66**	2.06	1.25*	1.14	2.13**
60	0.64	0.41**	0.47**	0.75**	0.77	1.83**	1.65**	1.60**	4.56	0.96	0.92	2.94**
	IEP				ITL				KRW			
1	0.01	1.02	1.01	1.03	0.02	1.00	1.00	1.04**	0.10	0.99	0.99	1.01
6	0.12	1.02	1.00	1.06	0.13	0.95	0.95	1.15**	0.82	0.93*	0.94*	1.02
12	0.26	1.02	0.99	1.13	0.30	0.87	0.88	1.26**	1.48	0.87**	0.89**	0.99
24	0.56	0.98	0.95	1.33**	0.63	0.71*	0.74**	1.35**	2.65	0.77**	0.81**	0.99
36	0.92	0.92	0.90	1.32	1.08	0.55**	0.61**	1.38**	3.00	0.74**	0.77**	1.17
60	1.33	0.93	0.91	0.94	1.09	0.41**	0.46**	1.65**	3.11	0.84	0.83	3.76
	NLG				NOK				PTE			
1	0.01	1.00	1.00	1.02	0.02	0.99	0.99	1.01	0.01	1.13*	1.08*	1.20**
6	0.05	0.98	0.97	1.05	0.12	0.95	0.96	1.00	0.05	1.45**	1.29**	1.75**
12	0.10	0.93	0.93	0.98	0.22	0.90*	0.92*	0.97	0.12	1.60**	1.39**	1.94**
24	0.20	0.82*	0.83**	0.86	0.25	0.94	0.93	1.07	0.23	2.07**	1.72**	1.94**
36	0.26	0.78*	0.79*	0.83	0.29	1.01	0.97	1.12	0.23	3.05**	2.44**	2.14**
60	0.40	0.73*	0.74**	0.72*	0.28	1.16	1.08	1.28*	0.38	3.84**	3.17**	2.19**
	SGD				ESP				SEK			
1	0.01	1.01	1.00	1.02*	0.01	1.02	1.01	1.04**	0.02	1.01	1.00	1.01
6	0.08	1.02	1.00	1.08*	0.06	1.02	0.99	1.17**	0.24	0.99	0.99	0.99
12	0.19	0.98	0.96	1.09	0.16	0.92	0.91	1.20**	0.52	0.98	0.97	0.96
24	0.52	0.82**	0.84**	1.04	0.37	0.74	0.76*	1.18*	0.95	0.99	0.97	0.96
36	0.85	0.69**	0.73**	0.97	0.55	0.57*	0.61**	1.03	1.09	1.18*	1.10	0.96
60	1.53	0.52**	0.57**	0.83**	0.84	0.48**	0.52**	0.82*	1.18	1.64**	1.48**	1.03

Notes: As in Table 1.

Figure 1: Real exchange rates (2010 = 100)

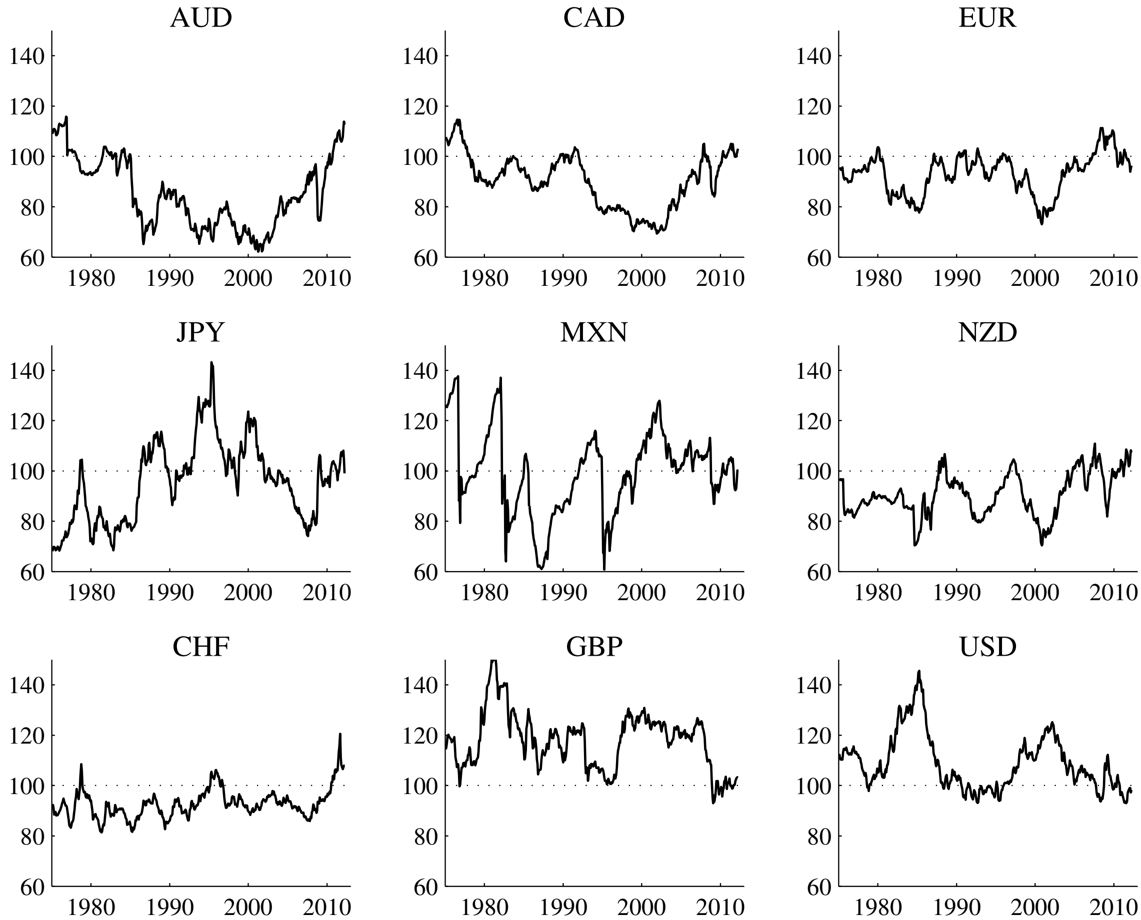
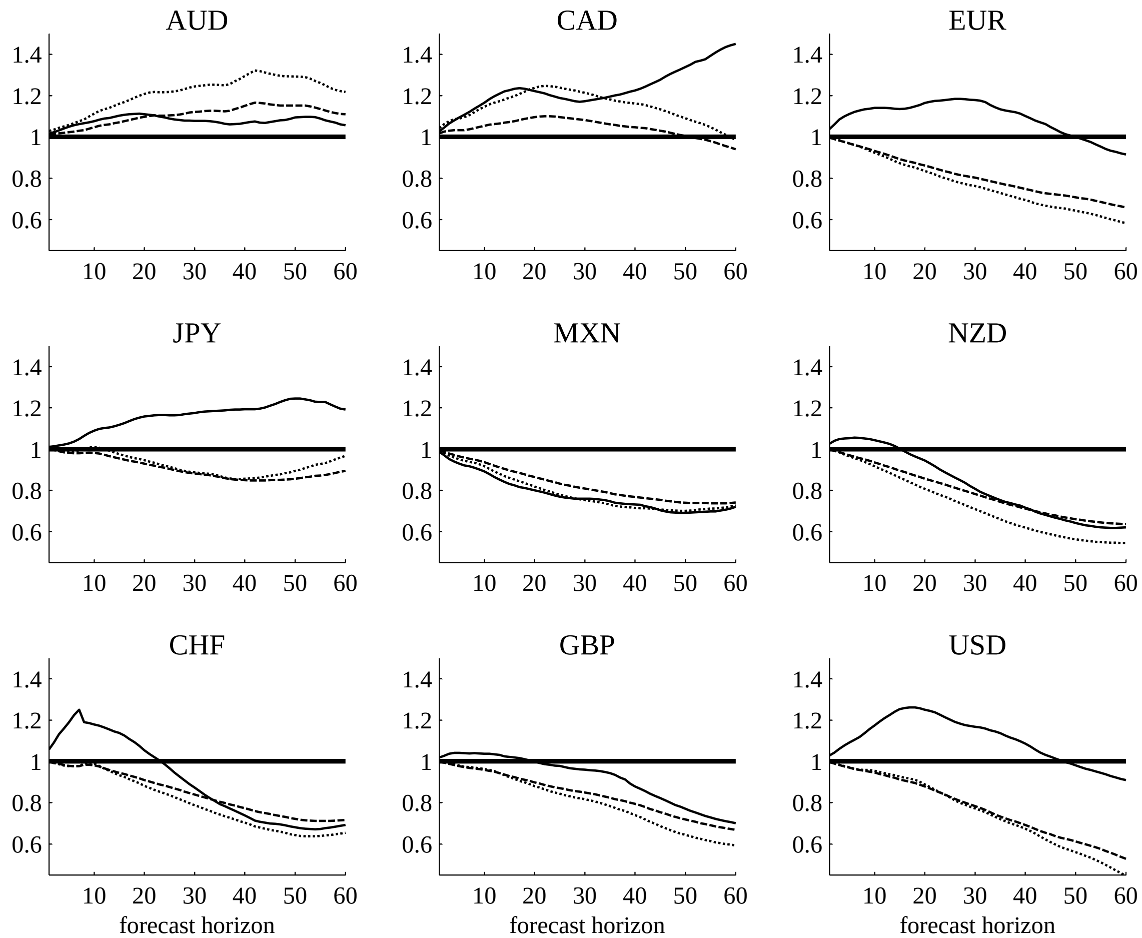
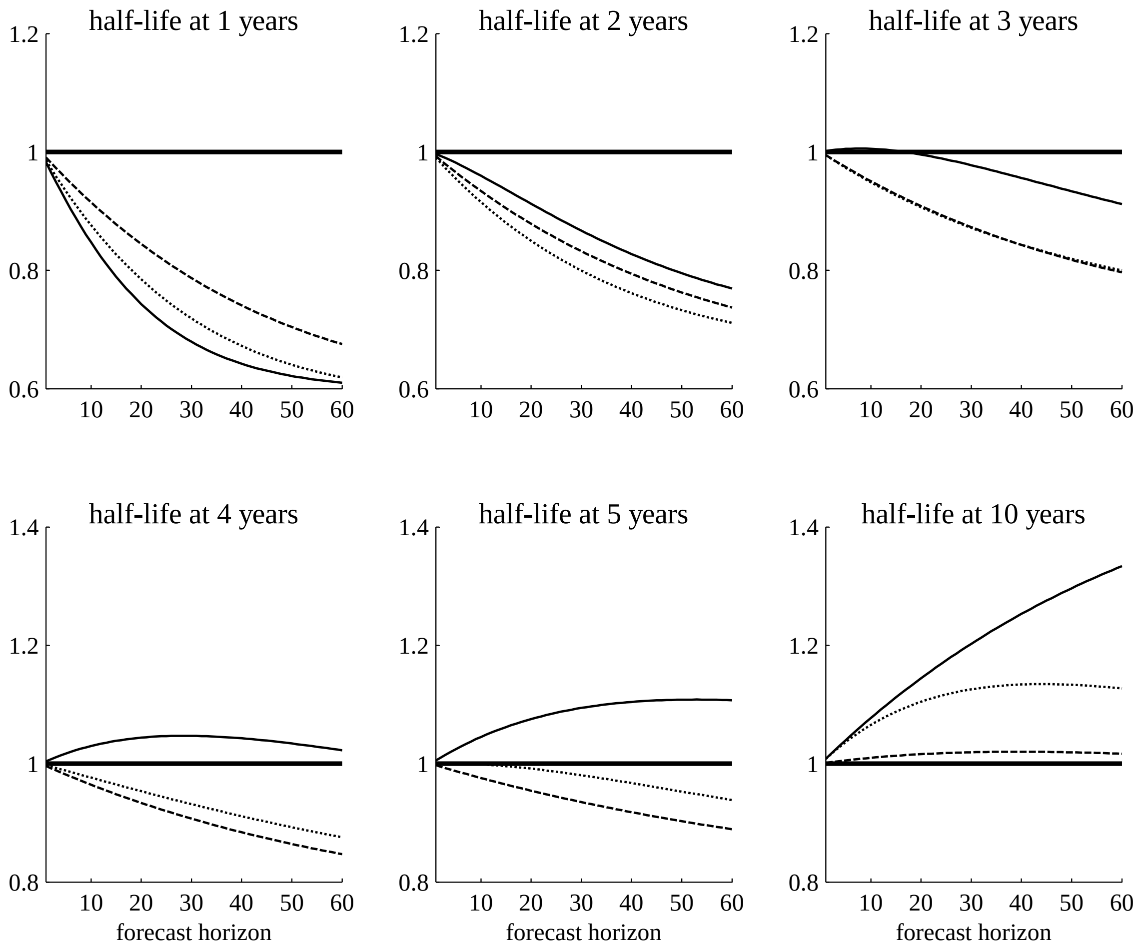


Figure 2: Mean Squared Forecast Errors



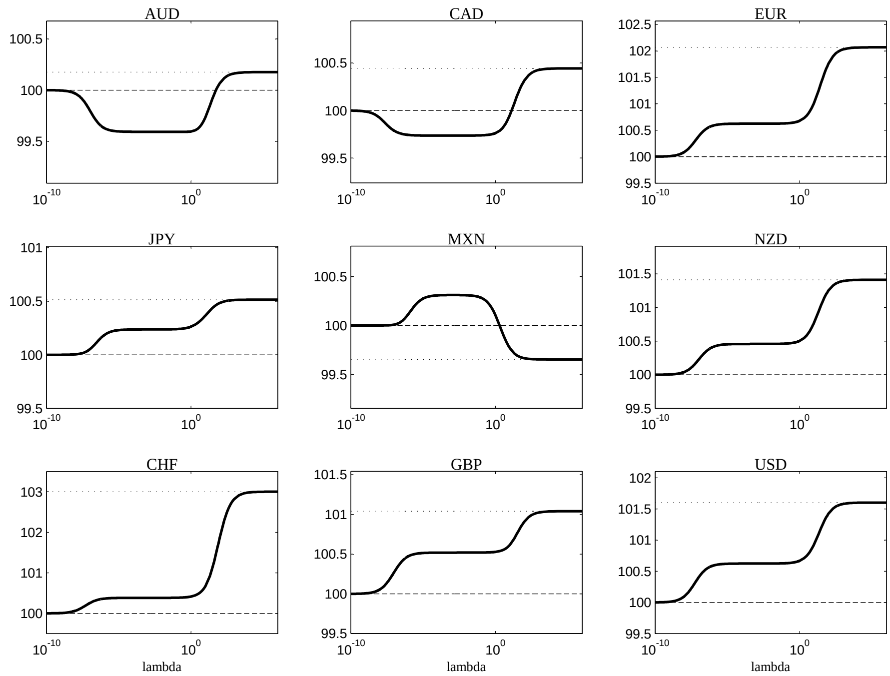
Notes: Each line represents the ratio of MSFE from a given method to MSFE from the random walk, where values below unity indicate better accuracy of point forecasts. The straight, dashed and dotted lines stand for AR, HL3 and HL5, respectively. The forecast horizon is expressed in months.

Figure 3: Theoretical Mean Squared Forecast Errors



Notes: Each line represents the ratio of MSFE from a given method to MSFE from the random walk, where values below unity indicate better accuracy of point forecasts. The straight, dashed and dotted lines stand for AR, HL3 and HL5, respectively. The forecast horizon is expressed in months.

Figure 4: Sensitivity analysis of MSFE on the λ (forecast horizon: 1 month)



Notes: Each line represents the ratio of MSFE from a given method to MSFE from the HL5 multiplied by 100, where values 100 unity indicate better accuracy of point forecasts. The straight, dashed and dotted lines stand for BAR, HL5 and AR1, respectively. The value of λ parameter is expressed using the logarithmic scale