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Exchange rate misalignment and external imbalances: what is the optimal monetary policy response?

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Abstract

How should monetary policy respond to excessive capital inflows that appreciate the currency and widen the external deficit? Using the workhorse two-country open-macro model, we derive a quadratic approximation of the utility-based global loss function in incomplete market economies, and solve for the optimal targeting rules under cooperation. The optimal monetary stance is expansionary if the exchange rate pass-through (ERPT) on import prices is complete, contractionary if nominal rigidities attenuate ERPT. Excessive capital inflows, however, may lead to currency undervaluation instead of overvaluation for some parameter values. The optimal stance is then invariably expansionary to support domestic demand.

Keywords: Currency misalignment, trade imbalances, asset markets and risk sharing, optimal targeting rules, international policy cooperation, exchange rate pass-through
JEL codes: E44, E52, E61, F41, F42

Non-technical summary

Cross-border capital flows raise widespread concerns about their potential adverse effects on domestic economies. Because of their impact on the exchange rate, domestic demand, and current account imbalances, inflows and outflows of capital may give rise to challenging policy trade-offs between internal objectives (inflation and output gap) and external objectives (competitiveness and trade). These concerns have generated a debate on the most appropriate tools for managing capital movements and their macroeconomic impact. They have also raised the need for a reconsideration of the role of monetary policy not just as a complement to other policy instruments (ranging from macroprudential policy to capital controls) but also as a first-line of defense in the absence of other readily implementable tools.

How should a central bank react to capital inflows that deteriorate the current account and appreciate the currency? One leading answer is that the natural rate still provides a reliable compass for monetary policy: to the extent that an external deficit raises the natural rate of interest, capital inflows should be systematically matched by a tighter monetary stance. However, this answer may not be appropriate in the presence of both financial market imperfections and nominal rigidities in price setting. In this context, recent literature has stressed that these frictions may result in capital flows that are inefficient and exchange rates that are misaligned (i.e., over/undervalued).[1] If a monetary contraction exacerbates currency overvaluation, for example, then the optimal response to a capital inflow may not even be a policy tightening. Is there a case for an expansionary monetary response that curbs the exchange rate overvaluation in such instances?

In this paper, we work out an analytically transparent characterization of the optimal monetary policy in the presence of inefficient capital flows, using the workhorse open economy monetary model—the two-country New Keynesian (NK) model under either complete or incomplete exchange rate pass-through (ERPT). As a standard and tractable way to introduce inefficient capital flows, we assume that the only internationally traded asset is an noncontingent bond. Thus, in the tradition of Obstfeld and Rogoff (1995), we capture the lack of efficient diversification in the data despite the number of seemingly available cross-border assets, by focusing on bond economies (see also Costinot et al. 2014). A notable property of the model for our purpose is that an inefficient capital inflow may cause currency over- or undervaluation, depending on some parameters values.

Our key result is that the optimal monetary policy response to inefficient capital flows depends on ERPT. In economies in which ERPT is complete–the case of producer currency pricing (PCP)–the optimal policy is invariably expansionary (contractionary) in response to an inflow (outflow). Under PCP, in the standard case when excessive inflows lead to currency overvaluation, losses in international price competitiveness result in expenditure switching effects redirecting global demand away from domestic goods and worsening the output gap—motivating

[1] In an economy with asset markets imperfections, cross-border financial flows in general deviate from their counterparts in the first best allocation, where complete asset markets insure idiosyncratic risk is fully insured, and are therefore "inefficient." In keeping with the literature, we will refer to capital flows in an economy where asset markets are incomplete, which deviate from the first best allocation, as “inefficient,” even if they result from optimal consumption smoothing by economic agents. By the same token, we will define exchange rate misalignment relative to the first best allocation. We note from the start that capital flows can be inefficient, and exchange rate misaligned, either because of asset markets frictions (in our model, incomplete asset markets), or because of nominal rigidities, or both.

policymakers to lean against an overvalued exchange rate even if this comes at the cost of some inflation. As a result, the optimal policy contains the magnitude of currency movements relative to the natural rate allocation (which in PCP economies could be achieved by implementing a strict policy of producer price stability). Real exchange rate movements are muted compared to those under the natural rate allocation. The optimal monetary stance remains expansionary also when capital inflows lead to excessive currency depreciation and hurt domestic consumers by reducing their purchasing power—a possibility specific to incomplete market economies, when purchasing power losses from adverse international price movements prevail over competitiveness gains (when parameter values result in sufficiently low price elasticities of trade and home bias in domestic demand). In this case, the expansionary monetary stance supports domestic demand, even if at the cost of exacerbating currency depreciation and feeding inflationary pressures. Moreover, real exchange rate movements are amplified compared to those in the natural rate allocation.

In economies where a low ERPT due to nominal rigidities in export and import prices mutes exchange rate expenditure switching effects on the output gap—a case dubbed local currency pricing (LCP) in the literature—the optimal monetary stance is always driven by the need to stabilize demand. In contrast with the PCP case, the optimal stance is contractionary when excessive inflows are associated with an overvalued currency and a domestic demand boom, leaning against the latter. The optimal stance is thus deflationary and exacerbates the currency misalignment. Under LCP, however, the sign of the optimal policy switches from contractionary to expansionary when inflows are associated with exchange rate undervaluation, as in PCP economies. In this case, monetary policy moves to support the inefficiently low domestic demand. As in PCP economies, this comes at the cost of further exchange rate depreciation and higher inflationary pressures. Overall, the optimal policy under LCP always amplifies real exchange rate volatility relative to a regime of strict CPI inflation targeting.

Moving forward, there are a number of directions of research. The interplay of domestic and cross-border financial frictions may strengthen the case for domestic stabilization at the cost of higher exchange rate movements under LCP. This would possibly be the case if a share of the residents in each country is excluded from financial markets, and thus operates under financial autarky. By the same token, allowing for gross foreign assets and liabilities would introduce valuation effects due to misalignment, on top and above the income effects of exchange rate movements stressed by our analysis (see e.g. Benigno 2009).

Strategic interactions among policymakers are another key issue. Inefficient capital flows have strong redistributive effects across borders. Cooperative policies attempt to redress these effects: in our analysis, when the optimal monetary policy at Home is either a contraction or an expansion, the Foreign monetary stance has the opposite sign. Without cooperation, however, these redistributive effects of capital inflows inherently create room for conflicts and strategic behavior.

Finally, in our analysis we abstract from the question of which export pricing strategy, PCP or LCP, is optimal from the vantage point of the firms, given the optimal policy. Thus, an important issue for future research is whether, in economic environments supporting the optimal choice of either PCP or LCP, the optimal stabilization rules would substantially deviate from the one we derive in this paper. Moreover, the evidence on the importance of pricing in vehicle (or dominant) currencies strongly motivates further work exploring the case of asymmetric pass-through. An important question is which direction monetary policy will take in the country which issues the dominant currency, when facing a capital inflow with currency overvaluation or undervaluation.

1 Introduction

Cross-border capital flows raise widespread concerns about their potential adverse effects on domestic economies. Because of their impact on the exchange rate, domestic demand, and current account imbalances, inflows and outflows of capital may give rise to challenging policy trade-offs between internal objectives (inflation and output gap) and external objectives (competitiveness and trade). These concerns have generated a debate on the most appropriate tools for managing capital movements and their macroeconomic impact. They have also raised the need for a reconsideration of the role of monetary policy not just as a complement to other policy instruments (ranging from macroprudential policy to capital controls) but also as a first-line of defense in the absence of other readily implementable tools.

How should a central bank react to capital inflows that deteriorate the current account imbalance and appreciate the currency? One leading answer is that the natural rate still provides a reliable compass for monetary policy: to the extent that an external deficit raises the natural rate of interest, capital inflows should be systematically matched by a tighter monetary stance (see, e.g., Obstfeld and Rogoff 2010).[2] However, this answer may not be appropriate in the presence of both financial market imperfections and nominal rigidities. In this context, as recently stressed by Farhi and Werning (2016), pecuniary and demand externalities result in capital flows that are inefficient and exchange rates that are misaligned (i.e., over/undervalued).[3] If a monetary contraction exacerbates misalignment, the optimal response to a capital inflow that overappreciates the currency may not even be a policy tightening. Is there a case for an expansionary monetary response that curbs the exchange rate overvaluation?

In this paper, we work out an analytically transparent characterization of the optimal monetary policy in the presence of inefficient capital flows, using the workhorse open economy monetary model—the two-country New Keynesian (NK) model under either complete or incomplete exchange rate pass-through (ERPT). As a standard and tractable way to introduce inefficient capital flows, we assume that the only internationally traded asset is an noncontingent bond (as in the seminal contribution by Obstfeld and Rogoff 1995; see also Costinot et al. 2014). A notable property of the model for our purpose is that inefficient capital inflows may cause currency over- or undervaluation, depending on parameters values.[4]

Our key result is that the optimal monetary policy response to inefficient capital inflows depends on ERPT. In economies in which ERPT is complete–the case of producer currency pricing (PCP)–the optimal policy is invariably expansionary. Under PCP, in the standard case

[2] "There is a case to be made that large current account deficits, other things equal, call for a tightening of monetary policy. Ferrero, Gertler, and Svensson (2008) present an example in which better macro performance comes from a monetary rule that recognizes how an external deficit raises the natural real rate of interest. The question deserves more research attention." Obstfeld and Rogoff (2010) p. 34. See also the recent discussion by Obstfeld (2020) stressing a similar point.

[3] In an incomplete market economy, cross-border financial flows in general deviate from their counterparts in the first best, where idiosyncratic risk is fully insured, and are therefore "inefficient." In keeping with the literature, we will refer to capital flows in a bond economy (or more generally in economies where markets are incomplete), which deviate from the first best allocation, as "inefficient," even if they result from optimal consumption smoothing by economic agents. By the same token, we will define exchange rate misalignment relative to the first best allocation. We note from the start that capital flows can be inefficient, and exchange rate misaligned, either because of financial frictions (in our model, incomplete asset markets), or because of nominal rigidities, or both.

[4] In the tradition of Obstfeld and Rogoff (1995), we capture the lack of efficient diversification in the data despite the number of seemingly available cross-border assets, by focusing on bond economies.

when excessive inflows lead to currency overvaluation, losses in international price competitiveness redirect global demand away from domestic goods and worsen the output gap—motivating policymakers to lean against an overvalued exchange rate even if this comes at the cost of positive inflation. As a result, the optimal policy contains the magnitude of currency movements relative to the natural rate allocation (which in PCP economies could be achieved by implementing a strict policy of price stability). Real exchange rate movements are muted compared to those under price stability. The optimal monetary stance remains expansionary also when capital inflows lead to excessive depreciation and hurt domestic consumers—a possibility specific to incomplete market economies, when a sufficiently low trade elasticities and home bias in demand cause the wealth effects from equilibrium international price movements to prevail over substitution effects. In this case, the expansionary monetary stance supports domestic demand, even if at the cost of exacerbating currency depreciation and feeding inflation. Real exchange rate movements are amplified compared to those under price stability.

In economies where a low ERPT due to nominal rigidities in export and import prices mutes exchange rate expenditure switching effects on the output gap—the case of local currency pricing (LCP)—the optimal monetary stance is always driven by the need to stabilize demand. In contrast with the PCP case, the optimal stance is contractionary when excessive inflows are associated with an overvalued currency and a domestic demand boom, to contain domestic demand. The optimal stance is thus deflationary and exacerbates the currency misalignment. Under LCP, however, the sign of the optimal policy switches from contractionary to expansionary when inflows are associated with exchange rate undervaluation. In this case, monetary policy moves to support the inefficiently low domestic demand. As in the PCP economy, this comes at the cost of fueling further exchange rate depreciation and inflationary pressures. Under LCP, the optimal policy always amplifies real exchange rate volatility relative to a policy regime of strict CPI stability.[5]

In developing our analysis, we make four novel contributions to the literature. First, we provide a second-order accurate approximation of the global welfare function for the standard New Keynesian two-country model with generically incomplete markets under PCP and LCP.[6] The function we derive is valid for an arbitrary number of assets—bond economies and financial autarky are obtained as special cases—without requiring restrictive assumptions on preferences (such that a unitary trade elasticity or an identical consumption basket across countries).

Second, we characterize the optimal targeting rules under cooperation and commitment for both PCP and LCP economies.[7] These rules hold for a wide range of shocks (including anticipated or unanticipated shocks to preferences, productivity, markups, etc.), but, unlike the global welfare function, are specific to a bond economy only.

[5] In our analysis we abstract from the question of which export pricing strategy, PCP or LCP, is optimal from the vantage point of the firms, given the optimal policy (see, e.g., recent work by Mukhin (2022)). An important issue for future research is whether, in economic environments supporting the optimal choice of either PCP or LCP, the optimal stabilization rules would substantially deviate from the one we derive in this paper.

[6] The loss function in our chapter for the Handbook of Monetary Economics is derived for the case of complete markets and for the case of financial autarky under PCP (see Corsetti et al. 2010). For LCP, Engel (2011) derives the loss function, but only for the case of complete markets. The loss function in this paper encompasses the above as special cases, and is the most general loss function for this class of models.

[7] We focus on the optimal policy under commitment and cooperation as it provides a useful benchmark against which to judge the optimal policy under non-cooperative behavior or discretion. See ongoing related work (Corsetti et al. 2021) for such a comparison under Dominant Currency Pricing.

Third, we show that a single welfare-relevant gap, a "wealth gap" combining cross-country demand misallocation with real exchange rate misalignment, indexes whether capital inflows are inefficiently high or low relative to the first-best allocation with full risk sharing. A positive (negative) "wealth gap" in response to capital inflows means that, because of imperfect insurance, domestic consumption is too high (low) relative to foreign, adjusting for purchasing power. In the targeting rules, this gap characterizes how risk sharing impinges on the trade-offs across (internal and external) objectives pursued by optimizing policymakers.[8]

Finally, we offer an analytical characterization of the macroeconomic dynamic response to inefficient flows under the optimal policy. In our analysis, without loss of generality we find it convenient to focus on "news shocks" (anticipation of future changes in fundamentals) as these typically generate capital flows that are excessive relative to the first best.[9] Notably, we show that in model specifications often adopted by the literature under LCP (see, e.g., Engel 2011), capital flows in response to (news and contemporaneous) shocks are *exogenous* to monetary policy. Not only this helps isolate the causal effects of inefficient flows. Also, it brings our analysis to bear directly on a case often debated in policy circles, where monetary policy can only mitigate the effects of inefficient capital flows on domestic macroeconomic dynamics, but cannot curb their size.[10]

Related literature Our analysis builds on a vast body of work that, over the last two decades, has reexamined a classic question in open economy macroeconomics, concerning the trade-offs between external and internal objective (see Benigno and Benigno 2003; Clarida, Galí and Gertler 2002; Corsetti and Pesenti 2005; Devereux and Engel 2003; Engel 2011; and Galí and Monacelli 2005, among others).[11] It is nonetheless useful to emphasize two strands of this literature that help highlight our contribution.

The first is the literature epitomized by Engel (2011), who studies optimal policy under complete markets contrasting LCP and PCP in the otherwise canonical open economy New Keynesian model developed by Clarida, Galí and Gertler (2002). A key result under LCP is that the optimal monetary policy supports an allocation with CPI-price stability and no exchange rate misalignment—which also implies no cross-country misallocation of demand–the demand gap defined in Section 3.1 below. Indeed, under the maintained assumption of complete markets, trade in financial assets ensures that real exchange rate misalignment and the demand gap are

[8] The wealth gap is akin to an endogenous and symmetric markup shock. While the exogenous markup shocks typically assumed in the monetary literature create aggregate global distortions, the inefficiencies from capital inflows have opposing effects on different economies, that cancel out in the aggregate. As we show below, a key implication is that, under the optimal policy, the Home and Foreign monetary stance will be symmetric but with the opposite sign. This is in contrast with the optimal response to the exogenous markup shocks commonly assumed by the monetary literature, which may be similar across borders, in particular under LCP, even when markup shocks are uncorrelated across countries (see e.g. our previous results in CDL 2010, page 902-904).

[9] See the seminal papers by Beaudry and Portier (2006), Beaudry et al. (2011) and Schmitt-Grohe and Uribe (2012). For the relevance of news shocks to future fundamentals in driving exchange rates, see Engel and West (2005) and Devereux and Engel (2007). While we focus on news in preferences or technology, news shocks impinging on savings may also stem from political risk (i.e., capital controls; see, e.g., Acharya and Bengui 2018), changes in the efficiency of financial intermediaries akin to UIP shocks (see, e.g., Gabaix and Maggiori 2015).

[10] These results are not affected by intermediation costs associated to the accumulation of net foreign asset position. Hence, barring additional algebraic complexity, they extend to economic environments similar to the one studied by Gabaix and Maggiori (2015).

[11] As discussed in Corsetti, Dedola, and Leduc (2010), most of the papers in the literature either assume complete markets or close to efficient capital flows because of particular restrictions on preference and technology parameters.

always proportional to each other—independently of whether ERPT is complete (PCP) or incomplete (LCP). This is where our results differ from, and complement, this literature. When markets are not complete, misalignment and demand gaps are not proportional to each other—monetary policy will not be able to close both of them simultaneously, facing trade-offs between competing internal and external objectives. To best illustrate the value added of our results, we keep the focus on the PCP and LCP economies, the cases that have long be center stage in the literature on the optimal design of monetary policy in open economies.[12]

The second strand of the literature includes a small number of contributions that, like ours, provide analytical characterizations of the optimal monetary policy in two-country models with incomplete financial markets.[13] Obstfeld and Rogoff (2003) and Devereux (2004) examine static frameworks without capital flows, and in which prices are set one period in advance—therefore, necessarily abstracting from the welfare implications of current account dynamics and inflation. Devereux and Sutherland (2008) study a dynamic setting similar to ours, but in which markets are effectively complete under flexible prices so that price stability also attains the first-best natural rate allocation.[14] Under PCP, Benigno (2009) emphasizes deviations from price stability, in economies in which net foreign asset holdings are asymmetrical in the nonstochastic steady state. However, the focus is on economies in which deviations from both purchasing power parity (PPP) and the law of one price are assumed away, in contrast with the analysis of real exchange rate misalignment at the core of optimal policy design analyzed in our paper. Our paper is also closely related to Farhi and Werning (2016), which provides a general characterization of optimal targeting rules in economies with nominal rigidities and financial market frictions. While this contribution focuses on the role of macroprudential policies when monetary policy is constrained, we focus on optimal monetary policy when macroprudential policies are not available—also explicitly taking into account standard welfare costs of inflation that stem from staggered price setting. Monetary policy with incomplete financial markets is also analyzed quantitatively by Rabitsch (2012), who revisits the benefits from international cooperation, and more recently by Senay and Sutherland (2019), who study the properties of instrument rules in a incomplete markets model with a portfolio of assets including bonds and equities.[15] Finally, the case of beggar-thy-self depreciations associated with low trade elasticities in incomplete market economies, which is the focus of Section 5 below, has been recently discussed by Auclert et al. (2021) in a model with heterogeneous agents within countries.

Additionally, our study is naturally related to the growing literature that emphasizes the role of pecuniary externalities under collateral constraints, financial accelerator (balance-sheet) effects and over- and underborrowing relative to the constrained-efficient allocation (see Benigno et al. 2011; Bianchi 2011; Bianchi and Mendoza 2010; Brunnermeier and Sannikov 2015; Dávila

[12] In a companion paper (Corsetti Dedola and Leduc 2021) we analyze the case of Dominant Currency Pricing (DCP) with asymmetric ERPT across borders recently emphasized by Gopinath (2016) and Gopinath et al. (2020). Casas et al. (2017) and Egorov and Mukhin (2023) study optimal monetary policy for this case, focusing on small open economies. Also, it would be interesting to examine how other sources of **incomplete** ERPT (such as those that we consider in earlier work, Corsetti et al. 2008b) may impact the optimal policy response.

[13] Other contributions have looked at similar issues in a small open economy framework—see e.g. De Paoli (2009) and Fanelli (2019).

[14] Tille (2005) assesses the welfare impact of integrating international asset markets with nominal rigidities and a stochastic component in monetary policy.

[15] A number of papers numerically solve open economy models under incomplete markets, and examine optimal policy often using ad hoc loss functions. See, for example, the early paper by Kollmann (2002).

and Korinek 2018; Jeanne and Korinek 2010; and Lorenzoni 2008, among others).[16] Devereux and Yu (2016) characterize optimal monetary policy under discretion in a small open economy with occasionally binding borrowing constraints. Relative to these papers, a distinct feature of our paper is a focus on monetary policy in a global equilibrium characterized by overborrowing (and obviously underborrowing in the other country) with respect to the first-best allocation.[17]

Finally, as regards the debate on the limits of monetary policy, our results are in line with Woodford (2009), showing that openness to foreign capital does not compromise monetary control, i.e., the ability of the central bank to pursue a desired monetary stance. Yet, as stressed by Farhi and Werning (2014) in a small open economy setting, inefficient capital flows may create adverse trade-offs across policy goals, hampering a central bank's ability to maintain the economy on an efficient path. We complement this work in that we inspect the monetary policy trade-offs created by capital flows, and characterize the optimal monetary response in the global cooperative equilibrium, when macroprudential policy and/or capital controls are not readily available.

The rest of the paper is organized as follows. The next section briefly goes over the standard two-good, two-country, New Keynesian model that we take as the framework for our analysis. Section 3 derives the global loss function, discussing each of its arguments in some detail, and characterizes the cooperative optimal targeting rules under PCP and LCP. In this section, we also analyze in detail how and why incomplete markets make a difference for monetary policy. In Section 4, we consider a baseline specification of the model that we dub the Cole and Obstfeld (CO) economy, where capital flows are exogenous to policy and independent of ERPT. We can therefore focus sharply on how the optimal monetary stance changes across LCP and PCP economies. In section 5, we go beyond the role of ERPT, and further study how the optimal monetary policy varies systematically depending on the equilibrium link between misalignment and capital flows. Section 6 concludes. The Appendix derives the loss function, the targeting rules, and the different allocations shown throughout the papers, and provides proofs for the propositions and lemmas stated in the text.

2 The model economy

The analysis builds on the standard open economy version of the workhorse model in monetary economics (see, e.g., Clarida, Galí and Gertler 2002 and Engel 2011), with well-known characteristics. The world economy consists of two countries of equal size, H and F. Each country specializes in one type of tradable good, produced in a number of varieties or brands defined over a continuum of unit mass. Brands of tradable goods are indexed by $h \in [0,1]$ in the Home country and $f \in [0,1]$ in the Foreign country. Firms producing the goods are monopolistic

[16] Cavallino (2016) examines foreign exchange interventions as a second instrument (in addition to conventional interest rate policy) available to the central bank to redress inefficient capital flows in an economy with borrowing constraints similar to those of Gabaix and Maggiori (2015).

[17] Key to our analysis is that, in equilibrium, the natural borrowing constraints in a bond economy depend on real exchange rate misalignment. Exchange rate movements drive differences in national wealth by affecting the relative value of a country's output (and thus the natural constraint on foreign borrowing), similarly to their valuation effects on outstanding foreign assets and liabilities already stressed by the literature (see, e.g., Gourinchas and Rey 2014). Since the relative value of output (and its present discounted value) reflect misalignment when financial markets are incomplete, real exchange rate movements induce an inefficient wealth wedge across countries.

suppliers of one brand only and use labor as the only input to production. These firms set prices either in local or producer currency units and in a staggered fashion as in Calvo (1983). Asset markets are complete at the national level, but incomplete internationally.

In what follows, we describe our setup focusing on the Home country, with the understanding that similar expressions also characterize the Foreign economy—variables referring to Foreign firms and households are marked with an asterisk.

2.1 The household's problem

2.1.1 Preferences

We consider a cashless economy in which the representative Home agent maximizes the expected value of her lifetime utility, where instantaneous utility U is a function of a consumption index, C, and (negatively) of labor effort L, specialized as follows:

$$U\left[C_t, L_t\right] = \zeta_{C,t}\frac{C_t^{1-\sigma}}{1-\sigma} - \kappa\frac{L_t^{1+\eta}}{1+\eta}, \qquad \sigma, \eta > 0 \tag{1}$$

whereas the model also allows for shocks to marginal utilities of consumption $\zeta_{C,t}$. Foreign agents' preferences are symmetrically defined. Households consume both domestically produced and imported goods. We define $C_t(h)$ as the Home agent's consumption as of time t of the Home good h; similarly, $C_t(f)$ is the Home agent's consumption of the imported good f. We assume that each good h (or f) is an an imperfect substitute for all other goods' varieties, with constant elasticity of substitution $\theta > 1$:

$$C_{\mathrm{H},t} \equiv \left[\int_0^1 C_t(h)^{\frac{\theta-1}{\theta}} dh\right]^{\frac{\theta}{\theta-1}}, \qquad C_{\mathrm{F},t} \equiv \left[\int_0^1 C_t(f)^{\frac{\theta-1}{\theta}} df\right]^{\frac{\theta}{\theta-1}}. \tag{2}$$

The full consumption basket, C_t, in each country, aggregates Home and Foreign goods according to the following standard CES function:

$$C_t \equiv \left[a_{\mathrm{H}}^{1/\phi} C_{\mathrm{H},t}^{\frac{\phi-1}{\phi}} + a_{\mathrm{F}}^{1/\phi} C_{\mathrm{F},t}^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}}, \qquad \phi > 0, \tag{3}$$

where a_{H} and a_{F} are the weights on the consumption of Home and Foreign traded goods, respectively, and ϕ is the constant (trade) elasticity of substitution between $C_{\mathrm{H},t}$ and $C_{\mathrm{F},t}$.

2.1.2 Price indexes

The price index of the Home goods is given by:

$$P_{\mathrm{H},t} = \left[\int_0^1 P_t(h)^{1-\theta} dh\right]^{\frac{1}{1-\theta}}, \tag{4}$$

and the price index associated with the consumption basket, C_t, is:

$$\mathbb{P}_t = \left[a_{\mathrm{H}} P_{\mathrm{H},t}^{1-\phi} + a_{\mathrm{F}} P_{\mathrm{F},t}^{1-\phi}\right]^{\frac{1}{1-\phi}}. \tag{5}$$

Let $\mathcal{E}_t$ denote the Home nominal exchange rate, expressed in units of Home currency per unit of Foreign currency. The real exchange rate (RER) is customarily defined as the ratio of CPIs expressed in the same currency, i.e., $\mathcal{Q}_t = \frac{\mathcal{E}_t \mathbb{P}_t^*}{\mathbb{P}_t}$. The terms of trade (TOT) are instead defined as the relative price of domestic imports in terms of exports: $\mathcal{T}_t = \frac{P_{\mathrm{F},t}}{\mathcal{E}_t P^*_{\mathrm{H},t}}$ if firms set prices in local currency and $\frac{\mathcal{E}_t P^*_{\mathrm{F},t}}{P_{\mathrm{H},t}}$ under producer currency pricing.

2.1.3 Budget constraints

Home and Foreign agents trade an international one-period bond, B_{H}, which pays in units of Home currency and is zero in net supply. Households derive income from working, $w_t L_t$, from domestic firms' profits, $\Pi(h)$, lump-sum transfers T_t, and from interest payments, $(1+i_t)B_{\mathrm{H},t}$, where i_t is the nominal bond's yield, paid at the beginning of period t but known at time $t-1$. Households use their disposable income to consume and invest in bonds. The individual flow budget constraint for the representative agent j in the Home country is therefore:

$$P_{\mathrm{H},t} C_{\mathrm{H},t} + P_{\mathrm{F},t} C_{\mathrm{F},t} + B_{\mathrm{H},t+1} \leq w_t L_t + (1+i_{t-1}) B_{\mathrm{H},t} + \int_0^1 \Pi(h) dh + T_t. \tag{6}$$

The household's problem thus consists of maximizing lifetime utility, defined by (1), subject to the constraint (6).

2.2 Firms

Firms employ domestic labor to produce a differentiated product h according to the following linear production function:

$$Y_t(h) = \zeta_Y L_t(h), \tag{7}$$

where $L(h)$ is the demand for labor by the producer of the good h and ζ_Y is a technology shock common to all producers in the Home country, which follows a statistical process to be specified below.

Firms are subject to nominal rigidities à la Calvo so that, at any time t, they keep their price fixed with probability α. We assume that when firms update their prices, they do so simultaneously in the Home and Foreign markets. Following the literature, we consider two models of nominal price distortions in the export markets. According to the first model, firms set prices in the currency of the destination (local) market — this is the LCP hypothesis. The maximization problem is then as follows:

$$Max_{\mathcal{P}(h),\mathcal{P}^*(h)} \; E_t \left\{ \sum_{k=0}^{\infty} p_{bt,t+k} \alpha^k \begin{pmatrix} \left[\mathcal{P}_t(h) D_{t+k}(h) + \mathcal{E}_t \mathcal{P}^*_t(h) D^*_{t+k}(h)\right] - \\ MC_{t+k}(h) \left[D_{t+k}(h) + D^*_{t+k}(h)\right] \end{pmatrix} \right\} \tag{8}$$

where $p_{bt,t+k}$ is the firm's stochastic nominal discount factor between t and $t+k$, and the firm's demand at Home and abroad is given by:

$$\begin{aligned} D_t(h) &= \int \left(\frac{\mathcal{P}_t(h)}{P_{\mathrm{H},t}} \right)^{-\theta} C_{\mathrm{H},t} dh \\ D_t^*(h) &= \int \left(\frac{\mathcal{P}_t^*(h)}{P_{\mathrm{H},t}^*} \right)^{-\theta} C_{\mathrm{H},t}^* dh \end{aligned}$$

In these expressions, $P_{\mathrm{H},t}$ and $P_{\mathrm{H},t}^*$ denote the price index of Home goods in the Home and Foreign countries — the latter expressed in Foreign currency.

By the first-order condition of the producer's problem, the optimal price $\mathcal{P}_t(h)$ in domestic currency charged to domestic customers is:

$$\mathcal{P}_t(h) = \frac{\theta}{\theta - 1} \frac{E_t \sum_{k=0}^{\infty} \alpha^k p_{bt,t+k} D_{t+k}(h) MC_{t+k}(h)}{E_t \sum_{k=0}^{\infty} \alpha^k p_{bt,t+k} D_{t+k}(h)}; \tag{9}$$

while the price (in foreign currency) charged to customers in the Foreign country is:

$$\mathcal{P}_t^*(h) = \frac{\theta}{\theta - 1} \frac{E_t \sum_{k=0}^{\infty} \alpha^k p_{bt,t+k} D_{t+k}^*(h) MC_{t+k}(h)}{E_t \sum_{k=0}^{\infty} \alpha^k p_{bt,t+k} \mathcal{E}_{t+k} D_{t+k}^*(h)}. \tag{10}$$

According to the alternative model, we posit that firms set prices in the producer currency — this is the PCP hypothesis. In this case, exchange rate pass-through is complete. Given that demand elasticities are assumed to be the same across markets, in domestic currency the price charged to foreign consumers is the same as the optimal price charged at Home: the law of one price holds: $\mathcal{P}_t^*(h) = \mathcal{P}_t(h)/\mathcal{E}_t$. The optimal price is similar to (9), whereas Home demand is replaced by global demand.

Since all the producers that can choose their price set it to the same value, we obtain the following equations for $P_{\mathrm{H},t}$ and $P_{\mathrm{H},t}^*$

$$\begin{aligned} P_{\mathrm{H},t}^{1-\theta} &= \alpha P_{\mathrm{H},t-1}^{1-\theta} + (1-\alpha)\, \mathcal{P}_t(h)^{1-\theta}, \\ P_{\mathrm{H},t}^{*1-\theta} &= \alpha P_{\mathrm{H},t-1}^{*1-\theta} + (1-\alpha)\, \mathcal{P}_t^*(h)^{1-\theta}. \end{aligned} \tag{11}$$

Similar relations hold for the Foreign firms.

2.3 Asset markets and exchange rate determination

In specifying the asset market structure, we restrict trade to one financial instrument only, a safe nominal bond. While capturing the notion that international financial markets do not provide efficient risk insurance against all shocks, intertemporal trade still implies forward-looking exchange rate determination, as a by-product of equilibrium in financial markets. Namely, by combining the Euler equations for the Home households

$$\frac{U_C\left(C_t, \zeta_{C,t}\right)}{\mathbb{P}_t} = \left(1+i_t\right) E_t\left[\beta \frac{U_C\left(C_{t+1}, \zeta_{C,t+1}\right)}{\mathbb{P}_{t+1}}\right]$$

and the Foreign households:

$$\begin{aligned}
\frac{U_C\left(C_t^*, \zeta_{C,t}^*\right)}{\mathbb{P}_t^*} &= \left(1+i_t^*\right) E_t\left[\beta \frac{U_C\left(C_{t+1}^*, \zeta_{C,t+1}^*\right)}{\mathbb{P}_{t+1}^*}\right], \\
\frac{U_C\left(C_t^*, \zeta_{C,t}^*\right)}{\mathcal{E}_t \mathbb{P}_t^*} &= \left(1+i_t\right) E_t\left[\beta \frac{U_C\left(C_{t+1}^*, \zeta_{C,t+1}^*\right)}{\mathcal{E}_{t+1} \mathbb{P}_{t+1}^*}\right];
\end{aligned}$$

efficient trade in the international bond will imply the following uncovered interest parity condition, which equates the nominal stochastic discount rates in expectations:

$$E_t\left[\beta \frac{U_C\left(C_{t+1}, \zeta_{C,t+1}\right)}{U_C\left(C_t, \zeta_{C,t}\right)} \frac{\mathbb{P}_t}{\mathbb{P}_{t+1}}\right] = E_t\left[\beta \frac{U_C\left(C_{t+1}^*, \zeta_{C,t+1}^*\right)}{U_C\left(C_t^*, \zeta_{C,t}^*\right)} \frac{\mathcal{E}_t \mathbb{P}_t^*}{\mathcal{E}_{t+1} \mathbb{P}_{t+1}^*}\right] \tag{12}$$

Solved forward, this equation pins down the equilibrium exchange rate.

Under complete markets, the condition (12) holds state-by-state, rather than in expectations, since agents trade in contingent assets up to the point when, at the margin, the valuation of an extra unit of money of currency is equalized across borders in all circumstances. When countries are symmetric, this implies that the relative utility value of wealth, denoted by $\mathcal{W}_t$,

$$\mathcal{W}_t \equiv \frac{U_C\left(C_t^*, \zeta_{C,t}^*\right) \frac{1}{\mathcal{E}_t \mathbb{P}_t^*}}{U_C\left(C_t, \zeta_{C,t}\right) \frac{1}{\mathbb{P}_t}} = \frac{U_C\left(C_t^*, \zeta_{C,t}^*\right)}{U_C\left(C_t, \zeta_{C,t}\right)} \frac{1}{\mathcal{Q}_t} \tag{13}$$

is identically equal to one (see, e.g., Gravelle and Rees 1992, Backus and Smith 1993and Obstfeld and Rogoff 2001). Note that the marginal utility of consumption across borders is adjusted for the respective prices of the consumption basket.

Under incomplete markets, however, the equilibrium condition (12) only holds in expectations: any shocks will induce a wedge in the (ex post) relative value of wealth across borders, so that in general $\mathcal{W}_t \neq 1$. As shown below, $\mathcal{W}_t$ defines a theoretically grounded and efficient measure of cross-border imbalances that arise due to asset markets imperfections in the policy problem—in line with the approach by Woodford (2010), who studies monetary trade-offs under financial frictions in a closed economy setting allowing for agent heterogeneity.

2.4 Log-linearized equilibrium

Throughout the paper, the model's equilibrium conditions and constraints will be written out in log-deviations from the non-stochastic steady state—we will assume a symmetric steady-state in which the net foreign asset position is zero and the markup distortion is eliminated with appropriate subsidies. Details on the log-linearized model equations are given in the Appendix.

Notation-wise, we denote steady-state values of variable with an upper bar, and write $\widehat{x}_t = \ln x_t / \overline{x}$ for deviations from steady state under sticky prices. While we will study different specifications of the model—PCP vs. LCP, with either unitary or generic trade elasticity— we will not denote variables differently across them, since each specification will be discussed in a separate section or subsection. We make two exceptions to this notation convention. First, we will use the superscript fb to denote variables in the unique "first-best" allocation, corresponding to the case of complete asset markets, flexible prices and no markup distortions. Second, in Sections 4 and 5, we will use the superscript na to denote variables in the "natural" (flex-price) allocation.

Before delving into the analysis, it is useful to characterize upfront the first-best allocation against which we will define our loss functions and the optimal policy rules, as well as discuss two key properties of the model under incomplete markets.

2.4.1 The first-best allocation benchmark

The first-best output in the Home and Foreign country, $\widehat{Y}^{fb}_{H,t}$ and $\widehat{Y}^{fb}_{F,t}$, together with the real exchange rate and the terms of trade are shown in Table 1.

Table 1

The first-best allocation

$\widehat{Y}^{fb}_{H,t} = \frac{2a_{\rm H}(1-a_{\rm H})(\sigma\phi-1)\left(\widehat{\mathcal{T}}^{fb}_t\right)-(1-a_{\rm H})\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}^*_{C,t}\right)+\widehat{\zeta}_{C,t}+(1+\eta)\widehat{\zeta}_{Y,t}}{\eta+\sigma}$
$\widehat{Y}^{fb}_{F,t} = \frac{2a_{\rm H}(1-a_{\rm H})(\sigma\phi-1)\left(-\widehat{\mathcal{T}}^{fb}_t\right)+(1-a_{\rm H})\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}^*_{C,t}\right)+\widehat{\zeta}^*_{C,t}+(1+\eta)\widehat{\zeta}^*_{Y,t}}{\eta+\sigma}$
$\widehat{\mathcal{Q}}^{fb}_t = (2a_{\rm H}-1)\,\widehat{\mathcal{T}}^{fb}_t = \sigma\left(\widehat{C}^{fb}_t - \widehat{C}^{*fb}_t\right)$
$\widehat{\mathcal{T}}^{fb}_t = \frac{\sigma\left(\widehat{Y}^{fb}_{H,t}-\widehat{Y}^{fb}_{F,t}\right)-(2a_{\rm H}-1)\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}^*_{C,t}\right)}{4(1-a_{\rm H})a_{\rm H}(\sigma\phi-1)+1}$

The table highlights a key feature of the first-best allocation, that we will extensively use in our analysis. Even though households are forward looking, the equilibrium relative prices and quantities depend only on the current-period (exogenous) fundamentals, not on their expected future realizations, in line with the well-known results in Barro and King (1984).[18] A notable implication is that, in the first best, neither the short-term real interest rate (given by the expected growth rates in marginal utility), nor the long-term interest rate (which is proportional to the current marginal utility of consumption under our preference assumptions, as in Woodford (2004)) depends on anticipated shocks.

The same applies to cross-border capital flows. To represent these flows in the efficient economy, we denote, with slight abuse of notation, by $\widehat{\mathcal{B}}^{fb}_t$ the "notional" real net foreign assets, simply defined as cumulated real net exports (i.e., consumption minus income). Furthermore, we scale real net foreign assets by steady-state output, so that $\widehat{\mathcal{B}}^{fb}_t \simeq \frac{\mathcal{B}^{fb}_t - \overline{\mathcal{B}}}{\overline{Y}^{fb}}$. The cross-border efficient financial flows, characterized up to first order, can then be written as:

$$\widehat{\mathcal{B}}^{fb}_t - \beta^{-1}\widehat{\mathcal{B}}^{fb}_{t-1} = (1-a_{\rm H})\,\sigma^{-1}\left[(2a_{\rm H}(\sigma\phi-1)+1-\sigma)\,\widehat{\mathcal{T}}^{fb}_t - \left(\widehat{\zeta}_{C,t}-\widehat{\zeta}^*_{C,t}\right)\right] \tag{14}$$

Importantly, only contemporaneous shocks appear on the right hand side of this expression. Thus, relative to this benchmark, any cross-border flow of capital that responds to anticipated

[18] Recall that in the workhorse monetary model we use in our analysis, preferences are time separable and there is no capital accumulation (see Devereux and Engel 2009 for an analysis of the optimal monetary response to news shocks under complete markets). Introducing capital accumulation and other sources of sluggish adjustment, such has habits or adjustment costs would change the results that follow, but mainly quantitatively.

future changes in fundamentals (or news shocks) under incomplete markets is entirely inefficient.

2.4.2 Two notable properties of the incomplete-market NK workhorse model

Under the model specification assuming trade in one noncontingent bond, a key property of the log-linearized equilibrium is that, by the uncovered interest parity condition (12), $\widehat{\mathcal{W}}_t$ follows a random walk:

$$E_t\widehat{\mathcal{W}}_{t+1} = \widehat{\mathcal{W}}_t. \tag{15}$$

Because of incomplete risk sharing, shocks will generally result in a unit root in the relative value of wealth across borders—corresponding to a unit root in net foreign assets. A comment is in order in this respect. In the text to follow, we will carry out our analysis of the bond economy allowing for this unit root in $\widehat{\mathcal{W}}_t$ (and net foreign wealth). This is a choice motivated by tractability and analytical transparency, without prejudice for the gist of our analysis—see last section in the Appendix for a discussion of how our results would change under stationary-inducing costs of holding bonds.

A second key property of the model worth emphasizing follows from our assumption that the initial steady state is symmetric with zero net foreign wealth outstanding, consistent with the overall symmetry of the model, and bonds are short-term. Up to first order, then, the dynamic of net foreign assets (and thus $\widehat{\mathcal{W}}_t$) does not respond to the ex post returns on internationally traded bonds. Specifically, with one-period bonds, real net foreign assets (defined as $B_t = \frac{B_{\mathrm{H},t+1}}{\mathbb{P}_t}$) are capitalized at the steady-state real interest rate β^{-1}— see Section 1.3.2 in the Appendix.[19]

3 Why and how do incomplete markets affect monetary policy?

Our main objective is to examine the monetary policy trade-offs brought about by inefficient capital flows in economies where asset markets are incomplete. In this section, we first discuss the welfare-relevant gaps shaping policy trade-offs in open economies, and reconsider how incomplete markets affect the monetary transmission to macroeconomic variables. We then derive a general quadratic policy loss function obtained from a second-order approximation of agents' utility for generic incomplete markets (i.e., without specifying the form of market incompleteness). Finally, we characterize the optimal cooperative policy under commitment, in terms of optimal targeting rules.

3.1 Welfare-relevant gaps in an open economy

A recurrent theme in policy debates concerns the possibility that international relative prices are misaligned and cross-border borrowing/lending is too high or too low—corresponding to either excessive or insufficient demand in different countries. Drawing on previous work of ours (Corsetti et al. 2010), and using the same logic underlying the definition of the welfare-relevant output gap, we now define five gaps that, together, account for these policy concerns. Four out

[19] The same property holds irrespective of whether the internationally traded bond were denominated in Foreing currency or in real terms. Variations in the ex-post interest rate can have a first-order impact on the allocation if net foreign assets in steady state are non-zero, as analyzed in Benigno (2009), or if there are more than one internationally traded asset.

of these five gaps may open in economies with either nominal rigidities or financial frictions, or both—but only one is specific to incomplete market economies.

As is customary in monetary stabilization analysis, we will write policy objectives and targeting rules in terms of welfare-relevant gaps, expressing relevant variables as deviations from their first-best allocation values. All gaps will be denoted with a tilde.

3.1.1 Misalignment: real exchange rate gaps

Three relative price gaps account for misalignment. According to the standard definition of gaps, exchange rates are misaligned when they deviate from the value they would take in the efficient allocation.[20] Since there are different measures of international relative prices, there are different (complementary) measures of misalignment. For the relative price of consumption across countries, the welfare-relevant gap is:

$$\widetilde{\mathcal{Q}}_t = \widehat{\mathcal{Q}}_t - \widehat{\mathcal{Q}}_t^{fb}. \tag{16}$$

Analogously, for the relative price of tradables, the terms-of-trade gap is:

$$\widetilde{\mathcal{T}}_t = \widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}_t^{fb}. \tag{17}$$

Finally, misalignment can also arise when nominal rigidities in local currency translate into cross-border deviations from the law of one price (henceforth LOOP). In this case, identical goods are inefficiently traded at different prices domestically and abroad. These price differences define another dimension of misalignment, which, measured on average for the basket of Home goods, is given by:

$$\widetilde{\Delta}_{\mathrm{H},t} = (\widehat{\mathcal{E}}_t + \widehat{P}^*_{\mathrm{H},t} - \widehat{P}_{\mathrm{H},t}) \tag{18}$$

where $\widetilde{\Delta}_{\mathrm{H},t}$ is equal to zero when the LOOP holds. Note that, to the extent that $P^*_{\mathrm{H},t}$ and $P_{\mathrm{H},t}$ are sticky, the law of one price is violated with any movement in the exchange rate. Specifically, domestic currency depreciation tends to increase the Home firms' receipts in Home currency from selling goods abroad, relative to the Home market: Home currency depreciation raises $\widetilde{\Delta}_{\mathrm{H},t}$. Similar considerations apply to $\widetilde{\Delta}_{\mathrm{F},t}$.

3.1.2 Demand misallocation: demand and wealth gaps

Inefficient external positions could be captured by tracing capital flows in excess of the financial flows in an efficient allocation, i.e., $\widehat{\mathcal{B}}_t - \widehat{\mathcal{B}}_t^{fb}$, a gap that may open in the presence of either nominal or real (financial) distortions.[21] However, there is a more informative set of measures from a welfare perspective, that also brings about substantial benefits in terms of tractability

[20] We stress that, conceptually, the first-best exchange rate is not necessarily (and in general will not be) identical to the "equilibrium exchange rate," traditionally studied by international and policy institutions, as a guide to policy-making. The efficient exchange rate is theoretically and conceptually defined, at any time horizon, in relation to a hypothetical economy in which all prices are flexible and markets are complete. In fact, our measure of misalignment (as the difference between current exchange rates and the efficient one) is constructed in strict analogy to the notion of a welfare-relevant output gap, as the difference between current output and the efficient level of output, which does not coincide with the natural rate (i.e., the level of output with flexible prices).

[21] It is worth stressing that this measure would be well defined also under financial autarky, whereas $\widehat{\mathcal{B}}_t = 0$.

of the targeting rule and the loss function.

The first of these measures is the "relative demand gap," denoted by $\widetilde{\mathcal{D}}_t$ and defined as the cross-country difference in private (consumption) demand relative to the first best:

$$\widetilde{\mathcal{D}}_t = \widetilde{C}_t - \widetilde{C}_t^*.$$

As stressed by Engel (2011) and Fahri and Werning (2016), this gap may open also in complete market economies, reflecting nominal distortions. Combined with the real exchange rate gap, $\widetilde{\mathcal{Q}}_t$, however, $\widetilde{\mathcal{D}}_t$ adds up to a gap that opens only in the presence of financial frictions (whether or not there are nominal rigidities). We define this second measure of misallocation as the "wealth" gap, $\widetilde{\mathcal{W}}_t$:

$$\widetilde{\mathcal{W}}_t = \sigma\widetilde{\mathcal{D}}_t - \widetilde{\mathcal{Q}}_t, \tag{19}$$

where $\widetilde{\mathcal{W}}_t$ is equal to log-deviations in the relative value of wealth (13). If markets are complete, $\widetilde{\mathcal{W}}_t = 0$ always, even when the overall allocation is not efficient because of nominal rigidities or other distortions. If markets are incomplete, instead, $\widetilde{\mathcal{W}}_t$ will generally not be zero, and can have either sign, with a straightforward interpretation. A positive (negative) gap $\widetilde{\mathcal{W}}_t > 0$ ($\widetilde{\mathcal{W}}_t < 0$) means that, given the relative price of consumption, the consumption of the Home (national representative) individual is inefficiently high (low) vis-à-vis foreign consumption. Or, given $\widetilde{\mathcal{D}}_t$, the currency is excessively strong (weak) in real terms (relative to first best). We will show below that, in a bond economy, anticipated shocks generally open a wealth gap: although borrowing for consumption smoothing purposes is optimal from an individual-agent perspective, from a global welfare perspective it results in a Home wealth that is too high (for $\widetilde{\mathcal{W}}_t > 0$) or too low (for $\widetilde{\mathcal{W}}_t < 0$).[22]

3.2 The wealth gap and monetary policy trade-offs with incomplete markets

The wealth gap defined in the previous subsection nicely captures the policy trade-offs created by financial markets imperfections in the design of optimal stabilization rules. Under complete markets, the demand gap $\widetilde{\mathcal{D}}_t$ and real exchange rate misalignment $\widetilde{\mathcal{Q}}_t$ can each be different from zero—depending on the effect of nominal rigidities. Yet, as a consequence of full risk sharing, they will always remain proportional to each other, i.e., $\widetilde{\mathcal{W}}_t = \sigma\widetilde{\mathcal{D}}_t - \widetilde{\mathcal{Q}}_t = 0$. Closing $\widetilde{\mathcal{Q}}_t$ will be tantamount to closing $\widetilde{\mathcal{D}}_t$, and vice versa. Under incomplete markets, instead, $\widetilde{\mathcal{W}}_t$ generally deviates from zero, defining a gap specific to imperfect risk sharing, which can pose trade-offs with other welfare-relevant objectives; $\widetilde{\mathcal{D}}_t$ and $\widetilde{\mathcal{Q}}_t$ are no longer proportional to each other. In general, the optimal monetary rule will not close any of these gaps completely, but will have to minimize them jointly with inflation and output gaps.

The wealth gap itself confronts monetary authorities with a fundamental trade-off. A monetary easing leans against real over-appreciation ($\widetilde{\mathcal{Q}}_t < 0$), which *per se* reduces the wealth gap; however, by stimulating a domestic demand boom, it also raises $\widetilde{\mathcal{D}}_t$, which increases the wealth gap. In Section 4, we first derive a useful and tractable case in which the wealth gap

[22] With incomplete markets, price movements are not generally efficient. While fully rational from an individual perspective, agents's decisions to borrow and lend do not internalize their effects on international prices. An appreciation of the real exchange rate associated with a Home consumption boom is a leading example of a pecuniary externality. Relative prices are no longer correct indicators of relative scarcity: consumption is higher where the price of the consumption bundle is also higher; see Geanakoplos and Polemarchakis (1986).

$\widetilde{\mathcal{W}}_t$ and the associated capital flows are exogenous to policy, so that these two channels must exactly offset each other. When this is case, monetary authorities will not be able to affect the combined inefficiencies arising from both the misallocation in demand and the real exchange rate misalignment, regardless of LCP and PCP. As shown in Section 4, monetary policy may nonetheless affect the relative size of the demand misallocation and currency misalignment. In Section 5, we relax the assumptions required for the wealth gap to be exogenous to policy and consider the more general case.

Further insight on these policy trade offs can be gained by recognizing that both misalignment and the wealth gap have substantial implications for inflation dynamics, as they affect real marginal costs. Specifically, equilibrium wages respond to imported inflation, hence to exchange rate misalignment, and to equilibrium consumption, in turn a function of borrowing and financial flows, hence of the wealth gap. Drawing on previous work of ours (Corsetti et al. 2010), we can write the Phillips Curves (four of them under LCP, collapsing into two under PCP), as a function of misalignment and wealth gaps, in addition to output gaps:

$$\pi_{H,t} - \beta E_t \pi_{H,t+1} = \tag{20}$$

$$\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\left[\begin{array}{c}(\sigma+\eta)\widetilde{Y}_{H,t} \\ -(1-a_{\rm H})\left[2a_{\rm H}(\sigma\phi-1)\left(\widetilde{\mathcal{T}}_t+\widetilde{\Delta}_t\right)-\left(\widetilde{\mathcal{W}}_t+\widetilde{\Delta}_t\right)\right]\end{array}\right]$$

$$\pi^*_{H,t} - \beta E_t \pi^*_{H,t+1} = \pi_{H,t} - \beta E_t \pi_{H,t+1} + \frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\widehat{\Delta}_t,$$

$$\pi^*_{F,t} - \beta E_t \pi^*_{F,t+1} =$$

$$\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\left[\begin{array}{c}(\sigma+\eta)\widetilde{Y}_{F,t} \\ (1-a_{\rm H})\left[2a_{\rm H}(\sigma\phi-1)\left(\widetilde{\mathcal{T}}_t+\widetilde{\Delta}_t\right)-\left(\widetilde{\mathcal{W}}_t+\widetilde{\Delta}_t\right)\right]\end{array}\right]$$

$$\pi_{F,t} - \beta E_t \pi_{F,t+1} = \pi^*_{F,t} - \beta E_t \pi^*_{F,t+1} - \frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\widetilde{\Delta}_t,$$

where we used the fact that, under symmetry, $\widetilde{\Delta}_{{\rm H},t} = \widetilde{\Delta}_{{\rm F},t} = \widetilde{\Delta}_t$, see Engel (2011).[23] By inspecting the expressions above, it is apparent that the wealth gap is isomorphic to inefficient exogenous markup shocks, typically included in the analysis of the Phillips Curves (see the discussion in CDL 2010). Via its effects on equilibrium wages, a positive wealth gap pushes Home inflation up and lowers output below its efficient level through a currency over-appreciation, as we show below. Thus, with incomplete markets, misalignment and wealth gaps naturally create, endogenously, a trade-off between inflation and output, without the need to assume exogenous cost-push disturbances. As discussed below, however, the policy implications of exogenous and endogenous cost-push disturbances are different.[24]

[23] We have written the Phillps curve as a function of the terms of trade to highlight one dimension through which exchange rate and misalignment impinge on inflation dynamics. However, note the terms of trade are a function of Home and Foreign output gaps, as well as the wealth gap and $\widetilde{\Delta}_t$, as apparent from the following equilibrium expression:

$$\left(\widetilde{\mathcal{T}}_t+\widetilde{\Delta}_t\right) = \frac{\sigma\left(\widetilde{Y}_{H,t}-\widetilde{Y}_{F,t}\right)-(2a_{\rm H}-1)\left(\widetilde{\mathcal{W}}_t+\widetilde{\Delta}_t\right)}{4a_{\rm H}(1-a_{\rm H})(\sigma\phi-1)+1}.$$

[24] When markets are incomplete, the distinction between "efficient" and "inefficient" shocks usually drawn by the closed-economy literature becomes less useful for the purpose of policy design. Also shocks to tastes

3.3 A general (quadratic) global policy loss function

From the model, we derive a second-order approximation of the equally weighted sum of the utility of the Home and Foreign national representative agents—written in terms of the gaps defined above, all in quadratic forms. As stated in Proposition 1, the policy loss functions in open economies include both "internal" objectives (inflation and output gaps), and "external" ones (relative price misalignment and the relative demand gap).

Proposition 1: *Under the assumption that appropriate subsidies offset firms' markups to deliver an efficient, non-distorted steady state, the period-by-period quadratic welfare function for incomplete market economies under LCP is as follows:*

$$\mathcal{L}_t^W - \left(\mathcal{L}_t^W\right)^{fb} \ltimes \tag{21}$$
$$-\frac{1}{2}\left\{\begin{array}{c}(\sigma+\eta)\left(\widetilde{Y}_{H,t}^2+\widetilde{Y}_{F,t}^2\right)+\dfrac{\alpha}{(1-\alpha\beta)(1-\alpha)}\theta\left(\mathbf{\Pi}_t^2+\mathbf{\Pi}_t^{*2}\right)\\ -\dfrac{2a_{\mathrm{H}}(1-a_{\mathrm{H}})}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\left[(\sigma\phi-1)\sigma\left(\widetilde{Y}_{H,t}-\widetilde{Y}_{F,t}\right)^2-\phi\left(\widetilde{\mathcal{W}}_t+\widetilde{\Delta}_t\right)^2\right]\end{array}\right\}$$
$$+t.i.p.,$$

where for notational convenience we define $\Pi_t^2 \equiv a_{\mathrm{H}}\pi_{H,t}^2 + (1-a_{\mathrm{H}})\pi_{F,t}^2$ *and* $\Pi_t^{*2} \equiv a_{\mathrm{H}}\pi_{F,t}^{*2} + (1-a_{\mathrm{H}})\pi_{H,t}^{*2}$.

Proof. See the Appendix.■

In writing the above loss, for analytical convenience, we have substituted out the terms-of-trade misalignment using its equilibrium relation with output gaps, deviations from the law of one price, and relative demand gaps. Observe also that the expression is written in terms of (components of) CPI inflation and includes deviations from the LOOP, so that it directly applies to the LCP economy. Yet, its PCP counterpart can be readily obtained by setting the LOOP deviations to zero ($\widetilde{\Delta}_t = 0$), and using the fact that, under complete ERPT, the inflation terms reduce to $\Pi_t^2 \equiv \pi_{H,t}^2$ and $\Pi_t^{*2} \equiv \pi_{F,t}^{*2}$.[25]

As shown in the Appendix, the expression for our loss function encompasses the cases of financial autarky (no asset is traded internationally), international trade in one bond, as well as international trade in any number of assets, including complete markets. In this sense, the above loss function generalizes and complements the ones derived in previous work of ours (CDL 2010) for the case of autarky (under PCP) and complete markets.[26] The key result to highlight is the last term in the loss function, which captures the cross-border (mis)allocation between production and demand specific to open economies. This misallocation reflects (symmetric) LOOP deviations $\widetilde{\Delta}_t$ and the distortions arising from incomplete financial markets and lack of international risk sharing—synthesized by the term $\widetilde{\mathcal{W}}_t$, which will generally be non-zero when markets are incomplete.

and technology (labelled "efficient") endogenously open a wealth gap and create misalignments—and thus raise meaningful policy trade-offs between output and inflation under both LCP and PCP.

[25] In Corsetti, Dedola, and Leduc (2021), we derived the loss-function under the case of asymmetric ERPT with DCP, as a particular case of the above loss-function under symmetric LCP.

[26] Gaps (other than output gaps and inflation) similar to the ones we use in our analysis also identify policy objectives arising from heterogeneity among sectors and agents in economies distorted by financial imperfections, in addition to nominal rigidities (see, e.g., Cúrdia and Woodford 2016 for an analysis in a closed economy).

3.4 Optimal targeting rules in bond economies

For the workhorse bond economy model we study—where the only asset traded across border is a non-contingent nominal bond under the maintained assumption of zero net foreign assets in steady state[27]—we now characterize the optimal cooperative policy under commitment in terms of targeting rules. The derivation of these rules is standard: we maximize the present discounted value of the sum of (21) over time, subject to the log-linearized equilibrium conditions and constraints characterizing the competitive equilibrium allocation in bond economies. In the interest of transparency and tractability, we adopt a timeless perspective (see, e.g., Woodford 2010and related literature based on the Calvo model, whereby time inconsistency stems from infrequent price adjustment).

Following a common practice in the literature, we synthesize the optimal cooperative policy in terms of two targeting rules: a global rule summing up inflation and output gaps across countries, and a cross-country rule, expressed in terms of differences in gaps across countries, which are presented in propositions 2 through 5.

Proposition 2: *From a global perspective, the optimal targeting rule under cooperation and commitment under LCP is given by*

$$\begin{aligned} 0 \;=\; & \left(\widetilde{Y}_{H,t}-\widetilde{Y}_{H,t-1}\right)+\left(\widetilde{Y}_{F,t}-\widetilde{Y}_{F,t-1}\right)+ \\ & \theta\left[a_{\mathrm{H}}\pi_{H,t}+\left(1-a_{\mathrm{H}}\right)\pi_{F,t}+a_{\mathrm{H}}\pi^{*}_{F,t}+\left(1-a_{\mathrm{H}}\right)\pi^{*}_{H,t}\right], \end{aligned} \tag{22}$$

while in the case of a PCP economy the inflation term becomes $[\pi_{H,t}+\pi^{*}_{F,t}]$ *— since, under PCP, world CPI and PPI inflation rates coincide.*

Proof. See the Appendix.■

From a global perspective, the optimal cooperative monetary policy stabilizes output gaps and inflation at the global level. An important implication is that, to the extent that world inflation is zero under the optimal policy, the sum of output gaps is also zero.[28] Moreover, in this case the optimal monetary stance will have the opposite sign across countries, unless shocks are global.

Deriving cross-country or country-specific rules generally involves solving a system of difference equations from the optimal policy problem, which differ across PCP and LCP economies. Tractable general expressions—comparable to the global rule—can be derived only for the PCP case. In LCP economies, tractability requires parameter restrictions.

3.4.1 Complete pass-through (PCP) economies

Under PCP, it is possible to derive a compact, general cross-country targeting rule for a bond economy, characterized in Proposition 3.

[27] As already mentioned at the end of Section 2, for tractability and transparency we do not formally ensure stationarity by introducing, e.g., costly intermediation—see Schmidt-Grohe and Uribe (2003). The expressions for targeting rules to follow are independent of these costs when $\widetilde{\mathcal{W}}_t$ is exogenous to monetary policy—in the Cole and Obstfeld economy specified in the next section and more in general under LCP under the restriction $\sigma = 1$.

[28] By adding up all the Phillips Curves in (20), it is clear that, to the extent that world inflation is zero under the optimal policy, the sum of output gaps is also zero. In addition, also consumption deviations sum up to zero, i.e., we can write $\widetilde{D}_t \equiv \widetilde{C}_t - \widetilde{C}^{*}_t = 2\widetilde{C}_t$. These results also hold in the natural rate allocation.

Proposition 3: *In the PCP bond-economy, the optimal policy under cooperation and commitment is characterized by the global rule (22) in conjunction with the following cross-country targeting rule:*

$$\begin{aligned} 0 \quad = \quad & \left[\theta\left(\pi_{H,t}-\pi^*_{F,t}\right)+\left(\widetilde{Y}_{H,t}-\widetilde{Y}_{H,t-1}\right)-\left(\widetilde{Y}_{F,t}-\widetilde{Y}_{F,t-1}\right)\right] \\ & +2\frac{2a_{\mathrm{H}}(1-a_{\mathrm{H}})\phi}{\sigma+\eta(4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1)}\frac{2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma}{2a_{\mathrm{H}}(\phi-1)+1}\left(\widetilde{\mathcal{W}}_t-\widetilde{\mathcal{W}}_{t-1}\right), \end{aligned} \tag{23}$$

which holds without the need to impose parametric restrictions on σ,η and ϕ.

Proof. See the Appendix.■

In a bond economy, the optimal cross-country targeting rule introduces a trade-off between output gaps and inflation rates on the one hand, and the wealth gap on the other hand, which is absent under complete markets. As shown for instance by Engel (2011) and CDL (2010), the cross-country targeting rule in this case is given by

$$0=\theta\left(\pi_{H,t}-\pi^*_{F,t}\right)+\left(\widetilde{Y}_{H,t}-\widetilde{Y}_{H,t-1}\right)-\left(\widetilde{Y}_{F,t}-\widetilde{Y}_{F,t-1}\right). \tag{24}$$

Combining the global and cross-country rules for bond economies, we can further derive country-specific (cooperative) rules. For the Home economy, this rule is:

$$0=\left[\theta\pi_{H,t}+\left(\widetilde{Y}_{H,t}-\widetilde{Y}_{H,t-1}\right)\right]+\frac{2a_{\mathrm{H}}(1-a_{\mathrm{H}})\phi}{\sigma+\eta(4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1)}\frac{2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma}{2a_{\mathrm{H}}(\phi-1)+1}\left(\widetilde{\mathcal{W}}_t-\widetilde{\mathcal{W}}_{t-1}\right)$$

from which we derive the following important corollary.

Corollary 1: *Under PCP, if either markets are complete ($\widetilde{\mathcal{W}}_t=0$) or setting $\sigma=\phi=1$ in a bond economy, the optimal policy can be characterized by a pair of country-specific rules, which are a function of purely domestic objectives. For the Home country, such rule is:*

$$0=\theta\pi_{H,t}+\left(\widetilde{Y}_{H,t}-\widetilde{Y}_{H,t-1}\right). \tag{25}$$

Proof. Set either $\widetilde{\mathcal{W}}_t=0$ or $\sigma=\phi=1$ in (23) and combine with (22).■

According to the rule spelled out in Corollary 1, each country would stabilize its own output gap and GDP-deflator inflation—a notable (and widely discussed) case of "isomorphism" of optimal policy in closed and open economies. With full insurance and absent exogenous markup shocks, stabilizing inflation would completely close all gaps too—an instance of "divine coincidence" in open economy. When markets are incomplete, however, the divine coincidence breaks down. With $\widetilde{\mathcal{W}}_t\neq 0$, even when $\sigma=\phi=1$ and the targeting rule simplifies to (25), monetary policy faces a significant trade-off between inflation and the output gap: stabilizing inflation will not close all gaps in the economy. We will see in Section 4 below that, since complete ERPT magnifies the effects of currency movements on the output gap, the optimal policy under PCP will weigh more on stabilizing misalignment and the output gap, at the expense of inflation stabilization.

3.4.2 Incomplete pass-through (LCP) economies

In the LCP case, tractable expressions for the cross-country rule can be derived only under restrictive assumptions. Notably, under the assumptions that markets are complete a tractable rule is derived by Engel (2011) positing that the labor elasticity is infinite ($\eta = 0$). In this paper we generalize this finding, showing that, as long as $\eta = 0$, a tractable cross-country targeting rule can be derived also under incomplete markets. This novel result is stated below, whereas we present the targeting rule explicitly writing out the demand gap $\widetilde{\mathcal{D}}$, as a way to offer a direct and meaningful comparison with the rule derived by Engel (2011) under complete markets.[29]

Proposition 4: *Under LCP, if $\eta = 0$, the optimal policy under cooperation and commitment is fully characterized by the general global rule (22) and the following cross-country (difference) rule:*

$$\begin{aligned} 0 &= \theta\left(\pi_t - \pi_t^*\right) + \widetilde{\mathcal{D}}_t - \widetilde{\mathcal{D}}_{t-1} \\ &+ \frac{4a_{\rm H}\left(1-a_{\rm H}\right)\phi}{2a_{\rm H}\left(\phi-1\right)+1}\frac{\left(\sigma-1\right)}{\sigma}\left[\left(\widetilde{\mathcal{W}}_t + \widetilde{\Delta}_t\right) - \left(\widetilde{\mathcal{W}}_{t-1} + \widetilde{\Delta}_{t-1}\right)\right], \end{aligned} \tag{26}$$

where $a_{\rm H}\pi_{H,t} + (1-a_{\rm H})\,\pi_{F,t} = \pi_t$ and $(1-a_{\rm H})\,\pi^*_{H,t} + a_{\rm H}\pi^*_{F,t} = \pi^*_t$.

Proof. See the Appendix.■

A remarkable property of LCP economies under incomplete markets (somehow missed by the literature so far) allows us to derive a simpler version of the above rule. Namely, for the case of complete markets, Engel (2011) shows that, as long as $\eta = 0$, the relative prices $\widetilde{\mathcal{T}}_t + \widetilde{\Delta}_t$ are exogenous with respect to monetary policy. We further establish that, under the additional restriction that agents have log-utility, i.e., $\sigma = 1$, the same result holds under incomplete markets. Most importantly, under the same restrictions, also capital flows and the wealth gap are unaffected by monetary policy.[30]

Proposition 5. *In LCP bond economies, as long as $\eta = 0$ and $\sigma = 1$, relative prices $\widetilde{\mathcal{T}}_t + \widetilde{\Delta}_t$, cross-border capital flows ($\widetilde{\mathcal{B}}_t$) and the wealth gap ($\widetilde{\mathcal{W}}_t$) are independent of monetary policy for any value of trade elasticities ϕ.*

Proof. See the Appendix.■

To gain insight on the economics of Proposition 5, recall that, under the assumption that $\eta = 0$, i.e., when the disutility of labor is linear and the labor supply infinitely elastic, wages and marginal costs are only affected by the marginal utility of consumption—not by the marginal disutility of labor. This implies that both cross-country marginal costs differentials and the relative price term $\widetilde{\mathcal{T}}_t+\widetilde{\Delta}_t$ are entirely determined by cross-country aggregate demand conditions. Under incomplete markets $\widetilde{\mathcal{W}}_t$ is a key driver of these conditions, but with log utility and $\sigma = 1$ $\widetilde{\mathcal{W}}_t$ is exogenous to policy, in turn implying that $\widetilde{\mathcal{T}}_t + \widetilde{\Delta}_t$ is also exogenous.

Note that, when $\widetilde{\mathcal{T}}_t+\widetilde{\Delta}_t$ is exogenous to policy, a Home monetary expansion that depreciates the Home currency simultaneously widens the LOOP gap $\widetilde{\Delta}_t$ and strengthens the terms of trade in the same proportion. Nominal exchange rates and terms of trade thus move opposite from each other. This sharply differentiates LCP from PCP economies, where a currency depreciation invariably results in weaker terms of trade.

As a corollary of our results so far, setting $\sigma = 1$, we can combine the global and the cross-

[29] Analytically, the difference in the coefficients in front of the wealth gap in the targeting rules under PCP and LCP stems from the fact that both the budget constraint and the Phillips Curve are different in the two models. The implications for the economics of the targeting rules are best appreciated through the analysis of the macro-dynamics in Section 4 and especially 5 below.

[30] The last term on the right-hand side of the optimal rule (26) drops out when $\sigma = 1$: the expression for the cross-country rule (26) is the same under both complete and incomplete markets. However, as explained in the text, it does not follow that monetary policy is the same in the two cases.

country rule, to rewrite the optimal (cooperative) policy in terms of two symmetric country-specific rules.[31]

Corollary 2. *In LCP bond economies, as long as $\eta = 0$, $\sigma = 1$, the targeting rule for the Home economy is as follows*

$$
\begin{aligned}
0 &= \theta\pi_t + \left(\widetilde{C}_t - \widetilde{C}_{t-1}\right), \\
0 &= \theta\pi_t + 1/2 \cdot \left[\left(\widetilde{\mathcal{W}}_t - \widetilde{\mathcal{W}}_{t-1}\right) + \left(\widetilde{\mathcal{Q}}_t - \widetilde{\mathcal{Q}}_{t-1}\right)\right] \qquad (27)
\end{aligned}
$$

where the last expression (27) follows under the maintained assumption of no markup shocks.

Proof. Set $\sigma = 1$ in (26) and combine with (22), noting that in equilibrium under symmetry $\widetilde{C}_t + \widetilde{C}_t^* = \widetilde{Y}_{H,t} + \widetilde{Y}_{F,t} = 0$, where the last equality holds absent markup shocks.■

When markets are complete ($\widetilde{\mathcal{W}}_t = 0$), the rule (27) reduces to the expression derived by Engel (2011): with full risk insurance, provided that shocks are "efficient" (i.e., they affect tastes and/or technology only), the optimal policy sets CPI inflation rates to zero. A zero inflation policy closes the consumption gap and eliminates real exchange rate misalignment at once—reflecting the fact that these gaps are proportional to each other.[32] This is not possible when markets are incomplete, since $\widetilde{\mathcal{W}}_t \neq 0$ creates a trade-off between stabilizing inflation and mitigating relative demand gaps and misalignment. We will see that, since a low pass-through mutes the effects of the exchange rate on the output gap, the optimal policy will focus on stabilizing demand rather than misalignment (in contrast to the case of PCP).

4 Exchange rate pass through and optimal policy trade-offs

In the rest of the paper, we analyze the optimal conduct of monetary policy in economies that experience inefficient capital inflows and study the macroeconomic dynamics resulting from the implementation of the optimal targeting rules spelled out in the previous section, contrasting PCP and LCP. We find it convenient to present our results in two steps. As a first step, in this section, we specify a bond economy with log-consumption utility ($\sigma = 1$) and linear disutility of labor ($\eta = 0$)—two restrictions motivated by tractability in the case of LCP—as well as a unitary trade elasticity ($\phi = 1$). Because of the latter assumption, we dub this model specification the "Cole and Obstfeld" or CO economy, after Cole and Obstfeld (1991). In this CO economy excessive capital inflows are invariably associated with overappreciation. Most crucially, $\sigma = \phi = 1$ and $\eta = 0$ ensure that capital inflows reflecting agents' saving choices are exogenous to monetary policy and independent of the specification of nominal rigidities in export pricing (PCP or LCP). This allows us to flesh out how optimal policy depends on ERPT, holding constant the size of the flows, which facilitates the comparison across PCP and LCP.

As a second step, in Section 5, we relax the parametric restriction on the trade elasticity ϕ and show that the optimal monetary policy prescriptions derived in the CO economy remain

[31] Recall that absent exogenous markup shocks, global inflation and global output gaps are both zero under the optimal policy.

[32] Under LCP closing the real exchange rate misalignment (i.e., setting $\widetilde{\mathcal{Q}}_t = 0$) does not necessarily eliminate exchange rate variability and deviations from the law of one price—nor prevent inefficient deviations from the law of one price $\widetilde{\Delta}_t$ from mapping into output gap fluctuations. Because of nominal distortions in import and export pricing in local currency, the optimal constrained allocation cannot be first best, whether or not risk sharing is complete.

valid in response to excessive capital inflows (outflows) that overappreciate (underappreciate) the currency. In addition, we also show that, given home bias in demand, the equilibrium link between inefficient capital flows and misalignment changes sign for a sufficiently low trade elasticity—i.e., inflows driven by news shocks become associated with currency undervaluation and reduce relative domestic consumption. In this case, irrespective of ERPT, the optimal policy places much more weight on supporting demand—the optimal stance is expansionary under both PCP and LCP.

For the sake of analytical clarity, with little loss of generality we will focus the analysis on "news" shocks. As shown in Section 2.4, in the first-best allocation, the current values of macro variables do not respond to news foreshadowing changes in future fundamentals: the response of "gaps" (in anticipation of future changes in technology and preferences) thus coincides with the response in the equilibrium allocation until the anticipated shock materializes—with obvious gains in tractability and analytical transparency.[33]

4.1 A "Cole and Obstfeld" economy with capital flows exogenous to policy

It is well understood that in an environment with a Cobb-Douglas aggregator of domestic and imported goods ($\phi = 1$), log consumption utility ($\sigma = 1$) and symmetric home bias, production risk is efficiently shared via endogenous terms-of-trade movements, regardless of whether financial markets are complete or not (this applies to, e.g., productivity and markup shocks). However, terms of trade movements do not necessarily provide insurance against other sources of risk, ranging from political risk (i.e., capital controls; see, e.g., Acharya and Bengui 2018), to shocks to financial intermediation (see, e.g., Gabaix and Maggiori 2015]) and/or preference for foreign assets (see, e.g., Cavallino 2019), or preference shocks impinging on savings. As many of these shocks have broadly similar analytical representations, we will consider shocks to preferences that affect the intertemporal valuation of consumption, thus resulting in a motive to save and lend across borders.

4.1.1 Exogeneity of capital flows and the wealth gap

As shown in Table 1, in the first-best allocation, no macro variable (but the long-term interest rate) responds to news shocks. In our CO economy specification with $\sigma = \phi = 1$, the expression for our (notional) measure of efficient flows across borders (14) simplifies to:

$$\widehat{\mathcal{B}}_t^{fb} - \beta^{-1}\widehat{\mathcal{B}}_{t-1}^{fb} = -(1 - a_{\mathrm{H}})\left(\widehat{\zeta}_{C,t} - \widehat{\zeta}_{C,t}^{*}\right). \tag{28}$$

Moreover, when $\eta = 0$, news shocks have no effect on the first-best responses of exchange rates and relative prices at any time:[34]

$$\widehat{\mathcal{Q}}_t^{fb} = (2a_{\mathrm{H}} - 1)\,\widehat{\mathcal{T}}_t^{fb} = 0$$

[33] Devereux and Engel (2009) further emphasize that the analysis of "news shocks" highlights the forward-looking nature of exchange rate determination.

[34] With $\sigma = \phi = 1$,but $\eta > 0$, Home preference shocks in favor of current consumption systematically result in an "efficient" Home currency real appreciation:

$$\widehat{\mathcal{Q}}_t^{fb} = -\frac{\eta}{1+\eta}\,(2a_{\mathrm{H}} - 1)^2\left(\widehat{\zeta}_{C,t} - \widehat{\zeta}_{C,t}^{*}\right)$$

With trade in bonds, then, any borrowing/lending and any exchange rate movement in response to news shocks will provide a direct measure of welfare-relevant gaps. Specifically, compare the notional capital flows in the first best (28) with the flow of net foreign assets, given by the following expression:

$$\widehat{\mathcal{B}}_t = \widehat{\mathcal{B}}_{t-1} + (1 - a_{\mathrm{H}})\,\beta \sum_{j=0}^{\infty} \beta^j E_t \left[\left(\widehat{\zeta}_{C,t+1+j} - \widehat{\zeta}^*_{C,t+1+j} \right) - \left(\widehat{\zeta}_{C,t+j} - \widehat{\zeta}^*_{C,t+j} \right) \right]. \qquad (29)$$

An anticipated future fall in the relative degree of impatience $\left(\widehat{\zeta}_{C,t+1+j} - \widehat{\zeta}^*_{C,t+1+j} < 0\right)$ would cause capital to flow into the Home country when agents trade bonds (recall that a negative $\widehat{\mathcal{B}}_t$ denotes inflows into the Home country), while triggering no (notional) efficient flows under perfect risk sharing. Note that the size of the inefficient borrowing and lending is increasing in openness (i.e., decreasing in home bias a_{H}).

Any inefficient capital flow in turn opens a wealth gap:

$$(1 - a_{\mathrm{H}})\,\widetilde{\mathcal{W}}_t = -\left(\widehat{\mathcal{B}}_t - \beta^{-1}\widehat{\mathcal{B}}_{t-1} \right) - (1 - a_{\mathrm{H}}) \left(\widehat{\zeta}_{C,t} - \widehat{\zeta}^*_{C,t} \right). \qquad (30)$$

The expressions (29) and (30) highlight two important properties of the CO economy. First, both $\widehat{\mathcal{B}}_t$ and $\widetilde{\mathcal{W}}_t$ are a function of the exogenous preference shocks only, and therefore independent of nominal rigidities and monetary policy.[35] Second, a capital inflow ($\widehat{\mathcal{B}}_t < 0$) driven by news shocks will invariably lead to a positive wealth gap ($\widetilde{\mathcal{W}}_t > 0$). As the Home economy accommodates a higher desire to save among Foreign residents, the relative Home demand $\widetilde{\mathcal{D}}_t$ is too large, and/or, the real exchange rate appreciates too much.[36]

4.1.2 Capital market imperfections distort the natural rate allocation

Before delving into our analysis of monetary policy, we find it appropriate to stress that, with imperfect insurance, inefficient capital flows open a wealth gap and result in misallocation independently of price stickiness. This is apparent in Table 2, showing the natural rate (flexible price) allocation for the CO economy. In this table, all variables are expressed in terms of deviations from the first best allocation—the "welfare-relevant gaps" in the natural allocation are denoted with a superscript "*na*."

In the CO economy, under flexible prices, output gaps, exchange rate misalignment and the relative demand gap are all proportional to the (exogenous) gap $\widetilde{\mathcal{W}}_t$. When $\widetilde{\mathcal{W}}_t > 0$ and $\widehat{\mathcal{B}}_t < 0$ (as is the case in response to news shocks), capital inflows result in a negative output gap, an overvalued real exchange rate, and an excessive level of domestic consumption, both in absolute terms, $\widetilde{C}^{na}_t$, and relative to Foreigners, $\widetilde{\mathcal{D}}^{na}_t$. Through their effects on $\widetilde{\mathcal{W}}_t$ the inefficiencies in

[35] In the Appendix, we also show that the exogeneity of $\widehat{\mathcal{B}}_t$ and $\widetilde{\mathcal{W}}_t$ remains unaffected if cross-border flows are subject to costly intermediation in the vein of Gabaix and Maggiori (2015) so that both $\widehat{\mathcal{B}}_t$ and $\widetilde{\mathcal{W}}_t$ do not display a unit root behavior—a result emphasized by Cavallino (2019). Therefore, optimal targeting rules for the CO economy are the same as derived in Section 3 under both PCP and LCP.

[36] From (29) and (30), it should also be clear that both $\widehat{\mathcal{B}}_t$ and $\widetilde{\mathcal{W}}_t$ can be negative in response to contemporaneous (as opposed to "news") taste shocks, which raise the utility of current Home consumption (and associated with a relative increase in efficient output, $\widehat{Y}^{fb}_{H,t} - \widehat{Y}^{fb}_{F,t} > 0$). In this case, although capital flows into the Home country, domestic consumption is inefficiently low relative to the foreign one: in this case the inflow is inefficiently low. A key difference between contemporaneous and news shocks to preferences is that, with the former, $\widehat{\mathcal{B}}_t$ and $\widetilde{\mathcal{W}}_t$ have the same sign, while with the latter they have the opposite sign. Nevertheless, optimal policy in the CO economy would be still determined by the sign of $\widetilde{\mathcal{W}}_t$.

the shock transmission are purely redistributive: the Foreign economy just mirrors the Home responses. Note that the equilibrium adjustment to shocks requires Home real appreciation as long as $a_{\rm H} > 1/2$. Intuitively, the capital inflow into Home amounts to a *transfer* of purchasing power from abroad. Because of home bias in demand, if relative prices did not adjust, the transfer would translate into an excess supply of Foreign goods. We will return on this consideration in Section 5.

Table 2.
The natural rate allocation in the CO economy

$\widetilde{Y}^{na}_{H,t} = -\widetilde{Y}^{na}_{F,t} = -(1-a_{\rm H})\,\widetilde{\mathcal{W}}_t$
$\widetilde{\mathcal{T}}^{na}_t = -\widetilde{\mathcal{W}}_t$
$\widetilde{\mathcal{Q}}^{na}_t = -(2a_{\rm H}-1)\,\widetilde{\mathcal{W}}_t$
$\widetilde{\mathcal{D}}^{na}_t = 2\,(1-a_{\rm H})\,\widetilde{\mathcal{W}}_t$
$\widetilde{C}^{na}_t = -\widetilde{C}^{*na}_t = \frac{1}{2}\widetilde{\mathcal{D}}^{na}_t = (1-a_{\rm H})\,\widetilde{\mathcal{W}}_t$

By the properties of the linearized equilibrium, while in response to news shocks all gaps widen on impact with $\widetilde{\mathcal{W}}_t \neq 0$, they remain constant thereafter—since $E_t\widetilde{\mathcal{W}}_{t+1} = \widetilde{\mathcal{W}}_t$.[37] As a result, in the time span between the arrival of the news and the future change in fundamentals, the short-term natural rate of interest (equal to the expected growth rate of consumption under flexible prices) is not affected by the news shock.[38]

As is well understood, the natural rate allocation corresponds to an allocation with price stability under PCP. It can be shown further that by virtue of the specific properties of our CO economy, the expressions for consumption demand and relative demand in Table 2 would also hold under LCP if monetary policy perfectly stabilize the CPI. This result will provide a useful benchmark for the analysis of LCP economies below.

4.2 The sign of the optimal monetary stance depends on pass-through

A comparative analysis of PCP and LCP economies is particularly suitable in the CO specification, since in response to identical shocks, the sign and size of the ensuing capital flows and wealth gap—that is, the expressions for $\widehat{\mathcal{B}}_t$ and $\widetilde{\mathcal{W}}_t$ in (30) and (29)— are exactly the same, regardless of ERPT. All our results to follow will thus be conditional on the same news shock resulting in the same capital inflow $\widehat{\mathcal{B}}_t < 0$ and the same positive wealth gap $\widetilde{\mathcal{W}}_t > 0$.

The key contribution of our comparative analysis consists of highlighting and clarifying the role of ERPT, hence the role of the exchange rate as a determinant of global and relative demand for domestic goods, in shaping the optimal cooperative policy response to capital inflows. Under PCP, the monetary stance will be expansionary and inflationary at Home, while contractionary and deflationary abroad. Under LCP, the response will be contractionary at Home and expansionary abroad.

[37] When fundamentals change in the future, of course, macroeconomic variables will change again, including both deviations $\widetilde{C}^{na}_{t+s}$ and efficient consumption $\widehat{C}^{fb}_{t+s}$ (but not $\widetilde{\mathcal{Q}}^{na}_t$ if $\eta = 0$).

[38] It follows that, in a monetary policy framework requiring the policy rate to be equal to the short-term natural rate in each period, the short term rate would be initially unresponsive to the capital inflows.

4.2.1 Exchange rate stabilization and misalignment with complete pass-through in PCP economies

Table 3 presents the Home allocation under the optimal cooperative monetary policy in the PCP economy—the Foreign allocation is the symmetric counterpart. In the table, $\varkappa_1$ and $\varkappa_2$ denote, respectively, stable and unstable eigenvalues, linked to each other as formally stated in the following Lemma 1.[39]

Lemma 1. *For a probability of price changes* $0 < \alpha < 1$, *the variables (eigenvalues)* $\varkappa_1$ *and* $\varkappa_2$ *are related as follows:*

$$0 < \varkappa_1 < 1 < \beta^{-1} + \frac{(1-\alpha\beta)(1-\alpha)}{\alpha\beta}\theta < \varkappa_2$$
$$0 < \frac{(\beta\varkappa_2 - 1)}{\beta\varkappa_2} < 1.$$

Table 3
Constrained-efficient allocation under PCP in the CO economy

$\widetilde{Y}_{H,t} = -(1-a_{\mathrm{H}})\frac{(\beta\varkappa_2 - 1)}{\beta\varkappa_2}\widetilde{\mathcal{W}}_t + \varkappa_1\widetilde{Y}_{H,t-1}$
$\theta\pi_{H,t} = (1-a_{\mathrm{H}})\frac{(\beta\varkappa_2 - 1)}{\beta\varkappa_2}\widetilde{\mathcal{W}}_t + (1-\varkappa_1)\widetilde{Y}_{H,t-1}$
$\widetilde{\mathcal{T}}_t = -\left(1 - \frac{2(1-a_{\mathrm{H}})}{\beta\varkappa_2}\right)\widetilde{\mathcal{W}}_t + 2\varkappa_1\widetilde{Y}_{H,t-1}$
$\widetilde{\mathcal{Q}}_t = -(2a_{\mathrm{H}}-1)\left[\left(1 - \frac{2(1-a_{\mathrm{H}})}{\beta\varkappa_2}\right)\widetilde{\mathcal{W}}_t - 2\varkappa_1\widetilde{Y}_{H,t-1}\right]$
$\widetilde{\mathcal{D}}_t = 2(1-a_{\mathrm{H}})\left[1 + \frac{(2a_{\mathrm{H}}-1)}{\beta\varkappa_2}\right]\widetilde{\mathcal{W}}_t + 2(2a_{\mathrm{H}}-1)\varkappa_1\widetilde{Y}_{H,t-1}$

Table 3 highlights two key results. First, *on impact*, the allocation is a function of $\widetilde{\mathcal{W}}_t$ only—because of staggered price stickiness, however, in the periods following the arrival of the new shocks the dynamics under the optimal policy will also respond to the evolution of the output gap. Second, a policy regime of strict GDP deflator stabilization ($\pi_{H,t} = 0$, supporting the natural rate allocation in Table 2) will not be efficient (see Table 1). Rather, the optimal policy will trade off higher inflation variability for greater stabilization of the output gap and misalignment—under PCP, output gaps and misalignment (of the real exchange rate and the terms of trade) are positively related. In other words, in response to an inefficiently large capital inflow, Home monetary authorities lean against the overvaluation of the real exchange rate so as to contain the negative impact on the output gap, at the cost of positive inflation and widening cross-border demand misallocation.

We summarize and prove the salient properties of the allocation under the optimal policy in the following proposition.

Proposition 6. *In the Cole and Obstfeld economy under PCP with* $\sigma = \phi = 1$ *and* $\eta = 0$,

[39] The eigenvalues are given by:

$$\varkappa_{1,2} = \frac{1 + \beta + \frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\theta \pm \sqrt{\left[1 + \beta + \frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\theta\right]^2 - 4\beta}}{2\beta}$$

It can be shown that $\varkappa_1$ is increasing, $\varkappa_2$ is decreasing in the degree of price stickiness α.

the optimal policy response to news shocks generating inefficient capital flows results in a muted impact responses of the real real exchange rate and of the output gap relative to a regime pursuing strict inflation stability; the impact responses of the relative demand gap and GDP deflator are instead amplified.

Proof. Consider news shocks that cause $\widetilde{\mathcal{B}}_{t_0} < 0$ and $\widetilde{\mathcal{W}}_{t_0} > 0$, without loss of generality. Given Lemma 1, the short-run (GDP deflator) inflation in Table 3 is positive under the optimal policy:

$$\pi_{H,t_0} = (1 - a_{\mathrm{H}}) \frac{(\beta \varkappa_2 - 1)}{\theta \beta \varkappa_2} \widetilde{\mathcal{W}}_{t_0} > 0.$$

Compared to a regime of strict inflation stability (i.e., compared with Table 2), where

$$\widetilde{\mathcal{Q}}^{na}_{t_0} = -(2a_{\mathrm{H}} - 1)\, \widetilde{\mathcal{W}}_{t_0},$$

the combination of Home expansion and foreign contraction mitigates, without reversing, the Home exchange rate appreciation and misalignment:

$$\widetilde{\mathcal{Q}}_{t_0} = -(2a_{\mathrm{H}} - 1)\left(1 - \frac{2(1 - a_{\mathrm{H}})}{\beta \varkappa_2}\right) \widetilde{\mathcal{W}}_{t_0} < 0, \tag{31}$$

since $\left(1 - \frac{2(1 - a_{\mathrm{H}})}{\beta \varkappa_2}\right) < 1$. It also makes the Home output gap

$$\widetilde{Y}_{H,t_0} = -(1 - a_{\mathrm{H}}) \frac{(\beta \varkappa_2 - 1)}{\beta \varkappa_2} \widetilde{\mathcal{W}}_{t_0} < \widetilde{Y}^{na}_{H,t} < 0$$

less negative compared to $\widetilde{Y}^{na}_{H,t} = -(1 - a_{\mathrm{H}})\, \widetilde{\mathcal{W}}_t$, since $\frac{(\beta \varkappa_2 - 1)}{\beta \varkappa_2} < 1$ by Lemma 1. The relative demand gap

$$\widetilde{\mathcal{D}}_{t_0} = 2(1 - a_{\mathrm{H}}) \left[1 + \frac{(2a_{\mathrm{H}} - 1)}{\beta \varkappa_2}\right] \widetilde{\mathcal{W}}_{t_0} > 0, \tag{32}$$

is larger than $\widetilde{\mathcal{D}}^{na}_t = 2(1 - a_{\mathrm{H}})\, \widetilde{\mathcal{W}}_t$, since $\left[1 + \frac{(2a_{\mathrm{H}} - 1)}{\beta \varkappa_2}\right] > 1$.■

4.2.2 Domestic demand stabilization with incomplete pass-through in LCP economies

Under LCP, nominal exchange rate movements have limited expenditure switching effects on global demand; capital inflows appreciating the currency do not result in a comparable fall in the relative price of Home goods, nor in a sharp redirection of domestic and foreign demand towards foreign goods. Below we show that in contrast to the case of PCP, monetary authorities will optimally focus on reducing the cross border demand gap combining a Home contraction with a Foreign expansion.

The Home constrained-efficient allocation for our LCP economy is characterized in Table 4, again as a function of the (exogenous) wealth gap (30)—the Foreign allocation is the symmetric counterpart. As in the PCP case, $\varkappa_1$ and $\varkappa_2$ represent stable and unstable eigenvalues, respectively. However, in the LCP economy we have two additional eigenvalues, denoted by

ν_1 (stable) and ν_2 (unstable).[40] We should note that the eigenvalues $\varkappa_2$ and ν_2 determine the discounted value of expectations of future fundamentals in driving the equilibrium dynamics of the real exchange rate and of relative prices $\widetilde{\mathcal{T}}_t + \widehat{\Delta}_t$. Higher values of ν_1 and $\varkappa_1$ (corresponding to higher price stickiness) imply slower adjustments of $\widetilde{\mathcal{T}}_t + \widetilde{\Delta}_t$, misalignment, $\widetilde{\mathcal{Q}}_t$, and the demand gap, $\widetilde{\mathcal{D}}_t$. We again state the relations between eigenvalues in a Lemma.

Lemma 2. *For $0 < \alpha < 1$, the variables (eigenvalues) $\varkappa_1, \nu_1$ and $\varkappa_2, \nu_2$ are related as follows:*

$$\varkappa_2 > \nu_2,$$
$$1 > \frac{(\beta\varkappa_2 - 1)}{\beta\varkappa_2} > \frac{(\beta\nu_2 - 1)}{\beta\nu_2} > 0.$$

Table 4.
Constrained-efficient allocation under LCP in the CO economy

$\widetilde{Y}_{H,t} = 2a_{\mathrm{H}}\left(1 - a_{\mathrm{H}}\right)\left(\widetilde{\mathcal{T}}_t + \widetilde{\Delta}_t\right) + 1/2 \cdot \left(2a_{\mathrm{H}} - 1\right)\widetilde{\mathcal{D}}_t$
$\theta\pi_t = -\left(1 - a_{\mathrm{H}}\right)\frac{(\beta\varkappa_2 - 1)}{\beta\varkappa_2}\widetilde{\mathcal{W}}_t + \frac{1}{2}\left[\frac{(\beta\varkappa_2 - 1)}{\beta\varkappa_2}\widetilde{\mathcal{W}}_{t-1} + (1 - \varkappa_1)\widetilde{\mathcal{Q}}_{t-1}\right]$
$\widetilde{\mathcal{T}}_t + \widetilde{\Delta}_t = -\frac{(\beta\nu_2 - 1)}{\beta\nu_2}\widetilde{\mathcal{W}}_t + \nu_1\left(\widetilde{\mathcal{T}}_{t-1} + \widetilde{\Delta}_{t-1}\right)$
$\widetilde{\mathcal{Q}}_t = -\left(2a_{\mathrm{H}} - 1\right)\frac{(\beta\varkappa_2 - 1)}{\beta\varkappa_2}\widetilde{\mathcal{W}}_t - \frac{1}{\beta\varkappa_2}\left(\widetilde{\mathcal{W}}_t - \widetilde{\mathcal{W}}_{t-1}\right) + \varkappa_1\widetilde{\mathcal{Q}}_{t-1}$
$\widetilde{\mathcal{D}}_t = 2\left(1 - a_{\mathrm{H}}\right)\frac{(\beta\varkappa_2 - 1)}{\beta\varkappa_2}\widetilde{\mathcal{W}}_t + \frac{1}{\beta\varkappa_2}\widetilde{\mathcal{W}}_{t-1} + \varkappa_1\widetilde{\mathcal{Q}}_{t-1}.$

The optimal monetary policy stance follows from assessing the impact response of inflation at Home:

$$\pi_{t_0} = -\left(1 - a_{\mathrm{H}}\right)\frac{(\beta\varkappa_2 - 1)}{\theta\beta\varkappa_2}\widetilde{\mathcal{W}}_{t_0}; \tag{33}$$

and its symmetric counterpart in Foreign, $\pi^*_{t_0} = -\pi_{t_0}$. In light of Lemma 2, the above establishes that, under the optimal cooperative policy, the monetary response to capital inflows into Home (leading to $\widetilde{\mathcal{W}}_{t_0} > 0$) is contractionary and deflationary at Home—expansionary and inflationary abroad—a combination that exacerbates the Home overappreciation. Relative to a regime of strict CPI stability, the optimal policy will thus trade off relative demand stabilization for inflation variability and a larger real exchange rate misalignment. We again summarize the salient properties of the allocation under the optimal policy in a proposition.

Proposition 7. *In the Cole and Obstfeld economy with $\sigma = \phi = 1$ and $\eta = 0$, under LCP,*

[40] Namely for $\nu_{1,2}$:

$$\nu_{1,2} = \frac{1 + \beta + \frac{(1-\alpha\beta)(1-\alpha)}{\alpha} \pm \sqrt{\left[1 + \beta + \frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\theta\right]^2 - 4\beta}}{2\beta}.$$

So $\nu_{1,2}$ differ from $x_{1,2}$only in that the term $\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}$ is not multiplied by θ. As a result, we have the following relations:

$$0 < \varkappa_1 < 1 < \beta^{-1} + \frac{(1-\alpha\beta)(1-\alpha)}{\alpha\beta}\theta < \varkappa_2$$
$$0 < \nu_1 < 1 < \beta^{-1} + \frac{(1-\alpha\beta)(1-\alpha)}{\alpha\beta} < \nu_2,$$
$$\varkappa_2 \geq \nu_2$$

in response to news shocks generating inefficient capital flows, the real exchange rate and CPI inflation react more under the optimal policy than in a regime pursuing strict CPI stability. Relative to this regime, the impact response of the relative demand gap is attenuated, while that of the output gap can be smaller or larger.

Proof. The proof follows from Table 4 and Lemma 2. The fact that Home CPI inflation is not stabilized follows from evaluating the impact response of inflation in Table 4. The impact response of the Home real exchange rate under the optimal policy follows from:

$$\widetilde{\mathcal{Q}}_{t_0} = -\left[(2a_{\mathrm{H}} - 1)\frac{(\beta\varkappa_2 - 1)}{\beta\varkappa_2} + \frac{1}{\beta\varkappa_2}\right]\widetilde{\mathcal{W}}_{t_0}. \tag{34}$$

Since the expression in square brackets is greater than one and thus greater than $(2a_{\mathrm{H}} - 1)$, the impact response is larger in absolute value than under CPI price stability—whereas the expression for the real exchange rate under CPI stability coincides with $\widetilde{\mathcal{Q}}_t^{na} = -(2a_{\mathrm{H}} - 1)\widetilde{\mathcal{W}}_t$ (see Section 4.1.2).

The optimal policy attenuates the impact response of relative demand $\widetilde{\mathcal{D}}_{t_0}$ compared to strict CPI stability, since

$$\left\|\widetilde{\mathcal{D}}_{t_0}\right\| = 2(1 - a_{\mathrm{H}})\frac{(\beta\varkappa_2 - 1)}{\beta\varkappa_2}\left\|\widetilde{\mathcal{W}}_{t_0}\right\| < \left\|\widetilde{\mathcal{D}}_{t_0}^{na}\right\| = 2(1 - a_{\mathrm{H}})\left\|\widetilde{\mathcal{W}}_{t_0}\right\| \tag{35}$$

whereas the first inequality holds since $\dfrac{(\beta\varkappa_2 - 1)}{\beta\varkappa_2} < 1$, and we use the fact that the expression for relative demand under CPI stability coincides with $\widetilde{\mathcal{D}}_t^{na}$.

Finally, to show that the response of the output gap can be smaller or larger than under strict CPI stability, we first rewrite the expression in Table 4 as follows:

$$\begin{aligned}
\widetilde{Y}_{H,t_0} &= 2a_{\mathrm{H}}(1 - a_{\mathrm{H}})\left(\widetilde{\mathcal{T}}_{t_0} + \widetilde{\Delta}_{t_0}\right) + 1/2 \cdot (2a_{\mathrm{H}} - 1)\widetilde{\mathcal{D}}_{t_0} \\
&= -(1 - a_{\mathrm{H}})\left[1 - 2a_{\mathrm{H}}\left(1 - \frac{(\beta\nu_2 - 1)}{\beta\nu_2}\right) + (2a_{\mathrm{H}} - 1)\left(1 - \frac{(\beta\varkappa_2 - 1)}{\beta\varkappa_2}\right)\right]\widetilde{\mathcal{W}}_{t_0},
\end{aligned}$$

noting that the output gap under strict CPI stability, $\widetilde{Y}_{H,t_0}^{CPI}$, is given by:

$$\widetilde{Y}_{H,t_0}^{CPI} = -(1 - a_{\mathrm{H}})\left[1 - 2a_{\mathrm{H}}\left(1 - \frac{(\beta\nu_2 - 1)}{\beta\nu_2}\right)\right]\widetilde{\mathcal{W}}_{t_0}.$$

The result directly follows from comparing the two expressions using Lemma 2 and noting that the term in square brackets in the latter expression can be positive or negative, while the last term in square brackets in the expression for $\widetilde{Y}_{H,t_0}$ (i.e. $(2a_{\mathrm{H}} - 1)\left(1 - \dfrac{(\beta\varkappa_2 - 1)}{\beta\varkappa_2}\right)$) is always positive. Simple algebra shows that the latter fact implies that $\widetilde{Y}_{H,t_0}$ is always larger than $\widetilde{Y}_{H,t_0}^{CPI}$ in absolute value when the following condition holds:

$$2a_{\mathrm{H}} < \frac{1 + \dfrac{(\beta\varkappa_2 - 1)}{\beta\varkappa_2}}{1 + \dfrac{(\beta\varkappa_2 - 1)}{\beta\varkappa_2} - 2\dfrac{(\beta\nu_2 - 1)}{\beta\nu_2}}.$$

■

The proposition illustrates how the targeting rule (27) works. In response to a capital inflow into the Home economy, the (constrained-) optimal contractionary stance at Home (matched by the expansion abroad) contains the inefficient surge in Home demand relative to the Foreign one. However, concerns about inflation stabilization imply that the cooperative policy falls short of fully closing the demand gap.

By Lemma 2 we can establish that, under LCP, the output gap response to capital inflows is not necessarily negative—neither under the optimal policy nor under strict CPI stabilization (i.e., $\widetilde{Y}_{H,t_0} \lesseqgtr 0$, and $\widetilde{Y}^{CPI}_{H,t_0} \lesseqgtr 0$). Intuitively, the output gap is non-negative on impact if the positive effect of the capital inflow on the relative demand gap, $\widetilde{\mathcal{D}}_{t_0}$ outweighs the negative (and exogenous) effect of the terms-of-trade gap and deviations from the LOOP, $\widetilde{\mathcal{T}}_{t_0} + \widetilde{\Delta}_{t_0}$. It is easy to see that, on impact, the output gap $\widetilde{Y}_{H,t_0}$ is positive if the following condition is satisfied:

$$2a_{\mathrm{H}} > \frac{\beta\varkappa_2 - 1}{\frac{\beta\varkappa_2}{\beta\nu_2} - 1}.$$

This condition is more likely to hold in economies that are relatively closed (i.e., economies with a high home bias a_{H})—in the expression for the output gap above, openness increases the relative weight of $\left(\widetilde{\mathcal{T}}_{t_0} + \widetilde{\Delta}_{t_0}\right)$ and decreases that of $\widetilde{\mathcal{D}}_{t_0}$.[41]

Finally, we can also shed light on how the optimal policy and the economic dynamics change with the degree of nominal rigidities (and thus ERPT) and openness. As stated in the following corollary, the real exchange rate responds more in LCP economies where good prices are more flexible and home bias a_{H} is higher (the economy is less open).

Corollary 3. *The impact response of the real exchange rate in (34) and of the relative demand gap in (35) are, respectively, increasing and decreasing in both* $0 < \alpha < 1$ *and* $1 > a_{\mathrm{H}} \geq 1/2$.

Intuitively, for a given exogenous wealth gap $\widetilde{\mathcal{W}}_{t_0}$, in economies that are less open (a higher a_{H}), the optimal monetary policy becomes more concerned with the relative demand gap fueled by inefficient capital flows, and tolerates a larger misalignment. Similarly, if prices are stickier (a higher α), ERPT is lower. The optimal monetary policy is less concerned with redressing misalignment, since exchange rate movements are less consequential for the domestic output gap. Therefore, it attaches a larger weight to mitigating the relative demand gap.

4.3 Summary and discussion

This section has been devoted to the analysis of CO economies, where capital flows are exogenous to monetary policy and excessive inflows are associated with overappreciation and a relative demand boom. We have shown that when pass-through is complete (under PCP) and thus relative prices greatly affects output gaps, the optimal monetary policy focuses on stabilizing misalignment, at the expense of larger movements in $\widetilde{\mathcal{D}}_t$ and domestic demand. When pass-through is incomplete (under LCP), however, the exchange rate has limited expenditure switching effects and thus a little impact on output gaps. Optimal monetary policy focuses on

[41] The inequality is always violated (for any degree of openness), in the limit case where prices are almost flexible ($\varkappa_2 \simeq \nu_2 \to \infty$). Observe that from the last part of Proposition 7, if $\widetilde{Y}_{H,t_0} > 0$ when $\widetilde{\mathcal{W}}_{t_0} > 0$, then it is also *smaller* and more stabilized than $\widetilde{Y}^{CPI}_{H,t_0}$.

stabilizing $\widetilde{\mathcal{D}}_t$ and domestic demand at the expense of larger movements in misalignment.

Figure 1 offers a synthetic comparison of macroeconomic dynamics under the optimal policy in the PCP and LCP economies. The figure plots the impulse responses of the relevant gaps to a preference shock anticipated to occur 20 quarters in the future (whose materialization is intentionally left out of the time scale of the graph),[42] causing an immediate inflow of capital in the Home economy. The shock is normalized to produce an initial capital inflow as high as 1 percent of Home GDP.[43]

Recall that in the CO economy, both the capital inflows and the wealth gap are exogenous to macroeconomic adjustment and policy, hence independent of the monetary policy stance under LCP and PCP. As shown by the first graph in the upper left corner, the stock of foreign debt increases exogenously along the optimal adjustment path. The size of capital flows is excessive: the wealth gap (shown in the graph in the upper right corner) jumps to a positive value and remains constant, according to (15).

The remaining graphs in the figure instead highlight the different endogenous responses in the LCP economy (continuous lines) and the PCP economy (dashed lines). The price response (lower left corner) shows that the monetary stance is relatively expansionary under PCP (GDP-deflator inflation is positive), contractionary under LCP (CPI inflation is negative). Given identical shocks and parameters (but for import price stickiness), under the optimal policy, the real exchange rate is always less volatile under PCP (where monetary authorities lean against appreciation) than under LCP (where monetary authorities exacerbate misalignment).

It is worth recalling that, by the properties of the CO specification, the real exchange rate response in the LCP economy under CPI targeting, is the same as in the PCP economy under GDP deflator targeting—and thus equal to the response in the natural rate allocation, $\widehat{\mathcal{Q}}_t^{na} = -(2a_{\mathrm{H}} - 1)\,\widetilde{\mathcal{W}}_t$. Relative to this benchmark, we have shown that the optimal policy mutes the real exchange rate movements under PCP, and amplifies them under LCP. Correspondingly, the real exchange rate always undershoots its long-run value under PCP—while it overshoots it under LCP. Note however that, because of the expenditure-switching effects of the exchange rate on demand, in the short run the output gap remains more negative under PCP than under LCP—in spite of the fact that the policy stance is expansionary and thus contains the overappreciation.[44]

To conclude our analysis of the CO economy, two comments are in orders. First, when discussing the Phillips curves (20), we stressed that the wealth gap is 'isomorphic' to exogenous markup shocks. By no means this implies that wealth gaps and markup shocks elicit the same monetary policy responses. From the literature, we know that the Home response to an exogenous inflationary markup shock that causes real appreciation is always contractionary,

[42] Recall that when shocks materialize, gaps no longer coincide with actual variables deviations from steady state, complicating the interpretation of the graphical analysis.

[43] The parameter values are as follows: $\eta = 0, \phi = \sigma = 1, a_{\mathrm{H}} = .75, \beta = .99, \alpha = .75, \theta = 3$.

[44] Analytically, this follows from comparing the expression for the output gaps under PCP and LCP in light of the fact that:

$$(1 - a_{\mathrm{H}})\,\frac{(\beta\varkappa_2 - 1)}{\beta\varkappa_2} > (1 - a_{\mathrm{H}})\left[1 - 2a_{\mathrm{H}}\left(1 - \frac{(\beta\nu_2 - 1)}{\beta\nu_2}\right)\right],$$

since $\nu_2 < \varkappa_2$.Under LCP, since the Home stance is more contractionary than in a regime of strict CPI stabilization, it mitigates the demand effects of capital inflows on the the Home output gap—which is always smaller than in a regime of strict CPI stabilization.

irrespective of LCP and PCP.[45] Conversely, from the analysis in this section, we have seen that the Home policy response to an appreciation following capital inflows is expansionary under PCP, but contractionary under LCP.

As for our second comment, observe that, while our results for the CO economy are derived under commitment and a timeless perspective, they can readily be brought to bear on the case of cooperation under discretion. In general, the analytical characterization of the targeting rules under discretion is complicated by the fact that optimal policy is a function of, and at the same time affects, the dynamic of foreign debt accumulation. However, when capital flows and wealth gaps are exogenous to monetary policy, as is the cased in the CO economy, the targeting rules under discretion are not a function of the dynamic evolution of debt. Thus, since time inconsistency stems from staggered price stickiness à la Calvo, these rules can be easily derived from those under commitment given above —simply crossing out lagged terms.

5 Optimal policy with over and underappreciation of the exchange rate

In this section, we relax our key parametric restriction on the trade elasticity. Without the CO restriction on the trade elasticity, capital flows will respond not only to shocks to preferences for saving (or changes in taxes or capital controls) but also to productivity shocks.[46]

Modelling a general value of the trade elasticity allows us to underscore two important results. First, unlike the CO economy in the previous section, capital flows and the wealth gap will not necessarily be exogenous to monetary policy and independent of macroeconomic conditions. We will be able to bring the model to bear on the conditions under which monetary policy affects capital flows and the wealth gap.

Second, and most crucially, we will show that the main policy insights from the CO economy remain unchanged for values of the trade elasticity ϕ (smaller than but close to one, or larger than one) typically assumed in the literature—whereas for these values excessive capital inflows remain associated with overappreciation and excessive demand in the Home economy. The degree of ERPT will still be the crucial determinant of the optimal monetary response—expansionary under PCP, contractionary under LCP. At the same time, we will also show that, for sufficiently low values of the trade elasticity, news shocks can generate capital inflows that are associated with undervaluation, rather than overappreciation of the currency. In this case, the optimal monetary policy will deviate from the CO economy—dictating a Home expansion in support of domestic demand in both PCP and LCP economies.

[45] This is a well-known result in the literature under complete markets, see, e.g., Engel (2011) or CDL (2010). Intuitively, exogenous markup shocks do not cancel out when summing the Phillips curve across countries and thus affect global inflation. This is in contrast with the wealth gap $\widetilde{\mathcal{W}}_{t_0}$, which enters the country specific Phillips curves with the opposite sign. Since the global output gap and inflation have to sum to zero in the optimal-constrained allocation, as shown by Proposition 2, an exogenous inflationary markup shock will make the global output gap positive eliciting a negative global inflation and a contractionary monetary policy at least in the country where the shock is stronger.

[46] We study this type of business cycle disturbances in detail in the Appendix, see subsections 2.1.4 and 2.2.3.

5.1 The response of the wealth gap and capital flows to monetary policy

In the CO economy studied in the previous section, the wealth gap and the capital flows are exogenous to monetary policy. In addition, in Proposition 5 we highlighted a key property of the workhorse incomplete market model—that, under LCP, provided consumption utility is logarithmic, capital flows and the wealth gap remain exogenous as in the CO specification, even if $\phi \neq 1$. However, it is usually accepted that, when capital flows into a country, a monetary expansion that reduces interest rates and depreciates the exchange rate reduces the incentive for foreign investors to lend to the country, hence reduces the size of the capital inflow. In general, if capital flows respond to monetary policy, so does the wealth gap. Bringing our model to bear on this feedback, we now provide analytical insight on what shapes the equilibrium response of capital flows and the wealth gap to a monetary expansion. In particular, we show that this response depends on a host of structural features which include, in addition to the trade elasticity, risk aversion, openness, and ERPT.

The following proposition states sufficient conditions under which an expansionary monetary policy always curbs the size of inefficient capital inflows, under either PCP or LCP, in line with the conventional view. We express these conditions in terms of threshold values of the trade elasticity ϕ as a function of σ and a_{H}. The proposition further establishes that, for elasticities above the thresholds, the effect of an expansion on relative demand always prevails on its effect on the exchange rate, hence $\widetilde{\mathcal{W}}_t$ unambiguously widens.

Proposition 8: *Under the maintained assumptions of home bias* $(1 > a_{\mathrm{H}} \geq 1/2)$ *and linear disutility of labor* $(\eta = 0)$*, monetary easing always widens* $\widetilde{\mathcal{W}}_t$ *but decreases inefficient capital inflows* $\widetilde{\mathcal{B}}_t$ *for a trade elasticity* ϕ *above the following thresholds, one derived under PCP:*

$$\phi > \frac{1 + \frac{2a_{\mathrm{H}}-1}{\sigma}}{2a_{\mathrm{H}}} > 0 \text{ for any } \sigma > 0;$$

the other derived under LCP:

$$\phi \geq 1 \text{ for } \sigma > 1$$

Proof. See the Appendix.■

The key result here is that, for a wide range of parameterizations of the workhorse model, irrespective of ERPT, monetary policy moves capital flows and the wealth gap (i.e., the deviation from efficient risk sharing) in opposite directions.

5.2 The equilibrium link between capital flows, misalignment and demand: Insight from the economics of the "transfer problem"

As already noted in the previous section, under incomplete markets, capital inflows result in a transfer of purchasing power from abroad, reflecting higher savings by Foreign residents or higher dissaving by Home residents. Since there is home bias in demand, if relative prices and incomes did not adjust, the transfer would translate into an excess supply of Foreign goods at global level. Equilibrium unavoidably requires adjustment in relative prices and incomes. The way this adjustment takes place depends on the relative strength of wealth and substitution effects from capital inflows, and thus, crucially, on the trade elasticity (a key parameter in the workhorse open macro model).

When trade elasticities are sufficiently large, substitution effects from real exchange rate movements are stronger than wealth effects. In equilibrium, adjustment to a transfer from Foreign to Home requires Home real appreciation. Because of the fall in the relative price of Foreign output, Foreign real income falls and Home real income rises by more than the size of the transfer at constant prices. It is worth noting that such mechanism lies at the core of the "transfer problem" discussed by Keynes in the classical controversy with Ohlin about the implications of war reparation payments for the terms of trade of a country (see Keynes 1929 and Ohlin 1929). In line with Keynes' concern, the appreciation compounds the rise in Home relative wealth from the transfer, strengthening the positive response of $\widetilde{\mathcal{W}}_t$ to inflows.

The equilibrium adjustment is different if wealth effects from relative price adjustment are stronger than substitution effects—which is the case when, given home bias in consumption, the complementarity between Home and Foreign goods is sufficiently strong (i.e., the trade elasticity is sufficiently below one). In response to Home capital inflows there is no equilibrium with Home appreciation/Foreign depreciation, because this would drive Foreign (income and) demand too low for the goods markets to clear at global level. Instead, equilibrium requires Foreign appreciation/Home depreciation, with the effect of reducing Home relative wealth—driving the Home wealth gap into negative territory ($\widetilde{\mathcal{W}}_t < 0$) in spite of the transfer (see, e.g., Corsetti et al. 2008a).

To appreciate how the interplay of wealth and substitution effects impinges on the equilibrium, a good starting point is a reconsideration of the natural rate allocation. Under flexible prices, when the trade elasticity is no longer constrained to be unity (but with $\eta = 0$ and $\sigma = 1$), the impact response of capital flows and the wealth gap to news shock (to either preferences or technology) obeys the following relation:

$$-\left(1-a_{\mathrm{H}}\right)\left[2a_{\mathrm{H}}\left(\phi-1\right)+1\right]\widetilde{\mathcal{W}}_{t_0}^{na} = \widehat{\mathcal{B}}_{t_0}^{na}.$$

In response to news shocks leading to capital inflows ($\widehat{\mathcal{B}}_{t_0}^{na} = \widetilde{\mathcal{B}}_{t_0}^{na} < 0$), the wealth gap may be positive or negative, depending on the value of the trade elasticity and openness. Specifically, $\widehat{\mathcal{B}}_{t_0}^{na}$ and $\widetilde{\mathcal{W}}_{t_0}^{na}$ have the opposite sign (i.e., $\widehat{\mathcal{B}}_{t_0}^{na} < 0$, and $\widetilde{\mathcal{W}}_{t_0}^{na} > 0$) for trade elasticities above the following threshold (when $1 > a_{\mathrm{H}} \geq 1/2$):

$$\phi > \frac{2a_{\mathrm{H}}-1}{2a_{\mathrm{H}}} < 1/2. \tag{36}$$

Remarkably, however, as long as $\eta = 0$ and $\sigma = 1$, the trade elasticity does not directly affect other relevant welfare gaps such as $\widetilde{\mathcal{T}}_t^{na}$, $\widetilde{\mathcal{Q}}_t^{na}$, $\widetilde{\mathcal{D}}_t^{na}$ or $\widetilde{C}_t^{na}$—the expressions for these variables coincide with those in Table 2. With the notable exception of the output gap, the above gaps depend on the elasticity ϕ only via the response of $\widetilde{\mathcal{W}}_{t_0}^{na}$.

For elasticities above the threshold (36), the wealth gap in the natural allocation is positive in the case of a capital inflow. As apparent from Table 2, capital inflows appreciate the exchange rate, the Home currency is overvalued and Home domestic demand is excessive. The opposite is true for elasticities below the threshold (36): with a negative wealth gap, capital inflows are associated with real depreciation and the Home real exchange rate is undervalued; Home demand is not high enough.[47]

The same applies to economies featuring nominal rigidities. As shown in the Appendix, under our parameterization the exact cutoff for the trade elasticity in PCP economies is the same as (36) independently of the type of shocks (taste vs. technology)—it depends on the type of the shocks in LCP economies. Below we show that the strength of income relative to substitution effects discussed in this subsection has crucial implications for the design of monetary policy.

5.3 Optimal policy

We now come to the core takeaway from our analysis. In economies where, in response to news shocks, $\widehat{\mathcal{B}}_t < 0$ is associated with $\widetilde{\mathcal{W}}_t > 0$, the optimal policy prescriptions are the same as the one derived for the CO economy—thus the sign of the policy stance depends on ERPT. Conversely, in economies where $\widehat{\mathcal{B}}_t < 0$ is associated with $\widetilde{\mathcal{W}}_t < 0$, sustaining domestic demand and output in response to capital flows (that depreciate the currency) becomes the overriding concern of monetary policy. The optimal monetary stance is invariably expansionary for any degree of ERPT.[48]

For the sake of space, we report analytical results only for the LCP economy (in this subsection) and offer a synthetic comparison of LCP and PCP relying on graphical analysis (in the next subsection)—analytical results for the PCP economy are in the Appendix. For the sake of transparency, in the text to follow we will maintain the restriction $\sigma = 1$. With this restriction, under our parameterization capital flows and the wealth gap (while depending on ϕ) in the LCP economy remain exogenous to monetary policy (as in the CO specification).

The constrained-efficient allocation in the LCP economy for a generic $\phi \geq 0$ is shown in Table 5 (again abstracting from contemporaneous shocks). The trade elasticity ϕ matters in determining whether $\widehat{\mathcal{B}}_t < 0$ translates into a positive or negative $\widetilde{\mathcal{W}}_t$ as discussed in Section 5.2. But, conditional on given $\widehat{\mathcal{B}}_t$ and $\widetilde{\mathcal{W}}_t$ stemming from news shocks (similar to the case of the natural allocation), ϕ does not enter the expressions for the response of inflation, the terms of trade, the real exchange rate, and the demand gaps. Only the expression for the output gap depends directly on ϕ. Most strikingly, the other expressions in the table are actually the same as in Table 4 of Section 4.2.

Table 5

Constrained-efficient allocation under LCP for $\phi \geq 0$

[47] In either case (i.e., regardless of the sign of the wealth gap), the output gap remains negative—either because of the overvaluation, or because domestic demand relative to foreign is too low.

[48] A variety of financial market imperfections and frictions can in principle generate capital inflows that result in a decrease in wealth, by strengthening income effects over substitution effects from exchange rate movements. It is worth stressing that the results in the text would not hold under complete markets: full risk diversification would eliminate any adverse income effects from shocks and exchange rate movements.

$\widetilde{Y}_{H,t} = (1 - a_{\rm H}) \left[(2a_{\rm H} - 1) \dfrac{(\beta\varkappa_2 - 1)}{\beta\varkappa_2} - 2a_{\rm H}\phi\frac{(\beta\nu_2-1)}{\beta\nu_2} \right] \widetilde{\mathcal{W}}_t$
$\theta\pi_t = -(1 - a_{\rm H}) \dfrac{(\beta\varkappa_2 - 1)}{\beta\varkappa_2} \widetilde{\mathcal{W}}_t + \frac{1}{2} \left[\dfrac{(\beta\varkappa_2 - 1)}{\beta\varkappa_2} \widetilde{\mathcal{W}}_{t-1} + (1 - \varkappa_1) \widetilde{\mathcal{Q}}_{t-1} \right]$
$\widetilde{\mathcal{T}}_t + \widetilde{\Delta}_t = -\dfrac{(\beta\nu_2 - 1)}{\beta\nu_2} \widetilde{\mathcal{W}}_t + \nu_1 \left(\widetilde{\mathcal{T}}_{t-1} + \widetilde{\Delta}_{t-1} \right)$
$\widetilde{\mathcal{Q}}_t = -(2a_{\rm H} - 1) \dfrac{(\beta\varkappa_2 - 1)}{\beta\varkappa_2} \widetilde{\mathcal{W}}_t - \dfrac{1}{\beta\varkappa_2} \left(\widetilde{\mathcal{W}}_t - \widetilde{\mathcal{W}}_{t-1} \right) + \varkappa_1 \widetilde{\mathcal{Q}}_{t-1}$
$\widetilde{\mathcal{D}}_t = 2(1 - a_{\rm H}) \frac{(\beta\varkappa_2-1)}{\beta\varkappa_2} \widetilde{\mathcal{W}}_t + \dfrac{1}{\beta\varkappa_2} \widetilde{\mathcal{W}}_{t-1} + \varkappa_1 \widetilde{\mathcal{Q}}_{t-1}$

As in the CO economy, in response to (news shocks that trigger) a capital inflow, $\widehat{\mathcal{B}}_t < 0$, associated with positive wealth gap, $\widetilde{\mathcal{W}}_t > 0$, Home monetary authorities tighten to curb relative Home demand, at the cost of letting inflation decline and exacerbate the Home real exchange rate overappreciation in the short run—in the Appendix, we show that a sufficient condition for this result in the case of anticipated shocks to preferences is that ϕ is above the threshold (36). Relative to the CO economy in Section 4.2, however, for $\widetilde{\mathcal{W}}_t > 0$ the optimal contractionary stance does not necessarily bring the output gap into negative territory.[49] Depending on ϕ, the impact output gap response to a positive wealth gap, rewritten as

$$\widetilde{Y}_{H,t_0} = -(1 - a_{\rm H}) \left[1 - 2a_{\rm H} \left(1 - \phi \frac{(\beta\nu_2 - 1)}{\beta\nu_2} \right) + (2a_{\rm H} - 1) \left(1 - \frac{(\beta\varkappa_2 - 1)}{\beta\varkappa_2} \right) \right] \widetilde{\mathcal{W}}_{t_0}, \quad (37)$$

may have either sign. From the above, it is easy to show that a sufficient condition for the the output gap response to be negative is that ϕ is sufficiently above 1.

Conversely, the optimal response to excessive capital inflows is expansionary when these lead to excessive depreciation and a negative wealth gap; this is the case when ϕ is below the thresholds derived in the Appendix. In this case, despite the "transfer" from abroad, Home consumption is inefficiently low: monetary authorities optimally resort to expansionary policy to further expand Home demand, at the cost of higher domestic inflation and larger undervaluation of the exchange rate.

It follows that the key results in Proposition 7, comparing the allocation under strict CPI stability and the optimal constrained allocation in the LCP economy, generalize for any ϕ.

Proposition 9. *Under LCP, with $\sigma = 1$, $\eta = 0$ and $\phi \geq 0$, the optimal response to news shocks generating inefficient capital flows stabilizes on impact the real exchange rate and CPI inflation less than under a regime pursuing strict CPI stability, while the demand gap is more stable; the impact output gap instead can be smaller or larger.*

Proof. As shown in the Appendix, the allocation is the same as the one derived in Table 4 but for the output gap; therefore the relevant parts of the proof of Proposition 7 also apply here. Comparing (37) with the output gap response under CPI price stability:

$$\widetilde{Y}^{CPI}_{H,t_0} = -(1 - a_{\rm H}) \left[1 - 2a_{\rm H} \left(1 - \phi \frac{(\beta\nu_2 - 1)}{\beta\nu_2} \right) \right] \widetilde{\mathcal{W}}_{t_0}$$

[49] In line with our earlier analysis, the extent to which the optimal policy response translates into a lower demand gap $\widetilde{\mathcal{D}}_t$ will depend on the degrees of openness and stickiness of import prices, i.e. on exchange rate pass-through.

the result that $\left\|\widetilde{Y}_{H,t_0}\right\| \gtrless \left\|\widetilde{Y}^{CPI}_{H,t_0}\right\|$ follows from noting again that the term in square brackets in the latter expression can be positive or negative, while the term $(2a_{\mathrm{H}}-1)\left(1-\frac{(\beta\varkappa_2-1)}{\beta\varkappa_2}\right)$ in (37) is always positive. Moreover, the latter fact also implies that $\left\|\widetilde{Y}_{H,t_0}\right\| > \left\|\widetilde{Y}^{CPI}_{H,t_0}\right\|$ when the following condition holds:

$$2a_{\mathrm{H}} < \frac{1+\frac{(\beta\varkappa_2-1)}{\beta\varkappa_2}}{1+\frac{(\beta\varkappa_2-1)}{\beta\varkappa_2}-2\phi\frac{(\beta\nu_2-1)}{\beta\nu_2}}.$$

■

5.4 Exchange rate, inflation and output gaps: a comparison of LCP and PCP economies

We conclude by providing, in Figure 2, a synthetic graphical illustration of our main findings, comparing the LCP economy analyzed above with the PCP economy analyzed in the Appendix. For the PCP economy, the figure highlights that the optimal monetary response to capital inflows remains expansionary when the wealth gap switches sign (from positive to negative) and the real exchange rate misalignment goes from over to undervaluation. In the case of overvaluation, the driver of the expansion is the need to mitigate the loss of global demand due to excessive appreciation, impinging on the output gap—same as in the CO economy. In the case of excessive depreciation, policy responds to the need to sustain Home demand (at the cost of higher inflation), as residents suffer significant losses in income and purchasing power due to the fall in the international price of their country output. As shown in the previous subsection, instead, under LCP the optimal monetary policy—invariably geared to stabilize relative demand—switches from contractionary to expansionary across the two cases.

Figure 2 compares the responses to capital inflows under the optimal policy in economies with a relatively high trade elasticity, such that $\widetilde{\mathcal{W}}_{t_0} > 0$ (left column), and in economies with a relatively low trade elasticities, such that $\widetilde{\mathcal{W}}_{t_0} < 0$ (right column). The left column show results for $\phi = 2$ (hence $\widetilde{\mathcal{W}}_{t_0} > 0$), the right column for $\phi = 0.3$ (hence $\widetilde{\mathcal{W}}_{t_0} < 0$); all other parameters are the same as in Figure 1. Specifically, to enhance comparability with the CO economy, the new figure is drawn for the same anticipated preference shocks as in Figure 1, also resulting in $\widehat{\mathcal{B}}_t < 0$ (although, with $\phi \neq 1$, the inflow underlying Figure 2 does not necessarily amount to 1% of GDP). Furthermore, under our parameterization with $\sigma = 1$, the capital inflow and the wealth gap are exogenous to policy in the LCP economy, endogenous in the PCP economy—whereby according to Proposition 8 a monetary expansion would reduce the size of the capital inflows for $\phi > 1$. In the figure, the solid blue lines and the dashed red lines trace the impulse responses of misalignment, the price (CPI or PPI) level and the output gap, respectively, in the LCP and PCP economy.

When $\widetilde{\mathcal{W}}_{t_0} > 0$—in the left column—, the response of the misalignment and the price level is closely in line with the CO economy: they move in opposite directions across the PCP and the LCP economy, reflecting the difference in the optimal monetary stance. Relative to Figure 1, however, a higher value of the trade elasticity translates into a more negative and volatile output gap. The size of the output gap is particularly large in the PCP economy, reflecting both a stronger real exchange rate appreciation and a higher expenditure switching effect of this appreciation due to a higher elasticity. Recall that, relative to the natural rate allocation, the Home relative expansionary stance always contains exchange rate overvaluation.

For $\widetilde{\mathcal{W}}_{t_0} < 0$—in the right column—the misalignment goes from over to undervaluation. The optimal monetary response to the inefficient capital inflow has the same sign in the LCP and PCP economies—the optimal stance sustains Home demand. Inflation is inefficiently high and, relative to the high-elasticity economy, the real exchange rate is underappreciated (the gap is positive): the optimal policy exacerbates misalignment. The expansion contains the size of the negative output gap on impact in the LCP economy, and actually changes the sign of the output gap in the PCP economy—the output gap turns from negative to positive. Indeed, comparing the two columns in Figure 2 shows that, with the negative wealth gap, the optimal monetary stimulus in PCP economies becomes substantial—causing massive exchange rate overshooting and a sizeable positive output gap (relative to the natural rate allocation).

6 Conclusions

Much research has been devoted to the policy tools and measures that can be activated to insulate national economies from the ebb and flows of cross-border capital flows. In this paper, we have taken the perspective of monetary policy decision making, and analyzed what monetary instruments can deliver when additional tools are not readily available and/or are of limited effectiveness. Our main question is how monetary policy could optimally respond to inefficient capital flows, impacting on domestic macroeconomic dynamic and welfare, by optimally trading off domestic and external objectives.

Our study provides key analytical insights into the efficient resolution of this trade-off. When international capital markets provide imperfect risk insurance (so that capital flows are associated with currency misalignment), the design of optimal monetary rules hinges on recognizing the direct and indirect relevance of exchange rates for domestic stabilization and welfare. The workhorse new Keynesian model delivers insightful prescriptions in this respect, showing that optimal monetary policy crucially depends on ERPT.

Under complete pass through (the PCP economy), the monetary response is always expansionary, but the reason differs depending on the equilibrium link between inefficient capital flows and the wealth gap. The optimal expansion aims to prevent excessive appreciation from opening a large output gap when capital inflows strengthen the currency and cause a demand boom. Conversely, an expansion is primarily meant to support domestic demand when the inflow is associated to depreciation that hurts domestic consumption (for a low trade elasticity).

With LCP, the optimal monetary stance is invariably geared to stabilize demand—since a low ERPT mutes the effects of exchange rate movements on global demand and hence on the domestic output gap. In this case, the equilibrium link between flows and the wealth gap matters for the sign of the optimal stance. The monetary stance is optimally contractionary in response to an inflow that appreciates the currency and translates into an inefficient demand boom. It becomes expansionary when demand falls with excessive depreciation (the case of a low elasticity).

Moving forward, there are a number of directions of research. The interplay of domestic and cross-border financial frictions may strengthen the case for domestic stabilization at the cost of higher exchange rate movements under LCP. This would possibly be the case if a share of the residents in each country is excluded from financial markets, and thus operates under financial autarky.[50] By the same token, allowing for gross foreign assets and liabilities would introduce valuation effects due to misalignment, on top and above the income effects of exchange rate movements stressed by our analysis (see Benigno 2009).

Strategic interactions among policymakers are another key issue. Inefficient capital flows have strong redistributive effects across borders. Cooperative policies attempt to redress these effects: in our analysis, when the optimal monetary policy at Home is either a contraction or an expansion, the Foreign monetary stance has the opposite sign. Without cooperation, however, these redistributive effects of capital inflows inherently create room for conflicts and strategic behavior.

Finally, while in this paper we focus on the benchmark cases of PCP and LCP, the evidence on the importance of pricing in vehicle (or dominant) currencies strongly motivates further work exploring the case of asymmetric pass-through. An important question is which direction monetary policy will take in the country which issues the dominant currency, when facing a capital inflow with currency overvaluation or undervaluation.

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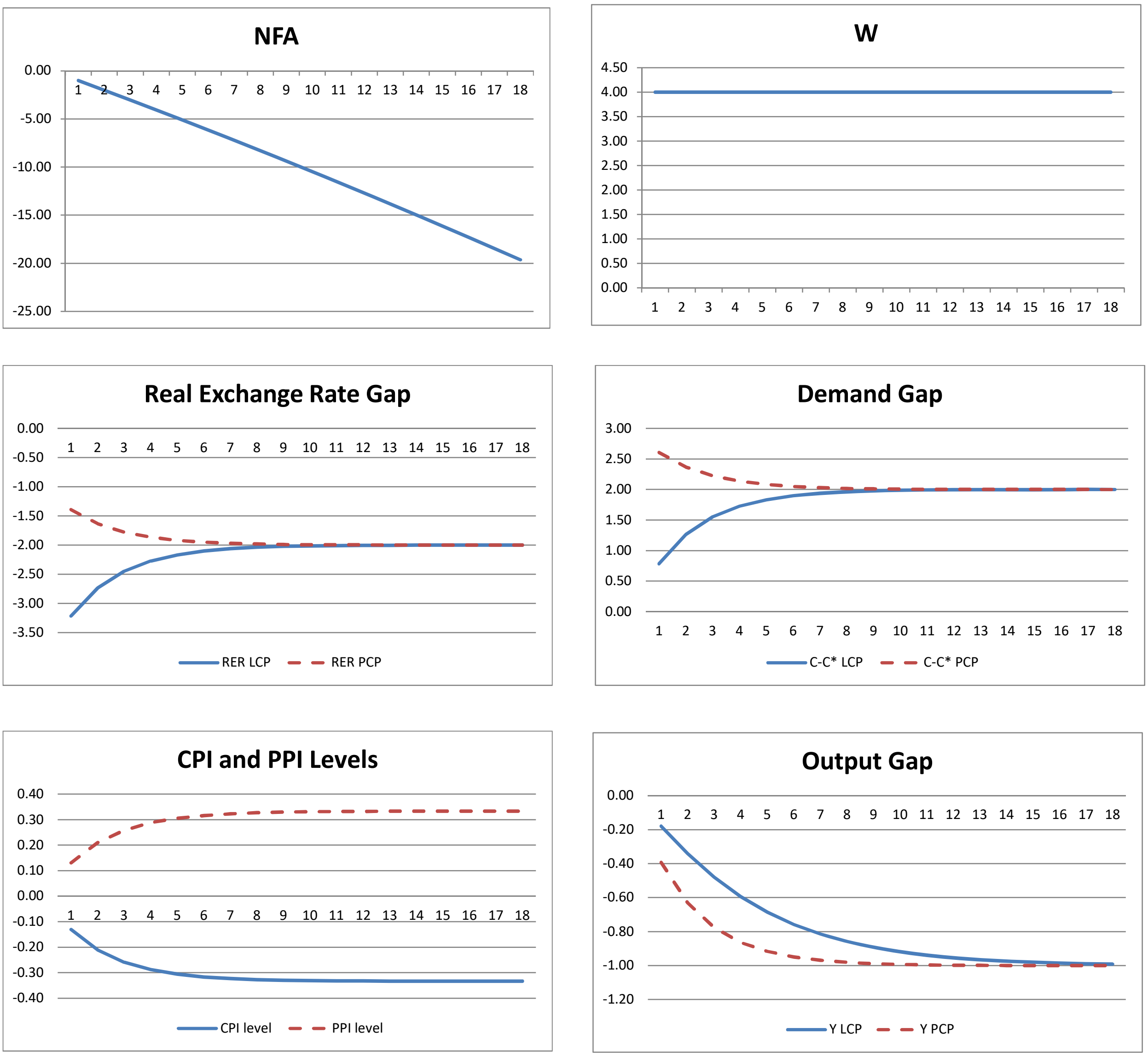
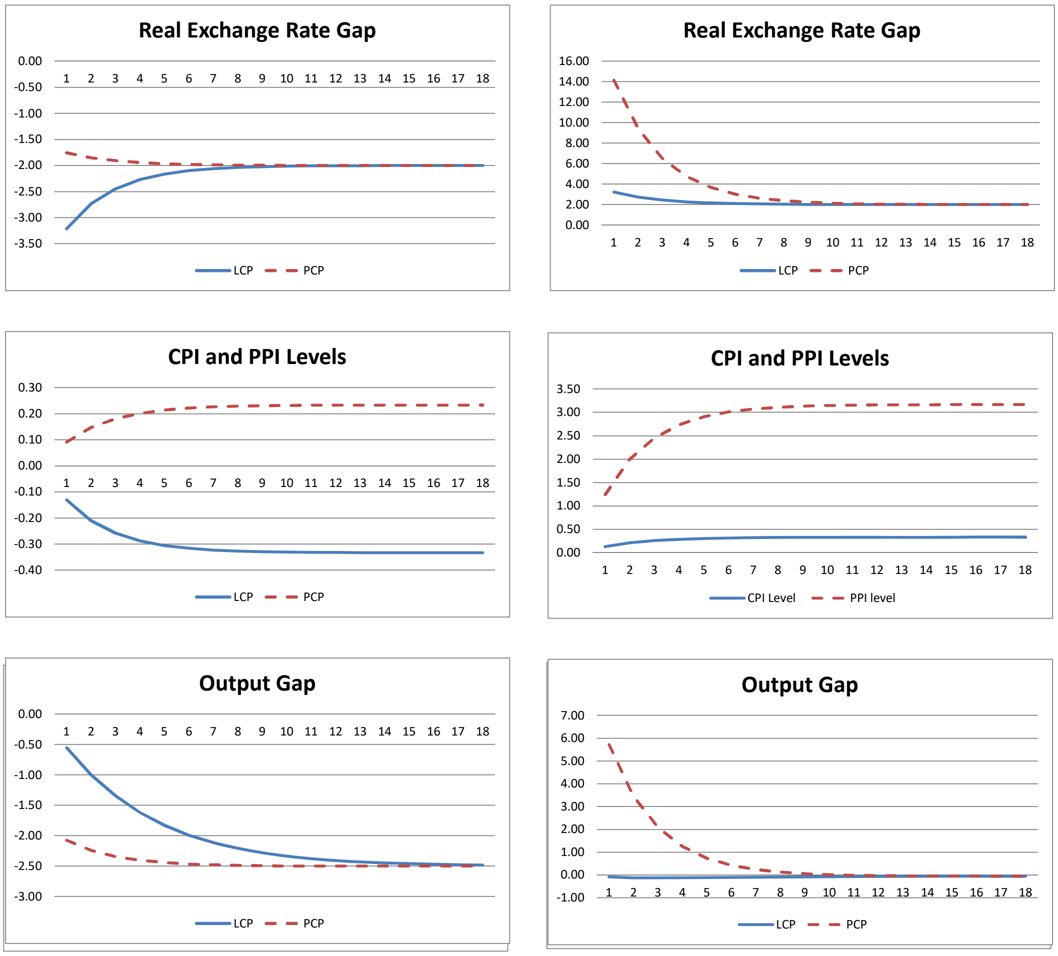
Figure 1



The figure is drawn for anticipated taste shocks that materialize after period 20 (not shown in the graphs). Parameter values are as follows: η=0, ϕ=σ=1, aH=.75, β=.99,α=.75 ,θ=3.

Figure 2

Capital Inflows with positive wealth gap | Capital Inflows with negative wealth gap



The figure is drawn for anticipated productivity shocks that materialize after period 20 (not shown in the graphs)
Parameter values are as follows: η=0, σ=1, aH=.75, β=.99,α=.75 ,θ=3 and ϕ=2 (left column) or ϕ=.3 (right column)

1 Quadratic loss function under LCP and generically incomplete markets: Proof of Proposition 1

In this section of the appendix we derive the quadratic loss function under LCP and generically incomplete markets. The PCP case can be understood as a special case where law of one price (LOOP) deviations are set to zero.

Write the one-period utility flow:

$$U(C) - V(L) = \zeta_C \frac{C^{1-\sigma} - 1}{1-\sigma} - \varpi \frac{L^{1+\eta}}{1+\eta},$$

Under the assumption of an efficient steady state with subsidy $\frac{(\theta-1)(1-\overline{\tau})}{\theta} = 1$, so that $U'(C) = -V'(L)$, the second order approximation of utility is as follows:

$$\widehat{C}_t - \widehat{Y}_{H,t} + \left(\frac{1-\sigma}{2}\widehat{C}_t + \widehat{\zeta}_{C,t}\right)\widehat{C}_t - (1+\eta)\left(\frac{1}{2}\widehat{Y}_{H,t} - \widehat{\zeta}_{Y,t}\right)\widehat{Y}_{H,t} +$$
$$-\frac{1}{2}\frac{\theta\alpha}{(1-\alpha\beta)(1-\alpha)}\left[a_{\mathrm{H}}\pi^2_{H,t} + (1-a_{\mathrm{H}})\pi^{*2}_{H,t}\right] + t.i.p. + o\left(\varepsilon^3\right),$$

where we have used the log-linear approximation to the aggregate production function: $\widehat{Y}_{H,t} = \widehat{\zeta}_{Y,t} + \widehat{L}_t$. Inflation rates appear in this expression because the second order approximation of labor effort is proportional to price dispersion, which in turn is a function of sectoral inflation rates under LCP and Calvo price-setting with symmetric probabilities α (see Engel (2009)).

Similarly, for the Foreign country we have,

$$\widehat{C}^*_t - \widehat{Y}_{F,t} + \left(\frac{1-\sigma}{2}\widehat{C}^*_t + \widehat{\zeta}_{C^*,t}\right)\widehat{C}^*_t - (1+\eta)\left(\frac{1}{2}\widehat{Y}_{F,t} - \widehat{\zeta}^*_{Y,t}\right)\widehat{Y}_{F,t} +$$
$$-\frac{1}{2}\frac{\theta\alpha}{(1-\alpha\beta)(1-\alpha)}\left[a_{\mathrm{H}}\pi^{*2}_{F,t} + (1-a_{\mathrm{H}})\pi^2_{F,t}\right] + t.i.p. + o\left(\varepsilon^3\right),$$

Under cooperation, the global policy objective function $\mathcal{L}^W_t$ will be the sum of the two country-specific terms.

$$\mathcal{L}^W_t = (\widehat{C}_t + \widehat{C}^*_t) - (\widehat{Y}_{H,t} + \widehat{Y}_{F,t}) + \left(\frac{1-\sigma}{2}(\widehat{C}_t + \widehat{\zeta}_{C,t}\right)\widehat{C}_t + \left(\frac{1-\sigma}{2}\widehat{C}^*_t + \widehat{\zeta}_{C^*,t}\right)\widehat{C}^*_t$$
$$-(1+\eta)\left(\frac{1}{2}\widehat{Y}_{H,t} - \widehat{\zeta}_{Y,t}\right)\widehat{Y}_{H,t} - (1+\eta)\left(\frac{1}{2}\widehat{Y}_{F,t} - \widehat{\zeta}^*_{Y,t}\right)\widehat{Y}_{F,t} +$$
$$-\frac{1}{2}\frac{\theta\alpha}{(1-\alpha\beta)(1-\alpha)}\left(\left[a_{\mathrm{H}}\pi^2_{H,t} + (1-a_{\mathrm{H}})\pi^{*2}_{H,t}\right] + \left[a_{\mathrm{H}}\pi^{*2}_{F,t} + (1-a_{\mathrm{H}})\pi^2_{F,t}\right]\right)$$
$$+t.i.p. + o\left(\varepsilon^3\right),$$

The objective of this appendix is to rewrite the above as a quadratic loss function in terms of gaps and misalignments.

1.1 Useful first order relationships

We begin by writing some useful relations. The real exchange rate is related to the terms of trade and deviations from the law of one price as follows:

$$\widehat{\mathcal{Q}}_t = (2a_{\mathrm{H}} - 1)\,\widehat{\mathcal{T}}_t + 2a_{\mathrm{H}}\widehat{\Delta}_t. \tag{1}$$

The first order approximations of $\widehat{C}_t$ and $\widehat{C}_t^*$, are given by,

$$\begin{aligned} \widehat{C}_t^* &= \widehat{C}_t - \sigma^{-1}\left[\widehat{\mathcal{Q}}_t + \widetilde{\mathcal{W}}_t + \left(\widehat{\zeta}_{C,t} - \widehat{\zeta}^*_{C,t}\right)\right] \\ \widehat{C}_t &= \frac{1}{2}\left\{\widehat{Y}_{H,t} + \widehat{Y}_{F,t} + \sigma^{-1}\left[\widehat{\mathcal{Q}}_t + \widetilde{\mathcal{W}}_t + \left(\widehat{\zeta}_{C,t} - \widehat{\zeta}^*_{C,t}\right)\right]\right\}, \end{aligned} \tag{2}$$

The first order approximations of $\widehat{C}_t$ and $\widehat{C}_t^*$ imply,

$$\begin{aligned} -(\widehat{C}_t - \widehat{Y}_{H,t}) &= \widehat{C}_t^* - \widehat{Y}_{F,t} = \\ &\quad \frac{1}{2}\left\{\widehat{Y}_{H,t} - \widehat{Y}_{F,t} - \sigma^{-1}\left[\widehat{\mathcal{Q}}_t + \widetilde{\mathcal{W}}_t + \left(\widehat{\zeta}_{C,t} - \widehat{\zeta}^*_{C,t}\right)\right]\right\} \end{aligned} \tag{3}$$

The first order approximation of aggregate demand yields,

$$\begin{aligned} \widehat{C}_t &= \widehat{Y}_{H,t} - (1 - a_{\mathrm{H}})\,\sigma^{-1}\left[\sigma\phi\widehat{\mathcal{T}}_t + (\sigma\phi - 1)\,\widehat{\mathcal{Q}}_t - \widetilde{\mathcal{W}}_t - \left(\widehat{\zeta}_{C,t} - \widehat{\zeta}^*_{C,t}\right)\right] \\ \widehat{C}_t^* &= \widehat{Y}_{F,t} + (1 - a_{\mathrm{H}})\,\sigma^{-1}\left[\sigma\phi\widehat{\mathcal{T}}_t + (\sigma\phi - 1)\,\widehat{\mathcal{Q}}_t - \widetilde{\mathcal{W}}_t - \left(\widehat{\zeta}_{C,t} - \widehat{\zeta}^*_{C,t}\right)\right] \end{aligned}$$

Combining the first order approximations of aggregate demand, we obtain,

$$\widehat{C}_t = \widehat{Y}_{H,t} - \frac{1 - a_{\mathrm{H}}}{\sigma}\left[2a_{\mathrm{H}}\phi\sigma\left(\widehat{\mathcal{T}}_t + \widehat{\Delta}_t\right) - \widehat{\mathcal{Q}}_t - \widetilde{\mathcal{W}}_t - \left(\widehat{\zeta}_{C,t} - \widehat{\zeta}^*_{C,t}\right)\right].$$

Combining the two expressions for consumption, we obtain the following expression for the terms of trade:

$$\begin{aligned} &[4a_{\mathrm{H}}\,(1 - a_{\mathrm{H}})\,(\sigma\phi - 1) + 1]\left(\widehat{\mathcal{T}}_t + \widehat{\Delta}_t\right) = \\ &\sigma\left(\widehat{Y}_{H,t} - \widehat{Y}_{F,t}\right) - (2a_{\mathrm{H}} - 1)\left[\widetilde{\mathcal{W}}_t + \widehat{\Delta}_t + \left(\widehat{\zeta}_{C,t} - \widehat{\zeta}^*_{C,t}\right)\right] \end{aligned} \tag{4}$$

In addition, shocks can be expressed in terms of efficient output and the terms of trade,

$$\begin{aligned} &\widehat{\zeta}_{C,t} + (1 + \eta)\,\widehat{\zeta}_{Y,t} = \\ &(\eta + \sigma)\,\widehat{Y}^{fb}_{H,t} - [2a_{\mathrm{H}}\,(1 - a_{\mathrm{H}})\,(\sigma\phi - 1)]\left(\widehat{\mathcal{T}}^{fb}_t\right) + (1 - a_{\mathrm{H}})\left(\widehat{\zeta}_{C,t} - \widehat{\zeta}^*_{C,t}\right) \end{aligned} \tag{5}$$

Next, using the first order approximation for domestic consumption, we can rewrite domestic marginal costs as follows,

$$\begin{aligned} &\sigma\widehat{C}_t - \widehat{\zeta}_{C,t} + \eta\widehat{Y}_{H,t} - (1 + \eta)\,\widehat{\zeta}_{Y,t} + (1 - a_{\mathrm{H}})\left(\widehat{\mathcal{T}}_t + \Delta_t\right) = \\ &(\eta + \sigma)\left(\widehat{Y}_{H,t} - \widehat{Y}^{fb}_{H,t}\right) + \\ &- (1 - a_{\mathrm{H}}) \cdot \left[(\sigma\phi - 1)\left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}^{fb}_t + \widehat{\mathcal{Q}}_t - \widehat{\mathcal{Q}}^{fb}_t\right) - \widetilde{\mathcal{W}}_t - \Delta_t\right] \end{aligned} \tag{6}$$

Rearranging,

$$\frac{\sigma}{2}\widehat{C}_t - \widehat{\zeta}_{C,t} + \frac{\eta}{2}\widehat{Y}_{H,t} - (1+\eta)\,\widehat{\zeta}_{Y,t} + \frac{1}{2}(1-a_{\mathrm{H}})\left(\widehat{\mathcal{T}}_t + \Delta_t\right) = \tag{7}$$

$$(\eta+\sigma)\left(\frac{1}{2}\widehat{Y}_{H,t} - \widehat{Y}^{fb}_{H,t}\right) - 2a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)\left(\frac{1}{2}\left(\widehat{\mathcal{T}}_t + \Delta_t\right) - \widehat{\mathcal{T}}^{fb}_t\right) +$$

$$\frac{1}{2}(1-a_{\mathrm{H}})\left(\widetilde{\mathcal{W}}_t + \Delta_t - \left(\widehat{\zeta}_{C,t} - \widehat{\zeta}^*_{C,t}\right)\right)$$

1.2 Derivation of the global loss function in terms of gaps and misalignments (Proof of Proposition 1)

To eliminate the linear terms from $\mathcal{L}^W_t$, we proceed as follows. First, we derive a second-order accurate expression for the sum of consumption across countries (the world aggregate demand) by summing up the budget constraints under LCP:

$$\frac{P_{\mathrm{H}}}{P}(C_{\mathrm{H}} + C^*_{\mathrm{H}}) + \frac{SP^*_{\mathrm{F}}}{P}(C_{\mathrm{F}} + C^*_{\mathrm{F}}) = \frac{P_{\mathrm{H}}}{P}Y_{\mathrm{H}} + \frac{SP^*_{\mathrm{F}}}{P}Y_{\mathrm{F}}$$

$$C + \mathcal{Q}C^* + \left(\frac{SP^*_{\mathrm{F}}}{P_{\mathrm{F}}} - 1\right)\frac{P_{\mathrm{F}}}{P}C_{\mathrm{F}} - \left(\frac{SP^*_{\mathrm{H}}}{P_{\mathrm{H}}} - 1\right)\frac{P_{\mathrm{H}}}{P}C^*_{\mathrm{H}} = \frac{P_{\mathrm{H}}}{P}Y_{\mathrm{H}} + \frac{P^*_{\mathrm{F}}}{P^*}\frac{SP^*}{P}Y_{\mathrm{F}}$$

$$C + \mathcal{Q}C^* + (1-a_{\mathrm{H}})\left[(\Delta_{\mathrm{F}} - 1)\left(\frac{P_{\mathrm{F}}}{P}\right)^{1-\phi}C + (\Delta^{-1}_{\mathrm{H}} - 1)\left(\frac{P^*_{\mathrm{H}}}{P^*}\right)^{1-\phi}\mathcal{Q}C^*\right] =$$

$$\frac{P_{\mathrm{H}}}{P}Y_{\mathrm{H}} + \frac{P^*_{\mathrm{F}}}{P^*}\mathcal{Q}Y_{\mathrm{F}}.$$

$$C + \mathcal{Q}C^* + (1-a_{\mathrm{H}})\left\{\begin{array}{c}(\Delta_{\mathrm{F}} - 1)\left[a_{\mathrm{H}}\mathcal{T}^{\phi-1}\Delta^{\phi-1}_{\mathrm{H}} + (1-a_{\mathrm{H}})\right]^{-1}C + \\ (\Delta^{-1}_{\mathrm{H}} - 1)\left[a_{\mathrm{H}}\mathcal{T}^{1-\phi}\Delta^{1-\phi}_{\mathrm{F}} + (1-a_{\mathrm{H}})\right]^{-1}\mathcal{Q}C^*\end{array}\right\} =$$

$$\left[a_{\mathrm{H}} + (1-a_{\mathrm{H}})\mathcal{T}^{1-\phi}\Delta^{1-\phi}_{\mathrm{H}}\right]^{-\frac{1}{1-\phi}}Y_{\mathrm{H}} +$$

$$\left[a_{\mathrm{H}} + (1-a_{\mathrm{H}})\mathcal{T}^{\phi-1}\Delta^{\phi-1}_{\mathrm{F}}\right]^{-\frac{1}{1-\phi}}\mathcal{Q}Y_{\mathrm{F}}.$$

The accurate second-order approximation to the world demand is:

$$
\begin{aligned}
& \widehat{C}_t + \widehat{C}_t^* + \tfrac{1}{2}\left(\widehat{C}_t^2 + \widehat{C}_t^{*2}\right) + \widehat{\mathcal{Q}}_t + \tfrac{1}{2}\widehat{\mathcal{Q}}_t^2 + \widehat{\mathcal{Q}}_t \widehat{C}_t^* + \\
& (1-a_{\mathrm{H}})\left[\begin{array}{c} \widehat{\Delta}_{\mathrm{F},t} + \tfrac{1}{2}\widehat{\Delta}_{\mathrm{F},t}^2 + \Delta_{\mathrm{F},t}\left(\widehat{C}_t + a_{\mathrm{H}}(1-\phi)\left(\widehat{\mathcal{T}}_t + \widehat{\Delta}_{\mathrm{H},t}\right)\right) - \\ \left(\widehat{\Delta}_{\mathrm{H},t} + \tfrac{1}{2}\widehat{\Delta}_{\mathrm{H},t}^2\right) + \widehat{\Delta}_{\mathrm{H},t}^2 - \widehat{\Delta}_{\mathrm{H},t}\left(\widehat{C}_t^* + \widehat{\mathcal{Q}}_t - a_{\mathrm{H}}(1-\phi)\left(\widehat{\mathcal{T}}_t + \widehat{\Delta}_{\mathrm{F},t}\right)\right) \end{array}\right] \\
= \; & \widehat{Y}_{H,t} + \widehat{Y}_{F,t} + \frac{1}{2}\left(\widehat{Y}_{H,t}^2 + \widehat{Y}_{F,t}^2\right) - (1-a_{\mathrm{H}})\left[\widehat{\mathcal{T}}_t + \widehat{\Delta}_{\mathrm{H},t} + \frac{1}{2}\left(\widehat{\mathcal{T}}_t^2 + \widehat{\Delta}_{\mathrm{H},t}^2\right)\right] - \\
& (1-a_{\mathrm{H}})\widehat{Y}_{H,t}\left(\widehat{\mathcal{T}}_t + \widehat{\Delta}_{\mathrm{H},t}\right) + (1-a_{\mathrm{H}})\left[\phi - 1 + (1-a_{\mathrm{H}})(1-\phi)\left(\frac{1}{1-\phi} + 1\right)\right]\widehat{\mathcal{T}}_t\widehat{\Delta}_{\mathrm{H},t} + \\
& \frac{1}{2}(1-a_{\mathrm{H}})\left[\phi + (1-a_{\mathrm{H}})(1-\phi)\left(\frac{1}{1-\phi} + 1\right)\right]\left(\widehat{\mathcal{T}}_t^2 + \widehat{\Delta}_{\mathrm{H},t}^2\right) + \\
& (1-a_{\mathrm{H}})\left[\widehat{\mathcal{T}}_t + \widehat{\Delta}_{\mathrm{F},t} + \frac{1}{2}\left(\widehat{\mathcal{T}}_t^2 + \widehat{\Delta}_{\mathrm{F},t}^2\right)\right] + \widehat{\mathcal{Q}}_t + \frac{1}{2}\widehat{\mathcal{Q}}_t^2 + \widehat{Y}_{F,t}\widehat{\mathcal{Q}}_t + (1-a_{\mathrm{H}})\widehat{Y}_{F,t}\left(\widehat{\mathcal{T}}_t + \widehat{\Delta}_{\mathrm{F},t}\right) + \\
& (1-a_{\mathrm{H}})\left(\widehat{\mathcal{T}}_t + \widehat{\Delta}_{\mathrm{F},t}\right)\widehat{\mathcal{Q}}_t + (1-a_{\mathrm{H}})\left[\phi - 1 + (1-a_{\mathrm{H}})(1-\phi)\left(\frac{1}{1-\phi} + 1\right)\right]\widehat{\mathcal{T}}_t\widehat{\Delta}_{\mathrm{F},t} + \\
& \frac{1}{2}(1-a_{\mathrm{H}})\left[(1-a_{\mathrm{H}})\left(\frac{1}{1-\phi} + 1\right)(1-\phi) + \phi - 2\right]\left(\widehat{\mathcal{T}}_t^2 + \widehat{\Delta}_{\mathrm{F},t}^2\right).
\end{aligned}
$$

As the linear terms in relative prices cancel out and under the maintained assumption of symmetry $\widehat{\Delta}_{\mathrm{H},t} = \widehat{\Delta}_{\mathrm{F},t} = \widehat{\Delta}_t$, we get:

$$
\widehat{C}_t + \widehat{C}_t^* + \frac{1}{2}\left(\widehat{C}_t^2 + \widehat{C}_t^{*2}\right) + (1-a_{\mathrm{H}})\left(\widehat{C}_t - \widehat{C}_t^* - \widehat{\mathcal{Q}}_t\right)\widehat{\Delta}_t =
$$

$$
\begin{aligned}
& \widehat{Y}_{H,t} + \widehat{Y}_{F,t} + \tfrac{1}{2}\left(\widehat{Y}_{H,t}^2 + \widehat{Y}_{F,t}^2\right) + \left(\widehat{Y}_{F,t} - \widehat{C}_t^*\right)\widehat{\mathcal{Q}}_t + \\
& (1-a_{\mathrm{H}})\left(\widehat{Y}_{F,t} - \widehat{Y}_{H,t}\right)\left(\widehat{\mathcal{T}}_t + \widehat{\Delta}_t\right) + a_{\mathrm{H}}(1-a_{\mathrm{H}})\phi\left(\widehat{\mathcal{T}}_t + \widehat{\Delta}_t\right)^2 + \\
& (1-a_{\mathrm{H}})\left(\left[1 - 2a_{\mathrm{H}}(1-\phi)\right]\widehat{\mathcal{T}}_t - 2a_{\mathrm{H}}(1-\phi)\widehat{\Delta}_t\right)\widehat{\Delta}_t,
\end{aligned}
$$

Second, we substitute in the approximation to the sum of consumption—in addition, we subtract $\frac{1}{2}(1-a_{\mathrm{H}})\widehat{\mathcal{T}}_t\left(\widehat{Y}_{H,t} - \widehat{Y}_{F,t}\right)$, $\left(\frac{\sigma}{2}\widehat{C}_t - \widehat{\zeta}_{C,t}\right)\widehat{Y}_{H,t}$ and $\left(\frac{\sigma}{2}\widehat{C}_t^* - \widehat{\zeta}_{C,t}^*\right)\widehat{Y}_{F,t}$ in order to have a second-order term in the product of output and marginal costs for each country.

$$
\begin{aligned}
&\mathcal{L}_t^W \ltimes \widehat{C}_t + \widehat{C}_t^* - \widehat{Y}_{H,t} - \widehat{Y}_{F,t} + \left(\frac{1-\sigma}{2}\widehat{C}_t + \widehat{\zeta}_{C,t}\right)\widehat{C}_t + \left(\frac{1-\sigma}{2}\widehat{C}_t^* + \widehat{\zeta}_{C,t}^*\right)\widehat{C}_t^* - \\
&(1+\eta)\left(\frac{1}{2}\widehat{Y}_{H,t} - \widehat{\zeta}_{Y,t}\right)\widehat{Y}_{H,t} - (1+\eta)\left(\frac{1}{2}\widehat{Y}_{F,t} - \widehat{\zeta}_{Y,t}^*\right)\widehat{Y}_{F,t} - \\
&\frac{1}{2}\frac{\theta\alpha}{(1-\alpha\beta)(1-\alpha)}\left[a_{\mathrm{H}}\pi_{H,t}^2 + (1-a_{\mathrm{H}})\pi_{H,t}^{*2} + a_{\mathrm{H}}\pi_{F,t}^{*2} + (1-a_{\mathrm{H}})\pi_{F,t}^2\right] \\
&+t.i.p. + o\left(\varepsilon^3\right) \\
&= -\left(\frac{\sigma}{2}\widehat{C}_t - \widehat{\zeta}_{C,t}\right)\widehat{C}_t - \left(\frac{\sigma}{2}\widehat{C}_t^* - \widehat{\zeta}_{C,t}^*\right)\widehat{C}_t^* + \left(\widehat{Y}_{F,t} - \widehat{C}_t^*\right)\widehat{\mathcal{Q}}_t - \\
&(1-a_{\mathrm{H}})\left(\widehat{C}_t - \widehat{C}_t^* - \widehat{\mathcal{Q}}_t\right)\widehat{\Delta}_t - \\
&\frac{1}{2}(1-a_{\mathrm{H}})\left(\widehat{Y}_{H,t} - \widehat{Y}_{F,t}\right)\left(\widehat{\mathcal{T}}_t + \widehat{\Delta}_t\right) + (1-a_{\mathrm{H}})a_{\mathrm{H}}\phi\left(\widehat{\mathcal{T}}_t + \widehat{\Delta}_t\right)^2 + \\
&(1-a_{\mathrm{H}})\left([1-2a_{\mathrm{H}}(1-\phi)]\widehat{\mathcal{T}}_t - 2a_{\mathrm{H}}(1-\phi)\widehat{\Delta}_t\right)\widehat{\Delta}_t - \\
&\left(\frac{\eta}{2}\widehat{Y}_{H,t} - (1+\eta)\widehat{\zeta}_{Y,t} + \frac{1}{2}(1-a_{\mathrm{H}})\left(\widehat{\mathcal{T}}_t + \widehat{\Delta}_t\right)\right)\widehat{Y}_{H,t} - \\
&\left(\frac{\eta}{2}\widehat{Y}_{F,t} - (1+\eta)\widehat{\zeta}_{Y,t}^* - \frac{1}{2}(1-a_{\mathrm{H}})\left(\widehat{\mathcal{T}}_t + \widehat{\Delta}_t\right)\right)\widehat{Y}_{F,t} - \\
&\frac{1}{2}\frac{\theta\alpha}{(1-\alpha\beta)(1-\alpha)}\left[a_{\mathrm{H}}\pi_{H,t}^2 + (1-a_{\mathrm{H}})\pi_{H,t}^{*2} + a_{\mathrm{H}}\pi_{F,t}^{*2} + (1-a_{\mathrm{H}})\pi_{F,t}^2\right] \\
&+t.i.p. + o\left(\varepsilon^3\right)
\end{aligned}
$$

$$
\begin{aligned}
&= -\left(\frac{\sigma}{2}\widehat{C}_t - \widehat{\zeta}_{C,t}\right)\left(\widehat{C}_t - \widehat{Y}_{H,t}\right) - \left(\frac{\sigma}{2}\widehat{C}_t^* - \widehat{\zeta}_{C,t}^* + \widehat{\mathcal{Q}}_t\right)\left(\widehat{C}_t^* - \widehat{Y}_{F,t}\right) - \\
&(1-a_{\mathrm{H}})\left(\widehat{C}_t - \widehat{C}_t^* - \widehat{\mathcal{Q}}_t\right)\widehat{\Delta}_t - \\
&\left(\frac{\sigma}{2}\widehat{C}_t - \widehat{\zeta}_{C,t} + \frac{\eta}{2}\widehat{Y}_{H,t} - (1+\eta)\widehat{\zeta}_{Y,t} + \frac{1}{2}(1-a_{\mathrm{H}})\left(\widehat{\mathcal{T}}_t + \widehat{\Delta}_t\right)\right)\widehat{Y}_{H,t} - \\
&\left(\frac{\sigma}{2}\widehat{C}_t^* - \widehat{\zeta}_{C,t}^* + \frac{\eta}{2}\widehat{Y}_{F,t} - (1+\eta)\widehat{\zeta}_{Y,t}^* - \frac{1}{2}(1-a_{\mathrm{H}})\left(\widehat{\mathcal{T}}_t + \widehat{\Delta}_t\right)\right)\widehat{Y}_{F,t} - \\
&\frac{1}{2}(1-a_{\mathrm{H}})\left(\widehat{\mathcal{T}}_t + \widehat{\Delta}_t\right)\left(\widehat{Y}_{H,t} - \widehat{Y}_{F,t}\right) + \\
&(1-a_{\mathrm{H}})a_{\mathrm{H}}\phi\left(\widehat{\mathcal{T}}_t + \widehat{\Delta}_t\right)^2 + (1-a_{\mathrm{H}})\left([1-2a_{\mathrm{H}}(1-\phi)]\widehat{\mathcal{T}}_t - 2a_{\mathrm{H}}(1-\phi)\widehat{\Delta}_t\right)\widehat{\Delta}_t - \\
&\frac{1}{2}\frac{\theta\alpha}{(1-\alpha\beta)(1-\alpha)}\left[a_{\mathrm{H}}\pi_{H,t}^2 + (1-a_{\mathrm{H}})\pi_{H,t}^{*2} + a_{\mathrm{H}}\pi_{F,t}^{*2} + (1-a_{\mathrm{H}})\pi_{F,t}^2\right] \\
&+t.i.p. + o\left(\varepsilon^3\right).
\end{aligned}
$$

Some more substitutions and algebra follows. Using the expressions for shocks (5) and domestic marginal costs (6) in terms of efficient output and terms of trade, we can express the loss in terms of output gaps, relative price misalignment, including Δ_t, and demand imbalances:

$$\mathcal{L}_t^W \ltimes -\left(\frac{\sigma}{2}\widehat{C}_t - \widehat{\zeta}_{C,t}\right)\left(\widehat{C}_t - \widehat{Y}_{H,t}\right) - \left(\frac{\sigma}{2}\widehat{C}_t^* - \widehat{\zeta}_{C,t}^* + \widehat{\mathcal{Q}}_t\right)\left(\widehat{C}_t^* - \widehat{Y}_{F,t}\right) -$$
$$(1-a_{\mathrm{H}})\left(\widehat{C}_t - \widehat{C}_t^* - \widehat{\mathcal{Q}}_t\right)\widehat{\Delta}_t -$$
$$\left[(\eta+\sigma)\left(\frac{1}{2}\widehat{Y}_{H,t} - \widehat{Y}_{H,t}^{fb}\right) - 2a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)\left(\frac{1}{2}\left(\widehat{\mathcal{T}}_t + \widehat{\Delta}_t\right) - \widehat{\mathcal{T}}_t^{fb}\right)\right]\widehat{Y}_{H,t} + -$$
$$\left[(\eta+\sigma)\left(\frac{1}{2}\widehat{Y}_{F,t} - \widehat{Y}_{F,t}^{fb}\right) + 2a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)\left(\frac{1}{2}\left(\widehat{\mathcal{T}}_t + \widehat{\Delta}_t\right) - \widehat{\mathcal{T}}_t^{fb}\right)\right]\widehat{Y}_{F,t} + -$$
$$\frac{1}{2}(1-a_{\mathrm{H}})\left[\left(\widehat{\mathcal{T}}_t + \widehat{\Delta}_t\right) + \widetilde{\mathcal{W}}_t + \widehat{\Delta}_t - \left(\widehat{\zeta}_{C,t} - \widehat{\zeta}_{C,t}^*\right)\right]\left(\widehat{Y}_{H,t} - \widehat{Y}_{F,t}\right) +$$
$$(1-a_{\mathrm{H}})a_{\mathrm{H}}\phi\left(\widehat{\mathcal{T}}_t + \widehat{\Delta}_t\right)^2 + (1-a_{\mathrm{H}})\left([1-2a_{\mathrm{H}}(1-\phi)]\widehat{\mathcal{T}}_t - 2a_{\mathrm{H}}(1-\phi)\widehat{\Delta}_t\right)\widehat{\Delta}_t -$$
$$\frac{1}{2}\frac{\theta\alpha}{(1-\alpha\beta)(1-\alpha)}\left[a_{\mathrm{H}}\pi_{H,t}^2 + (1-a_{\mathrm{H}})\pi_{H,t}^{*2} + a_{\mathrm{H}}\pi_{F,t}^{*2} + (1-a_{\mathrm{H}})\pi_{F,t}^2\right] + t.i.p. + o\left(\varepsilon^3\right),$$

Note that we have also collected all the terms multiplied by $\left(\widehat{C}_t^* - \widehat{Y}_{F,t}\right)$. Collecting the terms in output gaps and the terms multiplied by output differentials yields:

$$\mathcal{L}_t^W \ltimes -\left(\frac{\sigma}{2}\widehat{C}_t - \widehat{\zeta}_{C,t}\right)\left(\widehat{C}_t - \widehat{Y}_{H,t}\right) - \left(\frac{\sigma}{2}\widehat{C}_t^* - \widehat{\zeta}_{C,t}^* + \widehat{\mathcal{Q}}_t\right)\left(\widehat{C}_t^* - \widehat{Y}_{F,t}\right) -$$
$$(1-a_{\mathrm{H}})\left(\widehat{C}_t - \widehat{C}_t^* - \widehat{\mathcal{Q}}_t\right)\widehat{\Delta}_t -$$
$$(1-a_{\mathrm{H}})\frac{1}{2}\left[\left(\widehat{\mathcal{T}}_t + \widehat{\Delta}_t\right) + \left(\widetilde{\mathcal{W}}_t + \widehat{\Delta}_t\right) - \left(\widehat{\zeta}_{C,t} - \widehat{\zeta}_{C,t}^*\right)\right]\left(\widehat{Y}_{H,t} - \widehat{Y}_{F,t}\right) +$$
$$(1-a_{\mathrm{H}})a_{\mathrm{H}}\phi\left(\widehat{\mathcal{T}}_t + \widehat{\Delta}_t\right)^2 + (1-a_{\mathrm{H}})\left([1-2a_{\mathrm{H}}(1-\phi)]\widehat{\mathcal{T}}_t - 2a_{\mathrm{H}}(1-\phi)\widehat{\Delta}_t\right)\widehat{\Delta}_t +$$
$$2a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)\left(\frac{1}{2}\left(\widehat{\mathcal{T}}_t + \widehat{\Delta}_t\right) - \widehat{\mathcal{T}}_t^{fb}\right)\left(\widehat{Y}_{H,t} - \widehat{Y}_{F,t}\right) -$$
$$(\eta+\sigma)\left(\frac{1}{2}\widehat{Y}_{H,t} - \widehat{Y}_{H,t}^{fb}\right)\widehat{Y}_{H,t} - (\eta+\sigma)\left(\frac{1}{2}\widehat{Y}_{F,t} - \widehat{Y}_{F,t}^{fb}\right)\widehat{Y}_{F,t} -$$
$$\frac{1}{2}\frac{\theta\alpha}{(1-\alpha\beta)(1-\alpha)}\left[a_{\mathrm{H}}\pi_{H,t}^2 + (1-a_{\mathrm{H}})\pi_{H,t}^{*2} + a_{\mathrm{H}}\pi_{F,t}^{*2} + (1-a_{\mathrm{H}})\pi_{F,t}^2\right] + t.i.p. + o\left(\varepsilon^3\right).$$

Using (2) and (3), the first order approximations for $\widehat{C}_t$ and $\widehat{C}_t^*$, we can rearrange further,

$$
\begin{aligned}
&\mathcal{L}_t^W \ltimes \\
&\left[\frac{\sigma}{2}\left(\widehat{C}_t-\widehat{C}_t^*\right)-\widehat{\mathcal{Q}}_t-\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}_{C,t}^*\right)\right]\frac{1}{2}\left\{\widehat{Y}_{H,t}-\widehat{Y}_{F,t}-\sigma^{-1}\left[\widehat{\mathcal{Q}}_t+\widetilde{\mathcal{W}}_t+\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}_{C,t}^*\right)\right]\right\}+ \\
&(1-a_{\mathrm{H}})\left(\widehat{C}_t-\widehat{C}_t^*-\widehat{\mathcal{Q}}_t\right)\widehat{\Delta}_t- \\
&(1-a_{\mathrm{H}})\frac{1}{2}\left[\left(\widehat{\mathcal{T}}_t+\widehat{\Delta}_t\right)+\left(\widetilde{\mathcal{W}}_t+\widehat{\Delta}_t\right)-\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}_{C,t}^*\right)\right]\left(\widehat{Y}_{H,t}-\widehat{Y}_{F,t}\right)+ \\
&(1-a_{\mathrm{H}})a_{\mathrm{H}}\phi\left(\widehat{\mathcal{T}}_t+\widehat{\Delta}_t\right)^2+(1-a_{\mathrm{H}})\left([1-2a_{\mathrm{H}}(1-\phi)]\widehat{\mathcal{T}}_t-2a_{\mathrm{H}}(1-\phi)\widehat{\Delta}_t\right)\widehat{\Delta}_t+ \\
&2a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)\left(\frac{1}{2}\left(\widehat{\mathcal{T}}_t+\widehat{\Delta}_t\right)-\widehat{\mathcal{T}}_t^{fb}\right)\left(\widehat{Y}_{H,t}-\widehat{Y}_{F,t}\right)- \\
&(\eta+\sigma)\left(\frac{1}{2}\widehat{Y}_{H,t}-\widehat{Y}_{H,t}^{fb}\right)\widehat{Y}_{H,t}-(\eta+\sigma)\left(\frac{1}{2}\widehat{Y}_{F,t}-\widehat{Y}_{F,t}^{fb}\right)\widehat{Y}_{F,t}- \\
&\frac{1}{2}\frac{\theta\alpha}{(1-\alpha\beta)(1-\alpha)}\left[a_{\mathrm{H}}\pi_{H,t}^2+(1-a_{\mathrm{H}})\pi_{H,t}^{*2}+a_{\mathrm{H}}\pi_{F,t}^{*2}+(1-a_{\mathrm{H}})\pi_{F,t}^2\right]+t.i.p.+o\left(\varepsilon^3\right).
\end{aligned}
$$

Here is a key passage: using the definition of the demand gap $\widetilde{\mathcal{W}}_t=\sigma\left(\widehat{C}_t-\widehat{C}_t^*\right)-\widehat{\mathcal{Q}}_t-\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}_{C,t}^*\right)$, we can eliminate all the terms in consumption:

$$
\begin{aligned}
&\mathcal{L}_t^W \ltimes -\frac{1}{4}\sigma^{-1}\left[\widetilde{\mathcal{W}}_t^2-\left(\widehat{\mathcal{Q}}_t+\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}_{C,t}^*\right)\right)^2\right]+ \\
&\frac{1}{4}\left[\widetilde{\mathcal{W}}_t-\left(\widehat{\mathcal{Q}}_t+\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}_{C,t}^*\right)\right)\right]\left(\widehat{Y}_{H,t}-\widehat{Y}_{F,t}\right)- \\
&(1-a_{\mathrm{H}})\sigma^{-1}\left(\widehat{\mathcal{Q}}_t+\widetilde{\mathcal{W}}_t+\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}_{C,t}^*\right)-\sigma\widehat{\mathcal{Q}}_t\right)\widehat{\Delta}_t- \\
&(1-a_{\mathrm{H}})\frac{1}{2}\left[\left(\widehat{\mathcal{T}}_t+\widehat{\Delta}_t\right)+\left(\widehat{\mathcal{D}}_t+\widehat{\Delta}_t\right)-\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}_{C,t}^*\right)\right]\left(\widehat{Y}_{H,t}-\widehat{Y}_{F,t}\right)+ \\
&(1-a_{\mathrm{H}})a_{\mathrm{H}}\phi\left(\widehat{\mathcal{T}}_t+\widehat{\Delta}_t\right)^2+(1-a_{\mathrm{H}})[1-2a_{\mathrm{H}}(1-\phi)]\left(\widehat{\mathcal{T}}_t+\widehat{\Delta}_t\right)\widehat{\Delta}_t+ \\
&2a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)\left(\frac{1}{2}\left(\widehat{\mathcal{T}}_t+\widehat{\Delta}_t\right)-\widehat{\mathcal{T}}_t^{fb}\right)\left(\widehat{Y}_{H,t}-\widehat{Y}_{F,t}\right)- \\
&(\eta+\sigma)\left(\frac{1}{2}\widehat{Y}_{H,t}-\widehat{Y}_{H,t}^{fb}\right)\widehat{Y}_{H,t}-(\eta+\sigma)\left(\frac{1}{2}\widehat{Y}_{F,t}-\widehat{Y}_{F,t}^{fb}\right)\widehat{Y}_{F,t}- \\
&\frac{1}{2}\frac{\theta\alpha}{(1-\alpha\beta)(1-\alpha)}\left[a_{\mathrm{H}}\pi_{H,t}^2+(1-a_{\mathrm{H}})\pi_{H,t}^{*2}+a_{\mathrm{H}}\pi_{F,t}^{*2}+(1-a_{\mathrm{H}})\pi_{F,t}^2\right] \\
&+t.i.p.+o\left(\varepsilon^3\right).
\end{aligned}
$$

We then collect the terms in output differentials:

$$
\begin{aligned}
&\mathcal{L}_t^W \ltimes -\frac{1}{4}\sigma^{-1}\left[\widetilde{\mathcal{W}}_t^2-\left(\widehat{\mathcal{Q}}_t+\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}_{C,t}^*\right)\right)^2\right]- \\
&(1-a_{\mathrm{H}})\,\sigma^{-1}\left(\widehat{\mathcal{Q}}_t+\widetilde{\mathcal{W}}_t+\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}_{C,t}^*\right)-\sigma\widehat{\mathcal{Q}}_t\right)\Delta_t+ \\
&\frac{1}{4}\left[(2a_{\mathrm{H}}-1)\left(\left(\widetilde{\mathcal{W}}_t+\Delta_t\right)-\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}_{C,t}^*\right)\right)-\left(\widehat{\mathcal{T}}_t+\Delta_t\right)-2\Delta_t\right]\left(\widehat{Y}_{H,t}-\widehat{Y}_{F,t}\right)+ \\
&(1-a_{\mathrm{H}})\,a_{\mathrm{H}}\phi\left(\widehat{\mathcal{T}}_t+\Delta_t\right)^2+(1-a_{\mathrm{H}})\left(\left[1-2a_{\mathrm{H}}\,(1-\phi)\right]\widehat{\mathcal{T}}_t-2a_{\mathrm{H}}\,(1-\phi)\,\widehat{\Delta}_t\right)\widehat{\Delta}_t+ \\
&2a_{\mathrm{H}}\,(1-a_{\mathrm{H}})\,(\sigma\phi-1)\left(\frac{1}{2}\left(\widehat{\mathcal{T}}_t+\Delta_t\right)-\widehat{\mathcal{T}}_t^{fb}\right)\left(\widehat{Y}_{H,t}-\widehat{Y}_{F,t}\right)- \\
&(\eta+\sigma)\left(\frac{1}{2}\widehat{Y}_{H,t}-\widehat{Y}_{H,t}^{fb}\right)\widehat{Y}_{H,t}-(\eta+\sigma)\left(\frac{1}{2}\widehat{Y}_{F,t}-\widehat{Y}_{F,t}^{fb}\right)\widehat{Y}_{F,t}- \\
&\frac{1}{2}\frac{\theta\alpha}{(1-\alpha\beta)(1-\alpha)}\left[a_{\mathrm{H}}\pi_{H,t}^2+(1-a_{\mathrm{H}})\,\pi_{H,t}^{*2}+a_{\mathrm{H}}\pi_{F,t}^{*2}+(1-a_{\mathrm{H}})\,\pi_{F,t}^2\right]+t.i.p.+o\left(\varepsilon^3\right).
\end{aligned}
$$

and use the expression for the terms of trade (4) to obtain,

$$
\begin{aligned}
&\mathcal{L}_t^W \ltimes -\frac{1}{4}\sigma^{-1}\left[\widetilde{\mathcal{W}}_t^2-\left(\widehat{\mathcal{Q}}_t+\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}_{C,t}^*\right)\right)^2\right]- \\
&(1-a_{\mathrm{H}})\,\sigma^{-1}\left(\widehat{\mathcal{Q}}_t+\widetilde{\mathcal{W}}_t+\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}_{C,t}^*\right)-\sigma\widehat{\mathcal{Q}}_t\right)\widehat{\Delta}_t+ \\
&\frac{1}{4}\left[(2a_{\mathrm{H}}-1)\left(\left(\widetilde{\mathcal{W}}_t+\widehat{\Delta}_t\right)-\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}_{C,t}^*\right)\right)-\left(\widehat{\mathcal{T}}_t+\widehat{\Delta}_t\right)-2\widehat{\Delta}_t\right]\left(\widehat{Y}_{H,t}-\widehat{Y}_{F,t}\right)- \\
&-\frac{a_{\mathrm{H}}\,(1-a_{\mathrm{H}})\,(\sigma\phi-1)}{4a_{\mathrm{H}}\,(1-a_{\mathrm{H}})\,(\sigma\phi-1)+1}\,(2a_{\mathrm{H}}-1)\left[\left(\widetilde{\mathcal{W}}_t+\widehat{\Delta}_t\right)-\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}_{C,t}^*\right)\right]\left(\widehat{Y}_{H,t}-\widehat{Y}_{F,t}\right) \\
&(1-a_{\mathrm{H}})\,a_{\mathrm{H}}\phi\left(\widehat{\mathcal{T}}_t+\widehat{\Delta}_t\right)^2+(1-a_{\mathrm{H}})\left(\left[1-2a_{\mathrm{H}}\,(1-\phi)\right]\widehat{\mathcal{T}}_t-2a_{\mathrm{H}}\,(1-\phi)\,\widehat{\Delta}_t\right)\widehat{\Delta}_t- \\
&(\eta+\sigma)\left(\frac{1}{2}\widehat{Y}_{H,t}-\widehat{Y}_{H,t}^{fb}\right)\widehat{Y}_{H,t}-(\eta+\sigma)\left(\frac{1}{2}\widehat{Y}_{F,t}-\widehat{Y}_{F,t}^{fb}\right)\widehat{Y}_{F,t}- \\
&\frac{1}{2}\frac{\theta\alpha)}{(1-\alpha\beta)(1-\alpha)}\left[a_{\mathrm{H}}\pi_{H,t}^2+(1-a_{\mathrm{H}})\,\pi_{H,t}^{*2}+a_{\mathrm{H}}\pi_{F,t}^{*2}+(1-a_{\mathrm{H}})\,\pi_{F,t}^2\right]+ \\
&\frac{2a_{\mathrm{H}}\,(1-a_{\mathrm{H}})\,(\sigma\phi-1)\,\sigma}{4a_{\mathrm{H}}\,(1-a_{\mathrm{H}})\,(\sigma\phi-1)+1}\left(\frac{1}{2}\left(\widehat{Y}_{H,t}-\widehat{Y}_{F,t}\right)-\left(\widehat{Y}_{H,t}^{fb}-\widehat{Y}_{F,t}^{fb}\right)\right)\left(\widehat{Y}_{H,t}-\widehat{Y}_{F,t}\right) \\
&+t.i.p.+o\left(\varepsilon^3\right)
\end{aligned}
$$

The last three lines of the previous expression coincides with the loss function under complete markets, expressed in deviations from the first best ($\widetilde{x}_t = \widehat{x}_t - \widehat{x}_t^{fb}$) when also $\widehat{\Delta}_t = 0$—rewritten below for convenience:

$$\mathcal{L}_t^W - \left(\mathcal{L}_t^W\right)^{fb} \ltimes -\frac{1}{2}(\eta+\sigma)\left(\widetilde{Y}_{H,t}\right)^2 - \frac{1}{2}(\eta+\sigma)\left(\widetilde{Y}_{F,t}\right)^2$$
$$-\frac{1}{2}\frac{\theta\alpha}{(1-\alpha\beta)(1-\alpha)}\left[a_{\mathrm{H}}\pi_{H,t}^2 + (1-a_{\mathrm{H}})\pi_{H,t}^{*2} + a_{\mathrm{H}}\pi_{F,t}^{*2} + (1-a_{\mathrm{H}})\pi_{F,t}^2\right] +$$
$$\frac{a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)\sigma}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\left(\widetilde{Y}_{H,t} - \widetilde{Y}_{F,t}\right)^2 + t.i.p. + o\left(\varepsilon^3\right).$$

It follows that all the other terms in $\mathcal{L}_t^W$ above must cancel out when $\widetilde{\mathcal{W}}_t = \widehat{\Delta}_t = 0$. The final step in deriving the generic loss function consists of verifying this conjecture, and derive how our expression must change under incomplete markets and LOOP deviations.

Substitute out for $\widehat{\mathcal{Q}}_t$ in terms of $\widehat{\mathcal{T}}_t$ and $\widehat{\Delta}_t$ using (1):

$$-\frac{1}{4}\sigma^{-1}\left[\widetilde{\mathcal{W}}_t^2 - \left((2a_{\mathrm{H}}-1)\left(\widehat{\mathcal{T}}_t + \Delta_t\right) + \Delta_t + \left(\widehat{\zeta}_{C,t} - \widehat{\zeta}_{C,t}^*\right)\right)^2\right] -$$
$$(1-a_{\mathrm{H}})\sigma^{-1}\left[(2a_{\mathrm{H}}-1)\left(\widehat{\mathcal{T}}_t + \Delta_t\right) + \left(\Delta_t + \widetilde{\mathcal{W}}_t\right) + \left(\widehat{\zeta}_{C,t} - \widehat{\zeta}_{C,t}^*\right)\right]\Delta_t +$$
$$(1-a_{\mathrm{H}})\left((2a_{\mathrm{H}}-1)\left(\widehat{\mathcal{T}}_t + \Delta_t\right) + \Delta_t\right)\Delta_t +$$
$$\frac{1}{4}\left[(2a_{\mathrm{H}}-1)\left(\left(\widetilde{\mathcal{W}}_t + \Delta_t\right) - \left(\widehat{\zeta}_{C,t} - \widehat{\zeta}_{C,t}^*\right)\right) - \left(\widehat{\mathcal{T}}_t + \Delta_t\right) - 2\Delta_t\right]\left(\widehat{Y}_{H,t} - \widehat{Y}_{F,t}\right) -$$
$$\frac{a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}(2a_{\mathrm{H}}-1)\left[\left(\widetilde{\mathcal{W}}_t + \widehat{\Delta}_t\right) - \left(\widehat{\zeta}_{C,t} - \widehat{\zeta}_{C,t}^*\right)\right]\left(\widehat{Y}_{H,t} - \widehat{Y}_{F,t}\right) +$$
$$(1-a_{\mathrm{H}})a_{\mathrm{H}}\phi\left(\widehat{\mathcal{T}}_t + \widehat{\Delta}_t\right)^2 + (1-a_{\mathrm{H}})\left(\left[1 - 2a_{\mathrm{H}}(1-\phi)\right]\widehat{\mathcal{T}}_t - 2a_{\mathrm{H}}(1-\phi)\widehat{\Delta}_t\right)\widehat{\Delta}_t$$

and substitute out the output differential using (4), yielding,

$$
\begin{aligned}
= \quad & -\frac{1}{4}\sigma^{-1}\left[\widetilde{\mathcal{W}}_t^2-\left((2a_{\mathrm{H}}-1)\left(\widehat{\mathcal{T}}_t+\Delta_t\right)+\Delta_t+\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}^*_{C,t}\right)\right)^2\right]- \\
& (1-a_{\mathrm{H}})\,\sigma^{-1}\left[(2a_{\mathrm{H}}-1)\left(\widehat{\mathcal{T}}_t+\Delta_t\right)+\left(\Delta_t+\widetilde{\mathcal{W}}_t\right)+\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}^*_{C,t}\right)\right]\Delta_t- \\
& (1-a_{\mathrm{H}})\,\sigma^{-1}\sigma\left((2a_{\mathrm{H}}-1)\left(\widehat{\mathcal{T}}_t+\Delta_t\right)+\Delta_t\right)\Delta_t+ \\
& \frac{1}{4}\sigma^{-1}\left[\begin{array}{c}(2a_{\mathrm{H}}-1)\left(\left(\widetilde{\mathcal{W}}_t+\Delta_t\right)-\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}^*_{C,t}\right)\right)+ \\ -\left(\widehat{\mathcal{T}}_t+\Delta_t\right)-2\Delta_t\end{array}\right]\cdot \\
& \left(\begin{array}{c}\left[4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1\right]\left(\widehat{\mathcal{T}}_t+\Delta_t\right)+ \\ (2a_{\mathrm{H}}-1)\left[\widetilde{\mathcal{W}}_t+\Delta_t+\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}^*_{C,t}\right)\right]\end{array}\right)- \\
& \sigma^{-1}\frac{a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}(2a_{\mathrm{H}}-1)\left[\left(\widetilde{\mathcal{W}}_t+\widehat{\Delta}_t\right)-\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}^*_{C,t}\right)\right]\cdot \\
& \left(\begin{array}{c}\left[4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1\right]\left(\widehat{\mathcal{T}}_t+\Delta_t\right)+ \\ (2a_{\mathrm{H}}-1)\left[\widetilde{\mathcal{W}}_t+\Delta_t+\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}^*_{C,t}\right)\right]\end{array}\right)+ \\
& (1-a_{\mathrm{H}})\,a_{\mathrm{H}}\phi\left(\widehat{\mathcal{T}}_t+\widehat{\Delta}_t\right)^2+(1-a_{\mathrm{H}})\left(\left[1-2a_{\mathrm{H}}(1-\phi)\right]\widehat{\mathcal{T}}_t-2a_{\mathrm{H}}(1-\phi)\,\widehat{\Delta}_t\right)\widehat{\Delta}_t
\end{aligned}
$$

After some algebra, the above expression is reduced to:

$$
\begin{aligned}
= \quad & -\frac{a_{\mathrm{H}}(1-a_{\mathrm{H}})\phi}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\left(\widetilde{\mathcal{W}}_t+\Delta_t\right)^2+ \\
& (1-a_{\mathrm{H}})\left(\left[1-2a_{\mathrm{H}}(1-\phi)\right]\widehat{\mathcal{T}}_t-2a_{\mathrm{H}}(1-\phi)\,\widehat{\Delta}_t\right)\widehat{\Delta}_t- \\
& (1-a_{\mathrm{H}})\,\sigma^{-1}\left((1-\sigma)\left((2a_{\mathrm{H}}-1)\left(\widehat{\mathcal{T}}_t+\Delta_t\right)+\Delta_t\right)+\widetilde{\mathcal{W}}_t+\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}^*_{C,t}\right)\right)\Delta_t- \\
& \frac{1}{2}\sigma^{-1}\left(\left[4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1\right]\left(\widehat{\mathcal{T}}_t+\Delta_t\right)+(2a_{\mathrm{H}}-1)\left[\widetilde{\mathcal{W}}_t+\Delta_t+\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}^*_{C,t}\right)\right]\right)\Delta_t+ \\
& \frac{1}{2}\sigma^{-1}\left[(2a_{\mathrm{H}}-1)\left(\widehat{\mathcal{T}}_t+\Delta_t\right)+\widetilde{\mathcal{W}}_t+\Delta_t+\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}^*_{C,t}\right)\right]\Delta_t,
\end{aligned}
$$

which vanishes under complete markets and PCP. Collecting terms we get,

$$
\begin{aligned}
= \quad & -\frac{a_{\mathrm{H}}(1-a_{\mathrm{H}})\phi}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\left(\widetilde{\mathcal{W}}_t+\Delta_t\right)^2+ \\
& (1-a_{\mathrm{H}})\left(\left[1-2a_{\mathrm{H}}(1-\phi)\right]\widehat{\mathcal{T}}_t-2a_{\mathrm{H}}(1-\phi)\,\widehat{\Delta}_t\right)\widehat{\Delta}_t+ \\
& \frac{1}{2}\sigma^{-1}\left[2a_{\mathrm{H}}-1-4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)-1-2(1-a_{\mathrm{H}})(1-\sigma)(2a_{\mathrm{H}}-1)\right]\left(\widehat{\mathcal{T}}_t+\Delta_t\right)\Delta_t+ \\
& (1-a_{\mathrm{H}})\,\Delta_t^2+ \\
& \frac{1}{2}\sigma^{-1}\left[1-(2a_{\mathrm{H}}-1)-2(1-a_{\mathrm{H}})\right]\left(\widetilde{\mathcal{W}}_t+\Delta_t+\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}^*_{C,t}\right)\right)\Delta_t,
\end{aligned}
$$

which further simplifies as follows

$$
\begin{aligned}
= \quad & -\frac{a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\phi}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\left(\widetilde{\mathcal{W}}_t+\Delta_t\right)^2+ \\
& \left(1-a_{\mathrm{H}}\right)\left(\left[1-2a_{\mathrm{H}}\left(1-\phi\right)\right]\widehat{\mathcal{T}}_t-2a_{\mathrm{H}}\left(1-\phi\right)\widehat{\Delta}_t\right)\widehat{\Delta}_t+ \\
& \left(1-a_{\mathrm{H}}\right)\left[2a_{\mathrm{H}}\left(1-\phi\right)-1\right]\widehat{\mathcal{T}}_t\Delta_t+ \\
& \left(1-a_{\mathrm{H}}\right)2a_{\mathrm{H}}\left[1-\phi\right]\Delta_t^2.
\end{aligned}
$$

Given that the last three lines cancel out, we conclude that with generically incomplete market under LCP the loss function in deviations from the first best can be expressed as:

$$
\begin{aligned}
\mathcal{L}_t^W-\left(\mathcal{L}_t^W\right)^{fb} \ltimes & -\frac{1}{2}\left(\eta+\sigma\right)\left(\widetilde{Y}_{H,t}\right)^2-\frac{1}{2}\left(\eta+\sigma\right)\left(\widetilde{Y}_{F,t}\right)^2+ \\
-\frac{1}{2}\frac{\theta\alpha}{\left(1-\alpha\beta\right)\left(1-\alpha\right)} & \left[a_{\mathrm{H}}\pi_{H,t}^2+\left(1-a_{\mathrm{H}}\right)\pi_{H,t}^{*2}+a_{\mathrm{H}}\pi_{F,t}^{*2}+\left(1-a_{\mathrm{H}}\right)\pi_{F,t}^2\right]+ \\
\frac{a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1} & \left[\left(\sigma\phi-1\right)\sigma\left(\widetilde{Y}_{H,t}-\widetilde{Y}_{F,t}\right)^2-\phi\left(\widehat{\Delta}_t+\widetilde{\mathcal{W}}_t\right)^2\right] \\
& +t.i.p.+o\left(\varepsilon^3\right).
\end{aligned}
$$

This completes the derivation of the optimal monetary policy loss function in the LCP economy.

1.3 Generalizations

1.3.1 PCP economy

The loss function under PCP is a special case of the above in which all LOOP deviations $\widehat{\Delta}_t$ are set to zero, which also implies that the inflation term, $\left[a_{\mathrm{H}}\pi_{H,t}^2+\left(1-a_{\mathrm{H}}\right)\pi_{H,t}^{*2}+a_{\mathrm{H}}\pi_{F,t}^{*2}+\left(1-a_{\mathrm{H}}\right)\pi_{F,t}^2\right]$, is equal to $\pi_{H,t}^2+\pi_{F,t}^{*2}$.

1.3.2 Encompassing different specifications of market incompleteness

Observe that maximization of the world welfare subject to the implementability constraints characterizing the competitive equilibrium requires spelling out the exact form of market incompleteness. Taking the difference of the budget constraints for an economy with n traded assets we can generically arrive at the following expression:

$$
\begin{aligned}
& C_t-\mathcal{Q}_tC_t^*= \\
& \frac{P_{\mathrm{H},t}}{P_t}Y_{\mathrm{H},t}+\left(\frac{S_tP_{\mathrm{H},t}^*}{P_{\mathrm{H},t}}-1\right)\frac{P_{\mathrm{H},t}}{P_t}C_{\mathrm{H},t}^*-\left(\frac{P_{\mathrm{F},t}^*}{P_t^*}\mathcal{Q}_tY_{\mathrm{F},t}+\left(1-\frac{SP_{\mathrm{F},t}^*}{P_{\mathrm{F},t}}\right)\frac{P_{\mathrm{F},t}}{P_t}C_{\mathrm{F},t}\right)+ \\
& 2\left[\left(1+r_{t-1}\right)\mathcal{B}_{t-1}+\sum_i\alpha_{i,t-1}\left(R_{i,t}-\left(1+r_{t-1}\right)\right)-\mathcal{B}_t\right],
\end{aligned}
$$

$$
\begin{aligned}
&C_t - \mathcal{Q}_t C_t^* = \left[a_{\rm H} + (1-a_{\rm H})\,\mathcal{T}_t^{1-\phi}\Delta_{{\rm H},t}^{1-\phi}\right]^{-\frac{1}{1-\phi}} Y_{{\rm H},t} - \\
&\left[a_{\rm H} + (1-a_{\rm H})\,\mathcal{T}_t^{\phi-1}\Delta_{{\rm F},t}^{\phi-1}\right]^{-\frac{1}{1-\phi}} \mathcal{Q}_t Y_{{\rm F},t} + \\
&1 - \frac{P_{{\rm H},t}}{S_t P_{{\rm H},t}^*}\Bigg) \frac{P_{{\rm H},t}^*}{P_t^*}\frac{S_t P_t^*}{P_t} C_{{\rm H},t}^* + \left(\frac{S P_{{\rm F},t}^*}{P_{{\rm F},t}} - 1\right)\frac{P_{{\rm F},t}}{P_t} C_{{\rm F},t} + \\
&2\left[(1+r_{t-1})\,\mathcal{B}_{t-1} + \sum_i \alpha_{i,t-1}\left(R_{i,t} - (1+r_{t-1})\right) - \mathcal{B}_t\right]
\end{aligned}
$$

$$
\begin{aligned}
&C_t - \mathcal{Q}_t C_t^* = \left[a_{\rm H} + (1-a_{\rm H})\,\mathcal{T}_t^{1-\phi}\Delta_{{\rm H},t}^{1-\phi}\right]^{-\frac{1}{1-\phi}} Y_{{\rm H},t} - \\
&\left[a_{\rm H} + (1-a_{\rm H})\,\mathcal{T}_t^{\phi-1}\Delta_{{\rm F},t}^{\phi-1}\right]^{-\frac{1}{1-\phi}} \mathcal{Q}_t Y_{{\rm F},t} + \\
&(1-a_{\rm H})\left[\begin{array}{c} \dfrac{\Delta_{{\rm H},t}-1}{\Delta_{{\rm H},t}}\left(a_{\rm H}\mathcal{T}_t^{1-\phi}\Delta_{{\rm F},t}^{1-\phi} + (1-a_{\rm H})\right)^{1-\phi}\mathcal{Q}_t C_t^* + \\ (\Delta_{\rm F}-1)\left(a_{\rm H}\mathcal{T}_t^{\phi-1}\Delta_{{\rm H},t}^{\phi-1} + (1-a_{\rm H})\right)^{1-\phi} C_t \end{array}\right] + \\
&2\left[(1+r_{t-1})\,\mathcal{B}_{t-1} + \sum_i \alpha_{i,t-1}\left(R_{i,t} - (1+r_{t-1})\right) - \mathcal{B}_t\right]
\end{aligned}
$$

where all ex-post returns are expressed in terms of Home consumption prices — e.g. $1+r_{t-1} = \dfrac{1+i_t}{P_t/P_{t-1}}$ and $\sum_i \alpha_{i,t} = \mathcal{B}_t$. Around a symmetric steady state with zero real NFA ($\mathcal{B} = 0$) , the consumption differential, up to first order, is given by:

$$
\begin{aligned}
&\widehat{C}_t - \widehat{C}_t^* - \widehat{\mathcal{Q}}_t = \\
&\widehat{Y}_{H,t} - \widehat{Y}_{F,t} - \widehat{\mathcal{Q}}_t - 2\,(1-a_{\rm H})\,\widehat{\mathcal{T}}_t - (1-a_{\rm H})\left(\widehat{\Delta}_{{\rm F},t} + \widehat{\Delta}_{{\rm H},t}\right) + \\
&(1-a_{\rm H})\left(\widehat{\Delta}_{{\rm F},t} + \widehat{\Delta}_{{\rm H},t}\right) + 2\beta^{-1}\left(\widehat{\mathcal{B}}_{t-1} - \beta\widehat{\mathcal{B}}_t + \sum_i \frac{\omega_i}{Y}\left(\widehat{R}_{i,t} - \left(\widehat{1+r_{t-1}}\right)\right)\right).
\end{aligned}
$$

where NFA deviations are defined wrt to steady state output $\widehat{\mathcal{B}}_{t-1} = \dfrac{\mathcal{B}_{t-1} - 0}{Y}$, and ω_i represents the share of gross wealth invested in the i-th asset in the *stochastic* steady state.

For $\widehat{\Delta}_{{\rm H},t} = \widehat{\Delta}_{{\rm F},t} = \widehat{\Delta}_t$ under symmetry, we get:

$$
\begin{aligned}
\widehat{C}_t - \widehat{C}_t^* \quad = \quad & \widehat{Y}_{H,t} - \widehat{Y}_{F,t} - 2\,(1-a_{\rm H})\,\widehat{\mathcal{T}}_t + \\
& 2\beta^{-1}\left(\widehat{\mathcal{B}}_{t-1} - \beta\widehat{\mathcal{B}}_t + \sum_i \frac{\omega_i}{Y}\left(\widehat{R}_{i,t} - \left(\widehat{1+r_{t-1}}\right)\right)\right).
\end{aligned}
$$

Under financial autarky, since $\widehat{\mathcal{B}}_t = \widehat{\mathcal{B}}_{t-1} = 0$, we have the following:

$$
\widetilde{\mathcal{W}}_t = \sigma\left[\widehat{Y}_{H,t} - \widehat{Y}_{F,t} - 2\,(1-a_{\rm H})\,\widehat{\mathcal{T}}_t\right] - \widehat{\mathcal{Q}}_t - \left(\widehat{\zeta}_{C,t} - \widehat{\zeta}_{C,t}^*\right),
$$

whereas, in the case of a bond economy, the wealth gap $\widetilde{\mathcal{W}}_t$ will also reflect net capital flows:

$$\widetilde{\mathcal{W}}_t = \sigma \left[\begin{array}{c} -\left(\left(\widehat{\mathcal{B}}_t - \beta^{-1}\widehat{\mathcal{B}}_{t-1}\right) - \widehat{Y}_{H,t} + (1-a_{\mathrm{H}})\,\widehat{\mathcal{T}}_t\right) + \\ \left(\left(-\widehat{\mathcal{B}}_t - \beta^{-1}\left(-\widehat{\mathcal{B}}_{t-1}\right)\right) - \widehat{Y}_{F,t} - (1-a_{\mathrm{H}})\,\widehat{\mathcal{T}}_t\right) \end{array} \right] + \\ -\widehat{\mathcal{Q}}_t - \left(\widehat{\zeta}_{C,t} - \widehat{\zeta}^{*}_{C,t}\right).$$

Finally, rewriting in terms of gaps (useful when characterizing optimal policy)the wealth gap in a bond economy is given by,

$$\begin{aligned} \widetilde{\mathcal{W}}_t &= \sigma\left(\widetilde{C}_t - \widetilde{C}^*_t\right) - \widetilde{\mathcal{Q}}_t \\ &= \sigma\left[\widetilde{Y}_{H,t} - \widetilde{Y}_{F,t} + 2\beta^{-1}\left(\widehat{\mathcal{B}}_{t-1} - \beta\widehat{\mathcal{B}}_t\right)\right] - 2a_{\mathrm{H}}\widehat{\Delta}_t - \left[2(1-a_{\mathrm{H}})\,\sigma + (2a_{\mathrm{H}}-1)\right]\widetilde{\mathcal{T}}_t + \\ &\quad 2(1-a_{\mathrm{H}})\left[(2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma)\,\widehat{\mathcal{T}}^{fb}_t - \left(\widehat{\zeta}_{C,t} - \widehat{\zeta}^{*}_{C,t}\right)\right], \end{aligned}$$

and under autarky,

$$\begin{aligned} \widetilde{\mathcal{W}}_t &= \sigma\left(\widetilde{C}_t - \widetilde{C}^*_t\right) - \widetilde{\mathcal{Q}}_t \\ &= \sigma\left[\widetilde{Y}_{H,t} - \widetilde{Y}_{F,t}\right] - 2a_{\mathrm{H}}\widehat{\Delta}_t - \left[2(1-a_{\mathrm{H}})\,\sigma + (2a_{\mathrm{H}}-1)\right]\widetilde{\mathcal{T}}_t + \\ &\quad 2(1-a_{\mathrm{H}})\left[(2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma)\,\widehat{\mathcal{T}}^{fb}_t - \left(\widehat{\zeta}_{C,t} - \widehat{\zeta}^{*}_{C,t}\right)\right]. \end{aligned}$$

2 Characterizing optimal monetary targeting rules and optimal allocations under incomplete markets: Proofs of Propositions 2, 3, 4, 5, 9 [and 10?]

In this section we work out the constrained efficient allocation in our model economy—this is found by maximizing the expected discounted value of the following loss function in deviation from first best,

$$\begin{aligned}\mathcal{L}_t^W - \left(\mathcal{L}_t^W\right)^{fb} \ltimes & -\frac{1}{2}(\eta+\sigma)\left(\widehat{Y}_{H,t}-\widehat{Y}_{H,t}^{fb}\right)^2 - \frac{1}{2}(\eta+\sigma)\left(\widehat{Y}_{F,t}-\widehat{Y}_{F,t}^{fb}\right)^2 - \\ & \frac{1}{2}\frac{\theta\alpha}{(1-\alpha\beta)(1-\alpha)}\left[a_{\mathrm{H}}\pi_{H,t}^2+(1-a_{\mathrm{H}})\pi_{H,t}^{*2}+a_{\mathrm{H}}\pi_{F,t}^{*2}+(1-a_{\mathrm{H}})\pi_{F,t}^2\right] + \\ & \frac{a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)\sigma}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\left[\left(\widehat{Y}_{H,t}-\widehat{Y}_{H,t}^{fb}\right)-\left(\widehat{Y}_{F,t}-\widehat{Y}_{F,t}^{fb}\right)\right]^2 - \\ & \frac{a_{\mathrm{H}}(1-a_{\mathrm{H}})\phi}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\left(\widehat{\Delta}_t+\widehat{\mathcal{W}}_t\right)^2 + t.i.p. + o\left(\varepsilon^3\right),\end{aligned} \tag{8}$$

with respect to its arguments $\widehat{Y}_{H,t}, \widehat{Y}_{F,t}, \widehat{\mathcal{W}}_t, \widehat{\Delta}_t$ and $\pi_{H,t}, \pi_{F,t}, \pi^*_{H,t}, \pi^*_{F,t}$ subject to the NK Phillips curves, the equilibrium condition linking relative prices to output gap differentials and demand gaps, the definition of the wealth gap, and the Euler equation characterizing the evolution of the wealth gap. In the case of non-trivial portfolio decisions (not covered here), higher order Euler equations characterizing these choices would have also to be considered.

We treat the cases of PCP and LCP separately as some of the constraints differ significantly.

2.1 LCP economy

2.1.1 Proofs of propositions 2 and 4

In the LCP case, the monetary authority minimizes (1), with respect to its arguments $\widehat{Y}_{H,t}$, $\widehat{Y}_{F,t}$, $\widehat{\Delta}_t$, $\widehat{\mathcal{W}}_t$, and $\pi_{H,t}$, $\pi^*_{H,t}$, $\pi^*_{F,t}$, $\pi_{F,t}$, subject to the following constraints arising from the competitive equilibrium:

1. NK Phillips curves determining inflation rates

$$\begin{aligned}&\pi_{H,t} - \beta E_t \pi_{H,t+1} = \\ &\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\left[\begin{array}{c}(\sigma+\eta)\left(\widehat{Y}_{H,t}-\widehat{Y}_{H,t}^{fb}\right)+\widehat{\mu}_t+ \\ -(1-a_{\mathrm{H}})\left[2a_{\mathrm{H}}(\sigma\phi-1)\left(\widehat{\mathcal{T}}_t-\widehat{\mathcal{T}}_t^{fb}+\widehat{\Delta}_t\right)-\left(\widehat{\Delta}_t+\widehat{\mathcal{W}}_t\right)\right]\end{array}\right] \\ &= \pi^*_{H,t} - \beta E_t \pi^*_{H,t+1} + \frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\widehat{\Delta}_t,\end{aligned}$$

$$\pi^*_{F,t} - \beta E_t \pi^*_{F,t+1} =$$

$$\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\left[\begin{array}{c}(\sigma+\eta)\left(\widehat{Y}_{F,t} - \widehat{Y}^{fb}_{F,t}\right) + \widehat{\mu}^*_t + \\ (1-a_{\mathrm{H}})\left[2a_{\mathrm{H}}(\sigma\phi-1)\left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}^{fb}_t + \widehat{\Delta}_t\right) - \left(\widehat{\Delta}_t + \widehat{\mathcal{W}}_t\right)\right]\end{array}\right]$$

$$= \pi_{F,t} - \beta E_t \pi_{F,t+1} - \frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\widehat{\Delta}_t,$$

and the constraint on inflation differentials in the same currency:

$$\pi_{F,t} - \pi_{H,t} - \left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}_{t-1} + \widehat{\Delta}_t - \widehat{\Delta}_{t-1}\right) = 0,$$

where the equilibrium relations for first best outcomes $\widehat{Y}^{fb}_{H,t}$, $\widehat{Y}^{fb}_{F,t}$, $\widehat{\mathcal{T}}^{fb}_t$ in terms of fundamental shocks are as follows:

$$(\eta+\sigma)\widehat{Y}^{fb}_{H,t} =$$
$$[2a_{\mathrm{H}}\ (1-a_{\mathrm{H}})(\sigma\phi-1)]\left(\widehat{\mathcal{T}}^{fb}_t\right) - (1-a_{\mathrm{H}})\left(\widehat{\zeta}_{C,t} - \widehat{\zeta}^*_{C,t}\right) + \widehat{\zeta}_{C,t} + (1+\eta)\widehat{\zeta}_{Y,t}$$
$$(\eta+\sigma)\widehat{Y}^{fb}_{F,t} =$$
$$[2a_{\mathrm{H}}\ (1-a_{\mathrm{H}})(\sigma\phi-1)]\left(-\widehat{\mathcal{T}}^{fb}_t\right) + (1-a_{\mathrm{H}})\left(\widehat{\zeta}_{C,t} - \widehat{\zeta}^*_{C,t}\right) + \widehat{\zeta}^*_{C,t} + (1+\eta)\widehat{\zeta}^*_{Y,t},$$
$$[4(1-a_{\mathrm{H}})a_{\mathrm{H}}(\phi\sigma-1)+1]\widehat{\mathcal{T}}^{fb}_t = \sigma\left(\widehat{Y}^{fb}_{H,t} - \widehat{Y}^{fb}_{F,t}\right) - (2a_{\mathrm{H}}-1)\left(\widehat{\zeta}_{C,t} - \widehat{\zeta}^*_{C,t}\right);$$

2. the equilibrium condition linking relative prices to output gap differentials, $\widehat{\Delta}_t$ and demand gaps:

$$\widehat{\mathcal{T}}_t + \widehat{\Delta}_t - \widehat{\mathcal{T}}^{fb}_t = \frac{\sigma\left[\left(\widehat{Y}_{H,t} - \widehat{Y}_{F,t}\right) - \left(\widehat{Y}^{fb}_{H,t} - \widehat{Y}^{fb}_{F,t}\right)\right] - (2a_{\mathrm{H}}-1)\left(\widehat{\mathcal{W}}_t + \widehat{\Delta}_t\right)}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1};$$

3. the definition of wealth gap $\widetilde{\mathcal{W}}_t$ from the difference in budget constraints, depending also on net wealth $\widehat{\mathcal{B}}_t$:

$$\begin{aligned}\widetilde{\mathcal{W}}_t &= \widehat{\mathcal{W}}_t = \sigma\left[\left(\widehat{C}_t - \widehat{C}^*_t\right) - \left(\widehat{C}^{fb}_t - \widehat{C}^{*fb}_t\right)\right] - \left(\widehat{\mathcal{Q}}_t - \widehat{\mathcal{Q}}^{fb}_t\right) \\ &= \sigma\left[\begin{array}{c}\left(\widehat{Y}_{H,t} - \widehat{Y}_{F,t}\right) - \left(\widehat{Y}^{fb}_{H,t} - \widehat{Y}^{fb}_{F,t}\right) + \\ 2\beta^{-1}\left(\widehat{\mathcal{B}}_{t-1} - \beta\widehat{\mathcal{B}}_t + \sum_i \frac{\omega_i}{Y}\left(\widehat{R}_{i,t} - \left(\widehat{1+r_t}\right)\right)\right)\end{array}\right] + \\ &\quad - [2(1-a_{\mathrm{H}})\sigma + (2a_{\mathrm{H}}-1)]\left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}^{fb}_t\right) - 2a_{\mathrm{H}}\widehat{\Delta}_t + \\ &\quad 2(1-a_{\mathrm{H}})\left[(2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma)\widehat{\mathcal{T}}^{fb}_t - \left(\widehat{\zeta}_{C,t} - \widehat{\zeta}^*_{C,t}\right)\right],\end{aligned}$$

4. the Euler equation characterizing the evolution of $\widehat{\mathcal{W}}_t$ (and thus net wealth $\widehat{\mathcal{B}}_t$):

$$E_t\widehat{\mathcal{W}}_{t+1} - \widehat{\mathcal{W}}_t = 0.$$

Bond economy

Observe that in the case of a bond economy, the program amounts to choosing $\widehat{Y}_{H,t}$, $\widehat{Y}_{F,t}$, $\widehat{\Delta}_t$, $\widehat{\mathcal{W}}_t$, $\pi_{H,t}$, $\pi^*_{H,t}$, $\pi^*_{F,t}$, $\pi_{F,t}$, and $\widehat{\mathcal{B}}_t$, subject to the following expression for $\widehat{\mathcal{W}}_t$ in terms of differences of budget constraints, namely:

$$
\begin{aligned}
(1-a_{\mathrm{H}})\left[2a_{\mathrm{H}}(\phi-1)+1\right]\widehat{\mathcal{W}}_t &= \left[4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1\right]\left(\beta^{-1}\widehat{\mathcal{B}}_{t-1}-\widehat{\mathcal{B}}_t\right)+ \\
&(1-a_{\mathrm{H}})\left[2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma\right]\left[\left(\widehat{Y}_{H,t}-\widehat{Y}^{fb}_{H,t}\right)-\left(\widehat{Y}_{F,t}-\widehat{Y}^{fb}_{F,t}\right)\right]+ \\
&2a_{\mathrm{H}}(1-a_{\mathrm{H}})\left[2(1-a_{\mathrm{H}})(\sigma\phi-1)+1-\phi\right]\widehat{\Delta}_t+ \\
&(1-a_{\mathrm{H}})\left[4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1\right]\cdot \\
&\sigma^{-1}\left[\left(2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma\right)\widehat{\mathcal{T}}^{fb}_t-\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}^*_{C,t}\right)\right]
\end{aligned}
$$

The necessary FOC's with respect to inflation are given by:

$$
\begin{aligned}
\pi_{H,t} &: \quad 0=-\theta\frac{\alpha}{(1-\alpha\beta)(1-\alpha)}a_{\mathrm{H}}\pi_{H,t}-\gamma_{H,t}+\gamma_{H,t-1}-\gamma_t \\
\pi^*_{H,t} &: \quad 0=-\theta\frac{\alpha}{(1-\alpha\beta)(1-\alpha)}(1-a_{\mathrm{H}})\pi^*_{H,t}-\gamma^*_{H,t}+\gamma^*_{H,t-1} \\
\pi_{F,t} &: \quad 0=-\theta\frac{\alpha}{(1-\alpha\beta)(1-\alpha)}(1-a_{\mathrm{H}})\pi_{F,t}-\gamma_{F,t}+\gamma_{F,t-1}+\gamma_t \\
\pi^*_{F,t} &: \quad 0=-\theta\frac{\alpha}{(1-\alpha\beta)(1-\alpha)}a_{\mathrm{H}}\pi^*_{F,t}-\gamma^*_{F,t}+\gamma^*_{F,t-1},
\end{aligned}
$$

where $\gamma_{H,t}$, $\gamma_{F,t}$, $\gamma^*_{H,t}$and $\gamma^*_{F,t}$ are the multipliers associated with the Phillips curves — whose lags appear reflecting the assumption of commitment, implying the following solutions for the multipliers:

$$
\begin{aligned}
-\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\left(\gamma_{H,t}+\gamma_{F,t}\right) &= \theta\left(a_{\mathrm{H}}\widehat{p}_{H,t}+(1-a_{\mathrm{H}})\widehat{p}_{F,t}\right) \\
-\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\left(\gamma^*_{H,t}+\gamma^*_{F,t}\right) &= \theta\left(a_{\mathrm{H}}\widehat{p}^*_{F,t}+(1-a_{\mathrm{H}})\widehat{p}^*_{H,t}\right) \\
-2\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\gamma_t &= \theta\left[a_{\mathrm{H}}\left(\pi_{H,t}-\pi^*_{F,t}\right)+(1-a_{\mathrm{H}})\left(\pi^*_{H,t}-\pi_{F,t}\right)\right]+ \\
&-\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\begin{bmatrix}\left(-\gamma_{H,t}-\gamma^*_{H,t}+\gamma^*_{H,t-1}+\gamma_{H,t-1}\right)+ \\ -\left(-\gamma^*_{F,t}-\gamma_{F,t}+\gamma_{F,t-1}+\gamma^*_{F,t-1}\right)\end{bmatrix}.
\end{aligned}
$$

The FOC with respect to output is given by:

$$
\begin{aligned}
\widehat{Y}_{H,t} \quad : \quad & 0=(\sigma+\eta)\left(\widehat{Y}_{H,t}-\widehat{Y}_{H,t}^{fb}\right)+ \\
& -\frac{2a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)\sigma}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\left[\left(\widehat{Y}_{H,t}-\widehat{Y}_{H,t}^{fb}\right)-\left(\widehat{Y}_{F,t}-\widehat{Y}_{F,t}^{fb}\right)\right]+ \\
& \frac{2a_{\mathrm{H}}(1-a_{\mathrm{H}})\phi}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\frac{2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma}{2a_{\mathrm{H}}(\phi-1)+1}\left(\widehat{\Delta}_t+\widehat{\mathcal{W}}_t\right)+ \\
& -\left[\sigma+\eta-\frac{(1-a_{\mathrm{H}})(\sigma-1)}{2a_{\mathrm{H}}(\phi-1)+1}\right]\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\left(\gamma_{H,t}+\gamma_{H,t}^{*}\right)+ \\
& -\frac{(1-a_{\mathrm{H}})(\sigma-1)}{2a_{\mathrm{H}}(\phi-1)+1}\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\left(\gamma_{F,t}+\gamma_{F,t}^{*}\right)+ \\
& -\frac{1}{2a_{\mathrm{H}}(\phi-1)+1}\left(\beta E_t\gamma_{t+1}-\gamma_t\right)+ \\
& \frac{2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma}{2a_{\mathrm{H}}(\phi-1)+1}\left(\lambda_t-\beta^{-1}\lambda_{t-1}\right);
\end{aligned}
$$

$$
\begin{aligned}
\widehat{Y}_{F,t} \quad : \quad & 0=(\sigma+\eta)\left(\widehat{Y}_{F,t}-\widehat{Y}_{F,t}^{fb}\right)+ \\
& \frac{2a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)\sigma}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\left[\left(\widehat{Y}_{H,t}-\widehat{Y}_{H,t}^{fb}\right)-\left(\widehat{Y}_{F,t}-\widehat{Y}_{F,t}^{fb}\right)\right]+ \\
& -\frac{2a_{\mathrm{H}}(1-a_{\mathrm{H}})\phi}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\frac{2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma}{2a_{\mathrm{H}}(\phi-1)+1}\left(\widehat{\Delta}_t+\widehat{\mathcal{W}}_t\right)+ \\
& -\left[\sigma+\eta-\frac{(1-a_{\mathrm{H}})(\sigma-1)}{2a_{\mathrm{H}}(\phi-1)+1}\right]\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\left(\gamma_{F,t}+\gamma_{F,t}^{*}\right)+ \\
& -\frac{(1-a_{\mathrm{H}})(\sigma-1)}{2a_{\mathrm{H}}(\phi-1)+1}\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\left(\gamma_{H,t}+\gamma_{H,t}^{*}\right)+ \\
& \frac{1}{2a_{\mathrm{H}}(\phi-1)+1}\left(\beta E_t\gamma_{t+1}-\gamma_t\right)+ \\
& -\frac{2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma}{2a_{\mathrm{H}}(\phi-1)+1}\left(\lambda_t-\beta^{-1}\lambda_{t-1}\right);
\end{aligned}
$$

where we have used the fact that

$$
\begin{aligned}
\frac{\partial\widehat{\mathcal{W}}_t}{\partial\widehat{Y}_{H,t}} &= -\frac{\partial\widehat{\mathcal{W}}_t}{\partial\widehat{Y}_{F,t}}=\frac{2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma}{2a_{\mathrm{H}}(\phi-1)+1} \\
\frac{\partial\widehat{\mathcal{T}}_t}{\partial\widehat{Y}_{H,t}} &= \frac{\sigma-(2a_{\mathrm{H}}-1)\dfrac{\partial\widehat{\mathcal{W}}_t}{\partial\widehat{Y}_{H,t}}}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1} \\
&= -\frac{\partial\widehat{\mathcal{T}}_t}{\partial\widehat{Y}_{F,t}}=\frac{1}{2a_{\mathrm{H}}(\phi-1)+1};
\end{aligned}
$$

The FOC with respect to LOOP deviations is given by:

$$
\begin{aligned}
\widehat{\Delta}_t \quad : \quad & 0 = -\frac{2a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\phi}{2a_{\mathrm{H}}\left(\phi-1\right)+1}\left(\widehat{\Delta}_t+\widehat{\mathcal{W}}_t\right)+ \\
& \frac{\left(1-\alpha\beta\right)\left(1-\alpha\right)}{\alpha}\frac{1}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\phi\sigma-1\right)+1}\cdot \\
& \frac{1}{2}\left[\begin{array}{c}\left(4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\phi\sigma-1\right)+1\right)\left(\gamma_{H,t}+\gamma_{F,t}-\left(\gamma^*_{F,t}+\gamma^*_{H,t}\right)\right)- \\
\left(\left(2a_{\mathrm{H}}-1\right)-2\left(1-a_{\mathrm{H}}\right)\left[2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1\right]\frac{2a_{\mathrm{H}}\left[2\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1-\phi\right]}{2a_{\mathrm{H}}\left(\phi-1\right)+1}\right)\cdot \\
\left(\gamma_{H,t}+\gamma^*_{H,t}-\gamma_{F,t}-\gamma^*_{F,t}\right)\end{array}\right]- \\
& \frac{2a_{\mathrm{H}}-1}{2a_{\mathrm{H}}\left(\phi-1\right)+1}\left(\beta E_t\gamma_{t+1}-\gamma_t\right)- \\
& \frac{2a_{\mathrm{H}}\left[2\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1-\phi\right]}{2a_{\mathrm{H}}\left(\phi-1\right)+1}\left(\lambda_t-\beta^{-1}\lambda_{t-1}\right),
\end{aligned}
$$

where we have used the fact that:

$$
\begin{aligned}
\frac{\partial\widehat{\mathcal{W}}_t}{\partial\widehat{\Delta}_t} &= \frac{2a_{\mathrm{H}}\left[2\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1-\phi\right]}{2a_{\mathrm{H}}\left(\phi-1\right)+1} \\
&= -1+\frac{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}{2a_{\mathrm{H}}\left(\phi-1\right)+1} \\
\frac{\partial\widehat{\mathcal{T}}_t}{\partial\widehat{\Delta}_t} &= -1-\left(2a_{\mathrm{H}}-1\right)\frac{1+\frac{\partial\widehat{\mathcal{W}}_t}{\partial\widehat{\Delta}_t}}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1} \\
&= -1-\frac{\left(2a_{\mathrm{H}}-1\right)}{2a_{\mathrm{H}}\left(\phi-1\right)+1}=-\frac{2a_{\mathrm{H}}\phi}{2a_{\mathrm{H}}\left(\phi-1\right)+1}
\end{aligned}
$$

Finally, the FOC with respect to net wealth is given by:

$$
\begin{aligned}
\widehat{\mathcal{B}}_t \quad : \quad & 0 = 2a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\phi\left[E_t\widehat{\mathcal{W}}_{t+1}-\widehat{\mathcal{W}}_t\right]+ \\
& -\left(1-a_{\mathrm{H}}\right)\left[2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1\right]\frac{\left(1-\alpha\beta\right)\left(1-\alpha\right)}{\alpha}\cdot \\
& \left[\left(E_t\left(\gamma_{H,t+1}+\gamma^*_{H,t+1}\right)-\left(\gamma_{H,t}+\gamma^*_{H,t}\right)\right)-\left(E_t\left(\gamma_{F,t+1}+\gamma^*_{F,t+1}\right)-\left(\gamma_{F,t}+\gamma^*_{F,t}\right)\right)\right]+ \\
& \left(2a_{\mathrm{H}}-1\right)\left[\beta E_t\gamma_{t+2}-E_t\gamma_{t+1}+\beta E_t\gamma_{t+1}-\gamma_t\right]+ \\
& \left[4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1\right]\left[\left(E_t\lambda_{t+1}-\beta^{-1}\lambda_t\right)-\left(\lambda_t-\beta^{-1}\lambda_{t-1}\right)\right],
\end{aligned}
$$

which simplifies as follows:

$$
\begin{aligned}
0 \quad = \quad & -\left(1-a_{\mathrm{H}}\right)\left[2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1\right]\frac{\left(1-\alpha\beta\right)\left(1-\alpha\right)}{\alpha}\cdot \\
& \left[\left(E_t\left(\gamma_{H,t+1}+\gamma^*_{H,t+1}\right)-\left(\gamma_{H,t}+\gamma^*_{H,t}\right)\right)-\left(E_t\left(\gamma_{F,t+1}+\gamma^*_{F,t+1}\right)-\left(\gamma_{F,t}+\gamma^*_{F,t}\right)\right)\right]+ \\
& \left(2a_{\mathrm{H}}-1\right)\left[\beta E_t\gamma_{t+2}-E_t\gamma_{t+1}+\beta E_t\gamma_{t+1}-\gamma_t\right]+ \\
& \left[4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1\right]\left[\left(E_t\lambda_{t+1}-\beta^{-1}\lambda_t\right)-\left(\lambda_t-\beta^{-1}\lambda_{t-1}\right)\right].
\end{aligned}
$$

Proof of Proposition 2: Sum rule under LCP. By summing the FOCs for inflation rates and output the solution can be expressed in terms of a familiar sum rule for (the change in) world output gaps and CPI inflation rates (where observe that we have switched to the gap notation, e.g. $\widetilde{Y}_{H,t} = \widehat{Y}_{H,t} - \widehat{Y}^{fb}_{H,t}$):

$$\begin{aligned} 0 &= \widetilde{Y}_{H,t} + \widetilde{Y}_{F,t} + \theta\left(a_{\mathrm{H}}\widehat{p}_{H,t} + (1-a_{\mathrm{H}})\,\widehat{p}_{F,t} + a_{\mathrm{H}}\widehat{p}^{*}_{F,t} + (1-a_{\mathrm{H}})\,\widehat{p}^{*}_{H,t}\right) \\ &= \left[\widetilde{Y}_{H,t} - \widetilde{Y}_{H,t-1}\right] + \left[\widetilde{Y}_{F,t} - \widetilde{Y}_{F,t-1}\right] + \\ &\quad \theta\left[a_{\mathrm{H}}\pi_{H,t} + (1-a_{\mathrm{H}})\,\pi_{F,t} + a_{\mathrm{H}}\pi^{*}_{F,t} + (1-a_{\mathrm{H}})\,\pi^{*}_{H,t}\right], \end{aligned}$$

the same as under complete markets.

Proof of Proposition 4: Difference rule under LCP. The difference rule is difficult to characterize analytically, but for the special case of $\eta = 0$. From the FOC for output solve for the term $\left(\beta E_t\gamma_{t+1} - \gamma_t\right)$:

$$\begin{aligned} \frac{1}{2a_{\mathrm{H}}(\phi-1)+1}\left(\beta E_t\gamma_{t+1} - \gamma_t\right) &= (\sigma+\eta)\left(\widehat{Y}_{H,t} - \widehat{Y}^{fb}_{H,t}\right) \\ &\quad -\frac{2a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)\sigma}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\left[\left(\widehat{Y}_{H,t} - \widehat{Y}^{fb}_{H,t}\right) - \left(\widehat{Y}_{F,t} - \widehat{Y}^{fb}_{F,t}\right)\right] + \\ &\quad \frac{2a_{\mathrm{H}}(1-a_{\mathrm{H}})\phi}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\,\frac{2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma}{2a_{\mathrm{H}}(\phi-1)+1}\left(\widehat{\Delta}_t + \widehat{\mathcal{W}}_t\right) + \\ &\quad -\left\{\begin{array}{c} (\sigma+\eta)\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\left(\gamma_{H,t}+\gamma^{*}_{H,t}\right) + \\ \frac{(1-a_{\mathrm{H}})(\sigma-1)}{2a_{\mathrm{H}}(\phi-1)+1}\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\left[\left(\gamma_{F,t}+\gamma^{*}_{F,t}\right) - \left(\gamma_{H,t}+\gamma^{*}_{H,t}\right)\right] \end{array}\right\} + \\ &\quad \frac{2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma}{2a_{\mathrm{H}}(\phi-1)+1}\left(\lambda_t - \beta^{-1}\lambda_{t-1}\right) \end{aligned}$$

$$\begin{aligned} \frac{1}{2a_{\mathrm{H}}(\phi-1)+1}\left(\beta E_t\gamma_{t+1} - \gamma_t\right) &= -(\sigma+\eta)\left(\widehat{Y}_{F,t} - \widehat{Y}^{fb}_{F,t}\right) - \\ &\quad \frac{2a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)\sigma}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\left[\left(\widehat{Y}_{H,t} - \widehat{Y}^{fb}_{H,t}\right) - \left(\widehat{Y}_{F,t} - \widehat{Y}^{fb}_{F,t}\right)\right] + \\ &\quad \frac{2a_{\mathrm{H}}(1-a_{\mathrm{H}})\phi}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\,\frac{2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma}{2a_{\mathrm{H}}(\phi-1)+1}\left(\widehat{\Delta}_t + \widehat{\mathcal{W}}_t\right) + \\ &\quad (\sigma+\eta)\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\left(\gamma_{F,t}+\gamma^{*}_{F,t}\right) + \\ &\quad \frac{(1-a_{\mathrm{H}})(\sigma-1)}{2a_{\mathrm{H}}(\phi-1)+1}\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\left[\left(\gamma_{H,t}+\gamma^{*}_{H,t}\right) - \left(\gamma_{F,t}+\gamma^{*}_{F,t}\right)\right] + \\ &\quad \frac{2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma}{2a_{\mathrm{H}}(\phi-1)+1}\left(\lambda_t - \beta^{-1}\lambda_{t-1}\right); \end{aligned}$$

Summing up we obtain:

$$
\begin{aligned}
\frac{2}{2a_{\mathrm{H}}(\phi-1)+1}\left(\beta E_t\gamma_{t+1}-\gamma_t\right) \;=\;& (\sigma+\eta)\left[\left(\widehat{Y}_{H,t}-\widehat{Y}^{fb}_{H,t}\right)-\left(\widehat{Y}_{F,t}-\widehat{Y}^{fb}_{F,t}\right)\right]- \\
& \frac{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)\sigma}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\left[\left(\widehat{Y}_{H,t}-\widehat{Y}^{fb}_{H,t}\right)-\left(\widehat{Y}_{F,t}-\widehat{Y}^{fb}_{F,t}\right)\right]+ \\
& \frac{4a_{\mathrm{H}}(1-a_{\mathrm{H}})\phi}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\frac{2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma}{2a_{\mathrm{H}}(\phi-1)+1}\left(\widehat{\Delta}_t+\widehat{\mathcal{W}}_t\right)- \\
& \left\{(\sigma+\eta)-\tfrac{2(1-a_{\mathrm{H}})(\sigma-1)}{2a_{\mathrm{H}}(\phi-1)+1}\right\}\tfrac{(1-\alpha\beta)(1-\alpha)}{\alpha}\left[\left(\gamma_{H,t}+\gamma^*_{H,t}\right)-\left(\gamma_{F,t}+\gamma^*_{F,t}\right)\right]+ \\
& 2\frac{2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma}{2a_{\mathrm{H}}(\phi-1)+1}\left(\lambda_t-\beta^{-1}\lambda_{t-1}\right)
\end{aligned}
$$

Consider now the FOC wrt LOOP:

$$
\begin{aligned}
\widehat{\Delta}_t \;:\; 0 =& -\frac{2a_{\mathrm{H}}(1-a_{\mathrm{H}})\phi}{2a_{\mathrm{H}}(\phi-1)+1}\left(\widehat{\Delta}_t+\widehat{\mathcal{W}}_t\right)+ \\
& \frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\frac{1}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\phi\sigma-1)+1}\cdot \\
& \frac{1}{2}\left[\begin{array}{c}(4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\phi\sigma-1)+1)\left(\gamma_{H,t}+\gamma_{F,t}-\left(\gamma^*_{F,t}+\gamma^*_{H,t}\right)\right)+ \\ -\left[(2a_{\mathrm{H}}-1)-2(1-a_{\mathrm{H}})\left[2a_{\mathrm{H}}(\sigma\phi-1)+1\right]\frac{2a_{\mathrm{H}}[2(1-a_{\mathrm{H}})(\sigma\phi-1)+1-\phi]}{2a_{\mathrm{H}}(\phi-1)+1}\right]\cdot \\ \left(\gamma_{H,t}+\gamma^*_{H,t}-\gamma_{F,t}-\gamma^*_{F,t}\right)\end{array}\right]+ \\
& -\frac{2a_{\mathrm{H}}-1}{2a_{\mathrm{H}}(\phi-1)+1}\left(\beta E_t\gamma_{t+1}-\gamma_t\right)+ \\
& -\frac{2a_{\mathrm{H}}\left[2(1-a_{\mathrm{H}})(\sigma\phi-1)+1-\phi\right]}{2a_{\mathrm{H}}(\phi-1)+1}\left(\lambda_t-\beta^{-1}\lambda_{t-1}\right);
\end{aligned}
$$

From the FOC for $\widehat{\mathcal{B}}_t$

$$
\begin{aligned}
0 \;=\;& -(1-a_{\mathrm{H}})\left[2a_{\mathrm{H}}(\sigma\phi-1)+1\right]\tfrac{(1-\alpha\beta)(1-\alpha)}{\alpha}\cdot \\
& \left[E_t\left(\gamma_{H,t+1}+\gamma^*_{H,t+1}\right)-\left(\gamma_{H,t}+\gamma^*_{H,t}\right)-\left(E_t\left(\gamma_{F,t+1}+\gamma^*_{F,t+1}\right)-\left(\gamma_{F,t}+\gamma^*_{F,t}\right)\right)\right]+ \\
& (2a_{\mathrm{H}}-1)\left[\left(\beta E_t\gamma_{t+2}-E_t\gamma_{t+1}\right)-\left(\beta E_t\gamma_{t+1}-\gamma_t\right)\right]+ \\
& \left[4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1\right]\left[\left(E_t\lambda_{t+1}-\beta^{-1}\lambda_t\right)-\left(\lambda_t-\beta^{-1}\lambda_{t-1}\right)\right],
\end{aligned}
$$

we get the following solution for $\left(\lambda_t-\beta^{-1}\lambda_{t-1}\right)$:

$$
\begin{aligned}
\left[4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1\right]\left(\lambda_t-\beta^{-1}\lambda_{t-1}\right) \;=\;& -(2a_{\mathrm{H}}-1)\left(\beta E_t\gamma_{t+1}-\gamma_t\right) \\
& +(1-a_{\mathrm{H}})\left[2a_{\mathrm{H}}(\sigma\phi-1)+1\right]\tfrac{(1-\alpha\beta)(1-\alpha)}{\alpha}\cdot \\
& \left[\left(\gamma_{H,t}+\gamma^*_{H,t}\right)-\left(\gamma_{F,t}+\gamma^*_{F,t}\right)\right]
\end{aligned}
$$

where we have also used the fact that:

$$
\frac{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}{2a_{\mathrm{H}}(\phi-1)+1}+(2a_{\mathrm{H}}-1)\frac{2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma}{2a_{\mathrm{H}}(\phi-1)+1}=\sigma.
$$

Thus we can write

$$
\begin{aligned}
2\sigma\left(\beta E_t\gamma_{t+1}-\gamma_t\right) &= \left[\sigma+\eta\left[4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1\right]\right]\left[\left(\widehat{Y}_{H,t}-\widehat{Y}^{fb}_{H,t}\right)-\left(\widehat{Y}_{F,t}-\widehat{Y}^{fb}_{F,t}\right)\right]+\\
&\quad 4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\phi\frac{2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1-\sigma}{2a_{\mathrm{H}}\left(\phi-1\right)+1}\left(\widehat{\Delta}_t+\widehat{\mathcal{W}}_t\right)+\\
&\quad -\left(\sigma+\eta\left[4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1\right]\right)\frac{\left(1-\alpha\beta\right)\left(1-\alpha\right)}{\alpha}\\
&\quad \left[\left(\gamma_{H,t}+\gamma^*_{H,t}\right)-\left(\gamma_{F,t}+\gamma^*_{F,t}\right)\right]
\end{aligned}
$$

Set $\eta=0$ and solve for $\left(\gamma_{H,t}+\gamma^*_{H,t}-\gamma_{F,t}-\gamma^*_{F,t}\right)$

$$
\begin{aligned}
2\sigma\left(\beta E_t\gamma_{t+1}-\gamma_t\right) &= \sigma\left[\left(\widehat{Y}_{H,t}-\widehat{Y}^{fb}_{H,t}\right)-\left(\widehat{Y}_{F,t}-\widehat{Y}^{fb}_{F,t}\right)\right]+\\
&\quad 4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\phi\frac{2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1-\sigma}{2a_{\mathrm{H}}\left(\phi-1\right)+1}\left(\widehat{\Delta}_t+\widehat{\mathcal{W}}_t\right)+\\
&\quad -\sigma\frac{\left(1-\alpha\beta\right)\left(1-\alpha\right)}{\alpha}\left[\left(\gamma_{H,t}+\gamma^*_{H,t}\right)-\left(\gamma_{F,t}+\gamma^*_{F,t}\right)\right]
\end{aligned}
$$

also using the FOC for $\widehat{\Delta}_t$ after substituting out for $\left(\lambda_t-\beta^{-1}\lambda_{t-1}\right)$:

$$
\begin{aligned}
&2\frac{2a_{\mathrm{H}}-1}{\left[4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1\right]}\left(\beta E_t\gamma_{t+1}-\gamma_t\right)=-\frac{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\phi}{2a_{\mathrm{H}}\left(\phi-1\right)+1}\left(\widehat{\Delta}_t+\widehat{\mathcal{W}}_t\right)+\\
&\quad\frac{\left(1-\alpha\beta\right)\left(1-\alpha\right)}{\alpha}\left(\gamma_{H,t}+\gamma_{F,t}-\left(\gamma^*_{F,t}+\gamma^*_{H,t}\right)\right)-\left(2a_{\mathrm{H}}-1\right)\frac{\left(1-\alpha\beta\right)\left(1-\alpha\right)}{\alpha}\\
&\quad\frac{\left(\gamma_{H,t}+\gamma^*_{H,t}-\gamma_{F,t}-\gamma^*_{F,t}\right)}{\left[4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1\right]}.
\end{aligned}
$$

The following equality holds:

$$
\begin{aligned}
&\frac{2a_{\mathrm{H}}-1}{\left[4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1\right]}\sigma\left[\left(\widehat{Y}_{H,t}-\widehat{Y}^{fb}_{H,t}\right)-\left(\widehat{Y}_{F,t}-\widehat{Y}^{fb}_{F,t}\right)\right]+\\
&\frac{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\phi}{2a_{\mathrm{H}}\left(\phi-1\right)+1}\left[\left(2a_{\mathrm{H}}-1\right)\frac{2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1-\sigma}{\left[4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1\right]}+\sigma\right]\left(\widehat{\Delta}_t+\widehat{\mathcal{W}}_t\right)=\\
&\sigma\frac{\left(1-\alpha\beta\right)\left(1-\alpha\right)}{\alpha}\left(\gamma_{H,t}+\gamma_{F,t}-\left(\gamma^*_{F,t}+\gamma^*_{H,t}\right)\right),
\end{aligned}
$$

which further simplifies as follows:

$$
\begin{aligned}
&\frac{2a_{\mathrm{H}}-1}{\left[4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1\right]}\sigma\left[\left(\widehat{Y}_{H,t}-\widehat{Y}^{fb}_{H,t}\right)-\left(\widehat{Y}_{F,t}-\widehat{Y}^{fb}_{F,t}\right)\right]+\\
&\left[\left(2a_{\mathrm{H}}-1\right)\frac{2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1-\sigma}{\left[4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1\right]}+\sigma\right]\frac{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\phi}{2a_{\mathrm{H}}\left(\phi-1\right)+1}\left(\widehat{\Delta}_t+\widehat{\mathcal{W}}_t\right)\\
=\;&\frac{\left(1-\alpha\beta\right)\left(1-\alpha\right)}{\alpha}\sigma\left(\gamma_{H,t}+\gamma_{F,t}-\left(\gamma^*_{F,t}+\gamma^*_{H,t}\right)\right).
\end{aligned}
$$

In turn we can rewrite the left hand side of the above expression as follows:

$$\begin{aligned}
&(2a_{\mathrm{H}}-1)\left[\widehat{\mathcal{T}}_t-\widehat{\mathcal{T}}_t^{fb}+\widehat{\Delta}_t+\frac{(2a_{\mathrm{H}}-1)\left(\widehat{\mathcal{W}}_t+\widehat{\Delta}_t\right)}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\right]+\\
&\left[(2a_{\mathrm{H}}-1)\frac{2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma}{[4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1]}+\sigma\right]\frac{4a_{\mathrm{H}}(1-a_{\mathrm{H}})\phi}{2a_{\mathrm{H}}(\phi-1)+1}\left(\widehat{\Delta}_t+\widehat{\mathcal{W}}_t\right)\\
=\;&\widehat{\mathcal{Q}}_t-\widehat{\mathcal{Q}}_t^{fb}-\widehat{\Delta}_t+\left(\widehat{\mathcal{W}}_t+\widehat{\Delta}_t\right)+\frac{(2a_{\mathrm{H}}-1)^2-1-4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\left(\widehat{\mathcal{W}}_t+\widehat{\Delta}_t\right)+\\
&\left[(2a_{\mathrm{H}}-1)\frac{2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma}{[4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1]}+\sigma\right]\frac{4a_{\mathrm{H}}(1-a_{\mathrm{H}})\phi}{2a_{\mathrm{H}}(\phi-1)+1}\left(\widehat{\Delta}_t+\widehat{\mathcal{W}}_t\right)\\
=\;&\widehat{\mathcal{Q}}_t-\widehat{\mathcal{Q}}_t^{fb}-\widehat{\Delta}_t+\left(\widehat{\mathcal{W}}_t+\widehat{\Delta}_t\right)+\frac{-4a_{\mathrm{H}}(1-a_{\mathrm{H}})\sigma\phi}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\left(\widehat{\mathcal{W}}_t+\widehat{\Delta}_t\right)+\\
&\left[(2a_{\mathrm{H}}-1)\frac{2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma}{[4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1]}+\sigma\right]\frac{4a_{\mathrm{H}}(1-a_{\mathrm{H}})\phi}{2a_{\mathrm{H}}(\phi-1)+1}\left(\widehat{\Delta}_t+\widehat{\mathcal{W}}_t\right)\\
=\;&\widehat{\mathcal{Q}}_t-\widehat{\mathcal{Q}}_t^{fb}-\widehat{\Delta}_t+\left(\widehat{\mathcal{W}}_t+\widehat{\Delta}_t\right)+\frac{4a_{\mathrm{H}}(1-a_{\mathrm{H}})\phi(\sigma-1)}{2a_{\mathrm{H}}(\phi-1)+1}\left(\widehat{\mathcal{W}}_t+\widehat{\Delta}_t\right).
\end{aligned}$$

Finally, using the FOC for inflation to substitute out $\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\left(\gamma_{H,t}+\gamma_{F,t}-\left(\gamma^*_{F,t}+\gamma^*_{H,t}\right)\right)$, we arrive at the following expression for the optimal difference rule in levels:

$$\begin{aligned}
0=\;&\sigma\theta\left[\left(a_{\mathrm{H}}\widehat{p}_{H,t}+(1-a_{\mathrm{H}})\widehat{p}_{F,t}\right)-\left(a_{\mathrm{H}}\widehat{p}^*_{F,t}+(1-a_{\mathrm{H}})\widehat{p}^*_{H,t}\right)\right]+\\
&\widehat{\mathcal{Q}}_t-\widehat{\mathcal{Q}}_t^{fb}+\widehat{\mathcal{W}}_t+\frac{4a_{\mathrm{H}}(1-a_{\mathrm{H}})\phi}{2a_{\mathrm{H}}(\phi-1)+1}(\sigma-1)\left(\widehat{\Delta}_t+\widehat{\mathcal{W}}_t\right),
\end{aligned}$$

which is straightforward to write in terms of inflation and growth rates of the other variables as in Proposition 4:

$$\begin{aligned}
0=\;&\theta\left[\left(a_{\mathrm{H}}\pi_{H,t}+(1-a_{\mathrm{H}})\pi_{F,t}\right)-\left(a_{\mathrm{H}}\pi^*_{H,t}+(1-a_{\mathrm{H}})\pi^*_{H,t}\right)\right]+\\
&\left[\left(\widehat{C}_t-\widehat{C}^*_t\right)-\left(\widehat{C}^{fb}_t-\widehat{C}^{*fb}_t\right)\right]-\left[\left(\widehat{C}_{t-1}-\widehat{C}^*_{t-1}\right)-\left(\widehat{C}^{fb}_{t-1}-\widehat{C}^{*fb}_{t-1}\right)\right]+\\
&\frac{4a_{\mathrm{H}}(1-a_{\mathrm{H}})\phi}{2a_{\mathrm{H}}(\phi-1)+1}\frac{(\sigma-1)}{\sigma}\left(\widehat{\Delta}_t-\widehat{\Delta}_{t-1}+\widehat{\mathcal{W}}_t-\widehat{\mathcal{W}}_{t-1}\right).
\end{aligned}$$

An alternative way of expressing the targeting criterion. The targeting criterion could also be expressed as a combination of the CPI-inflation and consumption differentials:

$$\begin{aligned}
0=\;&\theta\left[\left(a_{\mathrm{H}}\pi_{H,t}+(1-a_{\mathrm{H}})\pi_{F,t}\right)-\left(a_{\mathrm{H}}\pi^*_{F,t}+(1-a_{\mathrm{H}})\pi^*_{H,t}\right)\right]+\\
&\left[\widehat{\mathcal{W}}_t-\widehat{\mathcal{W}}_{t-1}\right]+\left[\left(\widehat{\mathcal{Q}}_t-\widehat{\mathcal{Q}}^{fb}_t\right)-\left(\widehat{\mathcal{Q}}_{t-1}-\widehat{\mathcal{Q}}^{fb}_{t-1}\right)\right]\\
0=\;&\theta\left[\left(a_{\mathrm{H}}\pi_{H,t}+(1-a_{\mathrm{H}})\pi_{F,t}\right)-\left(a_{\mathrm{H}}\pi^*_{H,t}+(1-a_{\mathrm{H}})\pi^*_{H,t}\right)\right]+\\
&\left[\left(\widehat{C}_t-\widehat{C}^*_t\right)-\left(\widehat{C}^{fb}_t-\widehat{C}^{*fb}_t\right)\right]-\left[\left(\widehat{C}_{t-1}-\widehat{C}^*_{t-1}\right)-\left(\widehat{C}^{fb}_{t-1}-\widehat{C}^{*fb}_{t-1}\right)\right].
\end{aligned}$$

Taking again the difference in CPI inflation using the NKPC:

$$a_{\rm H}\pi_{H,t} + (1-a_{\rm H})\,\pi_{F,t} - \left(a_{\rm H}\pi^*_{F,t} + (1-a_{\rm H})\,\pi^*_{H,t}\right) -$$
$$\beta E_t\left(a_{\rm H}\pi_{H,t+1} + (1-a_{\rm H})\,\pi_{F,t+1}\right) - \beta E_t\left(a_{\rm H}\pi^*_{F,t+1} + (1-a_{\rm H})\,\pi^*_{H,t+1}\right) =$$
$$= \frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\left\{(2a_{\rm H}-1)\left[\begin{array}{c}\left(\widehat{Y}_{H,t}-\widehat{Y}^{fb}_{H,t}\right)-\left(\widehat{Y}_{F,t}-\widehat{Y}^{fb}_{F,t}\right)- \\ (2a_{\rm H}-1)\left(\widehat{\mathcal{D}}_t+\widehat{\Delta}_t\right)+\widehat{\mu}_t-\widehat{\mu}^*_t+ \\ -4a_{\rm H}(1-a_{\rm H})(\phi-1)\left(\widehat{\mathcal{T}}_t-\widehat{\mathcal{T}}^{fb}_t+\widehat{\Delta}_t\right) \\ +\left(\widehat{\Delta}_t+\widehat{\mathcal{D}}_t\right)\end{array}\right]+2(1-a_{\rm H})\,\widehat{\Delta}_t\right\}$$
$$= \frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\left\{\widehat{\mathcal{Q}_t}-\widehat{\mathcal{Q}}^{fb}_t+(2a_{\rm H}-1)\left[\widehat{\mu}_t-\widehat{\mu}^*_t+\widehat{\mathcal{D}}_t\right]\right\},$$

where we have used the following relation:

$$\begin{aligned}\widehat{\mathcal{Q}_t}-\widehat{\mathcal{Q}}^{fb}_t &= (2a_{\rm H}-1)\left(\widehat{\mathcal{T}}_t-\widehat{\mathcal{T}}^{fb}_t+\widehat{\Delta}_t\right)+\widehat{\Delta}_t \\ &= (2a_{\rm H}-1)\,\frac{\left[\left(\widehat{Y}_{H,t}-\widehat{Y}_{F,t}\right)-\left(\widehat{Y}^{fb}_{H,t}-\widehat{Y}^{fb}_{F,t}\right)\right]-(2a_{\rm H}-1)\left(\widehat{\mathcal{D}}_t+\widehat{\Delta}_t\right)}{4a_{\rm H}(1-a_{\rm H})(\phi-1)+1}+\widehat{\Delta}_t.\end{aligned}$$

In contrast to a complete markets (CM) economy, a policy that sets CPI inflation rates to zero in response to efficient shocks is not optimal.

Finally, notice that we can also write the CPI inflation differential as a function of consumption differentials:

$$a_{\rm H}\pi_{H,t} + (1-a_{\rm H})\,\pi_{F,t} - \left(a_{\rm H}\pi^*_{F,t} + (1-a_{\rm H})\,\pi^*_{H,t}\right) -$$
$$\beta E_t\left(a_{\rm H}\pi_{H,t+1} + (1-a_{\rm H})\,\pi_{F,t+1}\right) - \beta E_t\left(a_{\rm H}\pi^*_{F,t+1} + (1-a_{\rm H})\,\pi^*_{H,t+1}\right)$$
$$= \frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\left\{\left[\left(\widehat{C}_t-\widehat{C}^*_t\right)-\left(\widehat{C}^{fb}_t-\widehat{C}^{*fb}_t\right)\right]-2(1-a_{\rm H})\,\widehat{\mathcal{W}}_t+(2a_{\rm H}-1)\left[\widehat{\mu}_t-\widehat{\mu}^*_t\right]\right\}$$

2.1.2 Proof of Proposition 5

We start by first proving Proposition 5 in the text, namely that $\widehat{\mathcal{T}}_t-\widehat{\mathcal{T}}^{fb}_t+\widehat{\Delta}_t$, $\widehat{\mathcal{W}}_t$ and $\widehat{\mathcal{B}}_t$ are independent of monetary policy under the maintained parametric assumptions $\sigma=1$ and $\eta=0$. Next, we proceed to solve for the optimal allocations.

We can solve for net foreign assets $\widehat{\mathcal{B}}_t$ and (the permanent shift in) $\widehat{\mathcal{W}}_t$ by using the budget constraint:

$$\begin{aligned}\widehat{\mathcal{W}}_t &= \widehat{\mathcal{W}}_t-\widehat{\mathcal{W}}^{fb}_t = \\ &= \sigma\left[\left(\widehat{C}_t-\widehat{C}^*_t\right)-\left(\widehat{C}^{fb}_t-\widehat{C}^{*fb}_t\right)\right]-\left(\widehat{\mathcal{Q}}_t-\widehat{\mathcal{Q}}^{fb}_t\right) \\ &= \sigma\left[\left(\widehat{Y}_{H,t}-\widehat{Y}^{fb}_{H,t}\right)-\left(\widehat{Y}_{F,t}-\widehat{Y}^{fb}_{F,t}\right)+2\beta^{-1}\left(\widehat{\mathcal{B}}_{t-1}-\beta\widehat{\mathcal{B}}_t\right)\right]+ \\ &\quad -\left[2(1-a_{\rm H})\sigma+(2a_{\rm H}-1)\right]\left(\widehat{\mathcal{T}}_t-\widehat{\mathcal{T}}^{fb}_t\right)-2a_{\rm H}\widehat{\Delta}_t+ \\ &\quad 2(1-a_{\rm H})\left[(2a_{\rm H}(\sigma\phi-1)+1-\sigma)\,\widehat{\mathcal{T}}^{fb}_t-\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}^*_{C,t}\right)\right],\end{aligned}$$

where we used the fact that

$$\left(\widehat{\mathcal{Q}}_t - \widehat{\mathcal{Q}}_t^{fb}\right) = 2a_{\mathrm{H}}\left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}_t^{fb} + \widehat{\Delta}_t\right) - \left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}_t^{fb}\right),$$

and the link between the output gap and relative prices:

$$\sigma\left[\left(\widehat{Y}_{H,t} - \widehat{Y}_{H,t}^{fb}\right) - \left(\widehat{Y}_{F,t} - \widehat{Y}_{F,t}^{fb}\right)\right] = \left[4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)(\sigma\phi - 1) + 1\right]\left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}_t^{fb} + \widehat{\Delta}_t\right) + (2a_{\mathrm{H}} - 1)\left(\widehat{\mathcal{W}}_t + \widehat{\Delta}_t\right)$$

we obtain the following simplification:

$$\begin{aligned}(1-a_{\mathrm{H}})\,\widehat{\mathcal{W}}_t \;=\;& \sigma\beta^{-1}\left(\widehat{\mathcal{B}}_{t-1} - \beta\widehat{\mathcal{B}}_t\right) + (1-a_{\mathrm{H}})(\sigma-1)\,\widehat{\Delta}_t + \\ & (1-a_{\mathrm{H}})\left[2a_{\mathrm{H}}(\sigma\phi-1) - (\sigma-1)\right]\left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}_t^{fb} + \widehat{\Delta}_t\right) + \\ & (1-a_{\mathrm{H}})\left[(2a_{\mathrm{H}}(\sigma\phi-1) + 1 - \sigma)\,\widehat{\mathcal{T}}_t^{fb} - \left(\widehat{\zeta}_{C,t} - \widehat{\zeta}_{C,t}^{*}\right)\right];\end{aligned}$$

when $\sigma = 1$ the expression becomes:

$$\begin{aligned}(1-a_{\mathrm{H}})\,\widehat{\mathcal{W}}_t \;=\;& \beta^{-1}\left(\widehat{\mathcal{B}}_{t-1} - \beta\widehat{\mathcal{B}}_t\right) + \\ & 2a_{\mathrm{H}}(1-a_{\mathrm{H}})(\phi-1)\left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}_t^{fb} + \widehat{\Delta}_t\right) + \\ & (1-a_{\mathrm{H}})\left[2a_{\mathrm{H}}(\phi-1)\,\widehat{\mathcal{T}}_t^{fb} - \left(\widehat{\zeta}_{C,t} - \widehat{\zeta}_{C,t}^{*}\right)\right].\end{aligned}$$

Using the consumption Euler equation we get the following difference equation for NFAs:

$$\begin{aligned}&\beta^{-1}\left[E_t\left(\beta\widehat{\mathcal{B}}_{t+1} - \widehat{\mathcal{B}}_t\right) - \left(\beta\widehat{\mathcal{B}}_t - \widehat{\mathcal{B}}_{t-1}\right)\right] = \\ &2a_{\mathrm{H}}(1-a_{\mathrm{H}})(\phi-1)\,E_t\left(\left(\widehat{\mathcal{T}}_{t+1} - \widehat{\mathcal{T}}_{t+1}^{fb} + \widehat{\Delta}_{t+1}\right) - \left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}_t^{fb} + \widehat{\Delta}_t\right)\right) + \\ &(1-a_{\mathrm{H}})\left[2a_{\mathrm{H}}(\phi-1)\,E_t\left(\widehat{\mathcal{T}}_{t+1}^{fb} - \widehat{\mathcal{T}}_t^{fb}\right) - E_t\left(\left(\widehat{\zeta}_{C,t+1} - \widehat{\zeta}_{C,t+1}^{*}\right) - \left(\widehat{\zeta}_{C,t} - \widehat{\zeta}_{C,t}^{*}\right)\right)\right].\end{aligned}$$

In order to solve it, observe first that we can solve for the expression for $\left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}_t^{fb} + \widehat{\Delta}_t\right)$ by using the relation

$$\pi_{F,t} - \pi_{H,t} = \left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}_{t-1} + \widehat{\Delta}_t - \widehat{\Delta}_{t-1}\right),$$

and taking the difference between the NKPC for $\pi_{F,t} - \pi_{H,t}$ with $\sigma = 1$ and $\eta = 0$ to get the following difference equation:

$$\begin{aligned}&\pi_{F,t} - \pi_{H,t} - \beta E_t\left(\pi_{F,t+1} - \pi_{H,t+1}\right) = \\ &\left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}_{t-1} + \widehat{\Delta}_t - \widehat{\Delta}_{t-1}\right) - \beta E_t\left(\widehat{\mathcal{T}}_{t+1} - \widehat{\mathcal{T}}_t + \widehat{\Delta}_{t+1} - \widehat{\Delta}_t\right) = \\ &-\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\left[\begin{array}{c}\left(\widehat{Y}_{H,t} - \widehat{Y}_{H,t}^{fb}\right) - \left(\widehat{Y}_{F,t} - \widehat{Y}_{F,t}^{fb}\right) - \widehat{\Delta}_t + \\ -2(1-a_{\mathrm{H}})\left[2a_{\mathrm{H}}(\phi-1)\left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}_t^{fb} + \widehat{\Delta}_t\right) - \left(\widehat{\Delta}_t + \widehat{\mathcal{W}}_t\right)\right]\end{array}\right].\end{aligned}$$

Using again the equilibrium relation between the output gap and relative prices also when $\sigma = 1$ and $\eta = 0$:

$$\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}_t^{fb} + \widehat{\Delta}_t = \frac{\left[\left(\widehat{Y}_{H,t} - \widehat{Y}_{H,t}^{fb}\right) - \left(\widehat{Y}_{F,t} - \widehat{Y}_{F,t}^{fb}\right)\right] - (2a_{\mathrm{H}} - 1)\left(\widehat{\mathcal{W}}_t + \widehat{\Delta}_t\right)}{4a_{\mathrm{H}}(1 - a_{\mathrm{H}})(\phi - 1) + 1},$$

we can simplify the above difference equation as follows:

$$\begin{gathered}\beta E_t\left[\left(\widehat{\mathcal{T}}_{t+1} - \widehat{\mathcal{T}}_{t+1}^{fb} + \widehat{\Delta}_{t+1}\right) - \left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}_t^{fb} + \widehat{\Delta}_t\right)\right] - \\ \left[\left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}_t^{fb} + \widehat{\Delta}_t\right) - \left(\widehat{\mathcal{T}}_{t-1} - \widehat{\mathcal{T}}_{t-1}^{fb} + \widehat{\Delta}_{t-1}\right)\right] - \\ \frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}_t^{fb} + \widehat{\Delta}_t\right) = \\ \frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\widehat{\mathcal{W}}_t - E_t\left[\beta\left(\widehat{\mathcal{T}}_{t+1}^{fb} - \widehat{\mathcal{T}}_t^{fb}\right) - \left(\widehat{\mathcal{T}}_t^{fb} - \widehat{\mathcal{T}}_{t-1}^{fb}\right)\right].\end{gathered}$$

We solve this difference equation for $\left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}_t^{fb} + \widehat{\Delta}_t\right)$:

$$\begin{aligned}\left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}_t^{fb} + \widehat{\Delta}_t\right) &= \nu_1\left(\widehat{\mathcal{T}}_{t-1} - \widehat{\mathcal{T}}_{t-1}^{fb} + \widehat{\Delta}_{t-1}\right) - \frac{(1-\alpha\beta)(1-\alpha)}{\alpha\beta}\sum_{j=0}^{\infty}\nu_2^{-j-1}\widehat{\mathcal{W}}_t + \\ &\quad \sum_{j=0}^{\infty}\nu_2^{-j-1}E_t\left[\left(\widehat{\mathcal{T}}_{t+j+1}^{fb} - \widehat{\mathcal{T}}_{t+j}^{fb}\right) - \beta^{-1}\left(\widehat{\mathcal{T}}_{t+j}^{fb} - \widehat{\mathcal{T}}_{t+j-1}^{fb}\right)\right],\end{aligned}$$

where $0 < \nu_1 < 1 < \beta^{-1} < \nu_2$ are the eigenvalues of the difference equation, solving the standard characteristic equation:

$$\beta\nu^2 - \left[1 + \beta + \frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\right]\nu + 1 = 0,$$

namely

$$\nu = \frac{1 + \beta + \frac{(1-\alpha\beta)(1-\alpha)}{\alpha} \pm \sqrt{\left[1 + \beta + \frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\right]^2 - 4\beta}}{2\beta}.$$

We simplify further using the fact that $\widehat{\mathcal{W}}_t$ is a martingale:

$$\begin{gathered}\left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}_t^{fb} + \widehat{\Delta}_t\right) = \nu_1\left(\widehat{\mathcal{T}}_{t-1} - \widehat{\mathcal{T}}_{t-1}^{fb} + \widehat{\Delta}_{t-1}\right) - \frac{(\beta\nu_2 - 1)}{\beta\nu_2}\widehat{\mathcal{W}}_t + \\ \sum_{s=0}^{\infty}\nu_2^{-s-1}E_t\left[\left(\widehat{\mathcal{T}}_{t+j+1}^{fb} - \widehat{\mathcal{T}}_{t+j}^{fb}\right) - \beta^{-1}\left(\widehat{\mathcal{T}}_{t+j}^{fb} - \widehat{\mathcal{T}}_{t+j-1}^{fb}\right)\right],\end{gathered}$$

where we have also used the fact that: $\frac{(\beta\nu_2 - 1)}{\beta\nu_2} = \frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\frac{1}{\beta(\nu_2 - 1)}$.

Observe that we have only used equilibrium relations that are independent of monetary policy. Therefore, the three variables $\left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}_t^{fb} + \widehat{\Delta}_t\right)$, $\widehat{\mathcal{B}}_t$ and $\widehat{\mathcal{W}}_t$

are all related and can be solved independently of monetary policy as a function of exogenous shocks only.

To complete the proof of Proposition 5 we thus need to show that net foreign assets $\widehat{\mathcal{B}}_t$ do not depend on monetary policy. This is straightforward, as by using the consumption Euler equation and substituting out the solution for the terms involving $\left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}_t^{fb} + \widehat{\Delta}_t\right)$, namely

$$E_t\left(\left(\widehat{\mathcal{T}}_{t+1} - \widehat{\mathcal{T}}_{t+1}^{fb} + \widehat{\Delta}_{t+1}\right) - \left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}_t^{fb} + \widehat{\Delta}_t\right)\right) =$$
$$-\left(1-\nu_1\right)\left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}_t^{fb} + \widehat{\Delta}_t\right) +$$
$$-\frac{(\beta\nu_2 - 1)}{\beta\nu_2}\widehat{\mathcal{W}}_t + \sum_{s=0}^{\infty}\nu_2^{-s-1}E_t\left[\left(\widehat{\mathcal{T}}_{t+s+2}^{fb} - \widehat{\mathcal{T}}_{t+s+1}^{fb}\right) - \beta^{-1}\left(\widehat{\mathcal{T}}_{t+s+1}^{fb} - \widehat{\mathcal{T}}_{t+s}^{fb}\right)\right],$$

we get the following difference equation for $\widehat{\mathcal{B}}_t$ that we can solve explicitly for NFAs independently of monetary policy:

$$\widehat{\mathcal{B}}_t - \widehat{\mathcal{B}}_{t-1} = -2a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\phi-1\right)\cdot$$
$$\left[\beta\sum_{j=0}^{\infty}\beta^j E_t\left[\left(\widehat{\mathcal{T}}_{t+j+1} - \widehat{\mathcal{T}}_{t+j+1}^{fb} + \widehat{\Delta}_{t+j+1}\right) - \left(\widehat{\mathcal{T}}_{t+j} - \widehat{\mathcal{T}}_{t+j}^{fb} + \widehat{\Delta}_{t+j}\right)\right]\right] -$$
$$\left(1-a_{\mathrm{H}}\right)\left[2a_{\mathrm{H}}\left(\phi-1\right)\right]\beta\sum_{j=0}^{\infty}\beta^j E_t\left[\left(\widehat{\mathcal{T}}_{t+j+1}^{fb} - \widehat{\mathcal{T}}_{t+j}^{fb}\right)\right] +$$
$$\left(1-a_{\mathrm{H}}\right)\beta\sum_{j=0}^{\infty}\beta^j E_t\left[\left(\widehat{\zeta}_{C,t+j+1} - \widehat{\zeta}_{C,t+j+1}^{*}\right) - \left(\widehat{\zeta}_{C,t+j} - \widehat{\zeta}_{C,t+j}^{*}\right)\right].$$

We can further simplify the latter expression using the above solutions for relative price misalignments; namely we have that for $j \geq 0$:

$$E_t\left(\widehat{\mathcal{T}}_{t+j} - \widehat{\mathcal{T}}_{t+j}^{fb} + \widehat{\Delta}_{t+j}\right) = \nu_1^{j+1}\left(\widehat{\mathcal{T}}_{t-1} - \widehat{\mathcal{T}}_{t-1}^{fb} + \widehat{\Delta}_{t-1}\right) - \frac{1-\nu_1^{j+1}}{1-\nu_1}\frac{(\beta\nu_2-1)}{\beta\nu_2}\widehat{\mathcal{W}}_t$$
$$+\sum_{s=0}^{j}\nu_1^{j-s}\quad\sum_{h=0}^{\infty}\nu_2^{-h-1}E_t\left[\left(\widehat{\mathcal{T}}_{t+h+s+1}^{fb} - \widehat{\mathcal{T}}_{t+h+s}^{fb}\right) - \beta^{-1}\left(\widehat{\mathcal{T}}_{t+h+s}^{fb} - \widehat{\mathcal{T}}_{t+h+s-1}^{fb}\right)\right]\Bigg),$$

Putting the above together we can find the following solution for NFAs only as a function of exogenous shocks and $\widehat{\mathcal{W}}_t$, which is also independent of monetary policy:

$$\widehat{\mathcal{B}}_t - \widehat{\mathcal{B}}_{t-1} = 2a_{\rm H}\left(1-a_{\rm H}\right)\left(\phi-1\right)\left(1-\nu_1\right)\cdot$$
$$\beta \textstyle\sum_{j=0}^{\infty} \beta^j \nu_1^{j+1} \left(\widehat{\mathcal{T}}_{t-1} - \widehat{\mathcal{T}}_{t-1}^{fb} + \widehat{\Delta}_{t-1}\right) +$$
$$\beta \textstyle\sum_{j=0}^{\infty} \beta^j \left\{ \sum_{s=0}^{j} \nu_1^{j-s} \left(\sum_{h=0}^{\infty} \nu_2^{-h-1} E_t \left[\begin{array}{c} \left(\widehat{\mathcal{T}}_{t+h+s+1}^{fb} - \widehat{\mathcal{T}}_{t+h+s}^{fb}\right) \\ -\beta^{-1}\left(\widehat{\mathcal{T}}_{t+h+s}^{fb} - \widehat{\mathcal{T}}_{t+h+s-1}^{fb}\right) \end{array} \right] \right) \right\}$$
$$+2a_{\rm H}\left(1-a_{\rm H}\right)\left(\phi-1\right)\frac{\left(\beta\nu_2-1\right)\nu_1}{\nu_2\left(1-\beta\nu_1\right)}\widehat{\mathcal{W}}_t$$
$$-2a_{\rm H}\left(1-a_{\rm H}\right)\left(\phi-1\right)\cdot$$
$$\sum_{j=0}^{\infty}\beta^j\left[\sum_{s=0}^{\infty}\nu_2^{-s-1}E_t\left[\left(\widehat{\mathcal{T}}_{t+j+s+2}^{fb}-\widehat{\mathcal{T}}_{t+j+s+1}^{fb}\right)-\beta^{-1}\left(\widehat{\mathcal{T}}_{t+j+s+1}^{fb}-\widehat{\mathcal{T}}_{t+j+s}^{fb}\right)\right]\right]$$
$$-\left(1-a_{\rm H}\right)\left[2a_{\rm H}\left(\phi-1\right)\right]\beta\sum_{j=0}^{\infty}\beta^j E_t\left[\left(\widehat{\mathcal{T}}_{t+j+1}^{fb}-\widehat{\mathcal{T}}_{t+j}^{fb}\right)\right]$$
$$+\left(1-a_{\rm H}\right)\beta\sum_{j=0}^{\infty}\beta^j E_t\left[\left(\widehat{\zeta}_{C,t+j+1}-\widehat{\zeta}_{C,t+j+1}^{*}\right)-\left(\widehat{\zeta}_{C,t+j}-\widehat{\zeta}_{C,t+j}^{*}\right)\right].$$

This completes the proof of Proposition 5.

2.1.3 Elasticity thresholds in Section 5.3

Here we derive the thresholds under LCP discussed in Section 5.3, showing that indeed they are always below equation (36) in the text. Differently from the case of the natural allocation, it turns out that, under LCP, the threshold value of the trade elasticity above which $\widehat{\mathcal{B}}_t$ and $\widetilde{\mathcal{W}}_t$ have the opposite sign is conditional on which shocks hit the economy. Conditional on anticipated taste shocks, $\widehat{\mathcal{B}}_t$ and $\widetilde{\mathcal{W}}_t$ have the opposite sign when ϕ is above the following threshold:

$$\phi > \frac{2a_{\rm H} - \frac{\beta\nu_2}{(\beta\nu_2-1)}}{2a_{\rm H}},$$

which is a function of openness and nominal rigidities and is always bounded above by (36) in the main text. For anticipated productivity shocks, the equilibrium link between $\widehat{\mathcal{B}}_t$ and $\widetilde{\mathcal{W}}_t$ depends also on the specific process governing productivity. From Proposition 5 in the text, we know that, with LCP, under our parameter restrictions capital flows and the associated wealth gap remain exogenous to policy even if the trade elasticity is different from unity (the case of CO economies). This is also apparent from Table A3 below, where we show the equilibrium relation between capital flows and the wealth gap under LCP, together with the full solution for the dynamics of capital flows. The two expressions in the table depend only on exogenous shocks, and on the current and anticipated future evolution of relative prices in the first-best allocation through the term $\mathcal{Z}_t$, unaffected by policy.

Table A3: Capital flows under LCP and with news shocks for $\phi \geq 0$

$$(1-a_{\mathrm{H}})\left[1+2a_{\mathrm{H}}(\phi-1)\frac{(\beta\nu_2-1)}{\beta\nu_2}\right]\widetilde{\mathcal{W}}_t = -\left(\widehat{\mathcal{B}}_t-\beta^{-1}\widehat{\mathcal{B}}_{t-1}\right)+$$
$$2a_{\mathrm{H}}(1-a_{\mathrm{H}})(\phi-1)\sum_{j=0}^{\infty}\nu_2^{-j-1}E_t\left[\left(\widehat{\mathcal{T}}_{t+j+1}^{fb}-\widehat{\mathcal{T}}_{t+j}^{fb}\right)-\beta^{-1}\left(\widehat{\mathcal{T}}_{t+j}^{fb}-\widehat{\mathcal{T}}_{t+j-1}^{fb}\right)\right]$$

$$\widehat{\mathcal{B}}_t-\widehat{\mathcal{B}}_{t-1} = \frac{2a_{\mathrm{H}}(\phi-1)\frac{(\beta\nu_2-1)\nu_1}{\nu_2(1-\beta\nu_1)}}{1+2a_{\mathrm{H}}(\phi-1)\frac{(\beta\nu_2-1)}{\beta\nu_2(1-\beta\nu_1)}}\left(\beta^{-1}\widehat{\mathcal{B}}_{t-1}-\widehat{\mathcal{B}}_{t-1}\right)+\mathcal{Z}_t+$$
$$\left[\frac{1+2a_{\mathrm{H}}(\phi-1)\frac{(\beta\nu_2-1)}{\beta\nu_2}}{1+2a_{\mathrm{H}}(\phi-1)\frac{(\beta\nu_2-1)}{\beta\nu_2(1-\beta\nu_1)}}\right]\beta\sum_{j=0}^{\infty}\beta^j E_t\left[\left(\widehat{\zeta}_{C,t+j+1}-\widehat{\zeta}^*_{C,t+j+1}\right)-\left(\widehat{\zeta}_{C,t+j}-\widehat{\zeta}^*_{C,t+j}\right)\right]$$

$$\mathcal{Z}_t = 2a_{\mathrm{H}}(1-a_{\mathrm{H}})(\phi-1)\sum_{j=0}^{\infty}\nu_2^{-j-1}E_t\left[\left(\widehat{\mathcal{T}}_{t+j+1}^{fb}-\widehat{\mathcal{T}}_{t+j}^{fb}\right)-\beta^{-1}\left(\widehat{\mathcal{T}}_{t+j}^{fb}-\widehat{\mathcal{T}}_{t+j-1}^{fb}\right)\right]$$
$$-2a_{\mathrm{H}}(1-a_{\mathrm{H}})(\phi-1)\left[\frac{1+2a_{\mathrm{H}}(\phi-1)\frac{(\beta\nu_2-1)}{\beta\nu_2}}{1+2a_{\mathrm{H}}(\phi-1)\frac{(\beta\nu_2-1)}{\beta\nu_2(1-\beta\nu_1)}}\right]\cdot\left\{\beta\sum_{j=0}^{\infty}\beta^j E_t\left[\left(\widehat{\mathcal{T}}_{t+j+1}^{fb}-\widehat{\mathcal{T}}_{t+j}^{fb}\right)\right]+\right.$$
$$\left.\sum_{j=0}^{\infty}\beta^j\left[\begin{array}{c}\sum_{s=0}^{\infty}\nu_2^{-s-1}E_t\left[\left(\widehat{\mathcal{T}}_{t+j+s+1}^{fb}-\widehat{\mathcal{T}}_{t+j+s}^{fb}\right)-\beta^{-1}\left(\widehat{\mathcal{T}}_{t+j+s}^{fb}-\widehat{\mathcal{T}}_{t+j+s-1}^{fb}\right)\right]\\-(1-\nu_1)\beta\left[\sum_{s=0}^{j}\nu_1^{j-s}\left(\sum_{h=0}^{\infty}\nu_2^{-h-1}E_t\left[\begin{array}{c}\left(\widehat{\mathcal{T}}_{t+h+s+1}^{fb}-\widehat{\mathcal{T}}_{t+h+s}^{fb}\right)\\-\beta^{-1}\left(\widehat{\mathcal{T}}_{t+h+s}^{fb}-\widehat{\mathcal{T}}_{t+h+s-1}^{fb}\right)\end{array}\right]\right)\right]\end{array}\right]\right\}.$$

Inspection of the table establishes that the trade elasticity ϕ is a key determinant of the joint response of $\widehat{\mathcal{B}}_t$ and $\widetilde{\mathcal{W}}_t$ to news shocks in two respects. First, ϕ determines whether a given "news shock" translates into inefficient borrowing or lending; second, it determines whether $\widehat{\mathcal{B}}_t$ and $\widetilde{\mathcal{W}}_t$ have the same or the opposite sign, which is crucial for the optimal monetary stance.

Starting with anticipated taste shocks, Table 1 in the main text shows that with $\sigma = 1$ and $\eta = 0$, the terms-of-trade response to *(current or anticipated) taste shocks* in the first-best allocation is $\widetilde{\mathcal{T}}_t^{fb} = 0$. So, $\mathcal{Z}_t = 0$ in Table A3 and the expression linking the wealth gap and real net foreign assets simplifies as follows:

$$(1-a_{\mathrm{H}})\left[1+2a_{\mathrm{H}}(\phi-1)\frac{(\beta\nu_2-1)}{\beta\nu_2}\right]\widetilde{\mathcal{W}}_t = -\left(\widehat{\mathcal{B}}_t-\beta^{-1}\widehat{\mathcal{B}}_{t-1}\right)$$

$$\widehat{\mathcal{B}}_t-\widehat{\mathcal{B}}_{t-1} = \frac{2a_{\mathrm{H}}(\phi-1)\frac{(\beta\nu_2-1)\nu_1}{\nu_2(1-\beta\nu_1)}}{1+2a_{\mathrm{H}}(\phi-1)\frac{(\beta\nu_2-1)}{\beta\nu_2(1-\beta\nu_1)}}\left(\beta^{-1}\widehat{\mathcal{B}}_{t-1}-\widehat{\mathcal{B}}_{t-1}\right)$$
$$+\left[\frac{1+2a_{\mathrm{H}}(\phi-1)\frac{(\beta\nu_2-1)}{\beta\nu_2}}{1+2a_{\mathrm{H}}(\phi-1)\frac{(\beta\nu_2-1)}{\beta\nu_2(1-\beta\nu_1)}}\right]\beta\sum_{j=0}^{\infty}\beta^j E_t\left[\left(\widehat{\zeta}_{C,t+j+1}-\widehat{\zeta}^*_{C,t+j+1}\right)-\left(\widehat{\zeta}_{C,t+j}-\widehat{\zeta}^*_{C,t+j}\right)\right].$$

From this, it is easy to derive the threshold above. Note that the threshold is decreasing in openness ($a_{\mathrm{H}} \to 1/2, \phi \geq 0$) and the degree of price stickiness ($\nu_2 \to 1/\beta$, and $\frac{\beta\nu_2}{(\beta\nu_2-1)} \to 1, \phi \geq 0$), and is smaller than the threshold in the natural allocation shown in (36) in the main text.

Second, since first-best terms of trade $\widetilde{\mathcal{T}}_{t+s}^{fb}$ are different from zero for productivity shocks, deriving a threshold requires taking a stand on the term Z_t in Table A3. Specifically, under anticipated productivity shocks $\widehat{\mathcal{B}}_t < 0$ only if $Z_t < 0$, which in turn implies the following restrictions on parameters and productivity shocks:

$$\sum_{j=0}^{\infty}\nu_2^{-j-1}E_t\left[\left(\widehat{\mathcal{T}}_{t+j+1}^{fb}-\widehat{\mathcal{T}}_{t+j}^{fb}\right)-\beta^{-1}\left(\widehat{\mathcal{T}}_{t+j}^{fb}-\widehat{\mathcal{T}}_{t+j-1}^{fb}\right)\right] <$$
$$\left[\frac{1+2a_{\mathrm{H}}(\phi-1)\frac{(\beta\nu_2-1)}{\beta\nu_2}}{1+2a_{\mathrm{H}}(\phi-1)\frac{(\beta\nu_2-1)}{\beta\nu_2(1-\beta\nu_1)}}\right]\cdot\left\{\beta\sum_{j=0}^{\infty}\beta^jE_t\left[\left(\widehat{\mathcal{T}}_{t+j+1}^{fb}-\widehat{\mathcal{T}}_{t+j}^{fb}\right)\right]+\right.$$
$$\left.+\sum_{j=0}^{\infty}\beta^j\left[\begin{array}{c}\sum_{s=0}^{\infty}\nu_2^{-s-1}E_t\left[\left(\widehat{\mathcal{T}}_{t+j+s+1}^{fb}-\widehat{\mathcal{T}}_{t+j+s}^{fb}\right)-\beta^{-1}\left(\widehat{\mathcal{T}}_{t+j+s}^{fb}-\widehat{\mathcal{T}}_{t+j+s-1}^{fb}\right)\right]\\-(1-\nu_1)\beta\left[\sum_{s=0}^{j}\nu_1^{j-s}\left(\sum_{h=0}^{\infty}\nu_2^{-h-1}E_t\left[\begin{array}{c}\left(\widehat{\mathcal{T}}_{t+h+s+1}^{fb}-\widehat{\mathcal{T}}_{t+h+s}^{fb}\right)\\-\beta^{-1}\left(\widehat{\mathcal{T}}_{t+h+s}^{fb}-\widehat{\mathcal{T}}_{t+h+s-1}^{fb}\right)\end{array}\right]\right)\right]\end{array}\right]\right\};$$

observe that the coefficient in square brackets on the second line is positive only if

$$\phi > \frac{2a_{\mathrm{H}}-\frac{\beta\nu_2}{(\beta\nu_2-1)}(1-\beta\nu_1)}{2a_{\mathrm{H}}} > \frac{2a_{\mathrm{H}}-\frac{\beta\nu_2}{(\beta\nu_2-1)}}{2a_{\mathrm{H}}}.$$

namely ϕ is larger than the threshold derived above for preference shocks.

Using the expression for $\widetilde{\mathcal{W}}_t$ in Table A3, we can derive an expression highlighting the conditions under which a capital inflow due to anticipated productivity shocks leads to a positive or a negative wealth gap:

$$(1-a_{\mathrm{H}})\widetilde{\mathcal{W}}_t=\left[\frac{2a_{\mathrm{H}}(1-a_{\mathrm{H}})(\phi-1)}{1+2a_{\mathrm{H}}(\phi-1)\frac{(\beta\nu_2-1)}{\beta\nu_2(1-\beta\nu_1)}}\right]\cdot\left\{\beta\sum_{j=0}^{\infty}\beta^jE_t\left[\left(\widehat{\mathcal{T}}_{t+j+1}^{fb}-\widehat{\mathcal{T}}_{t+j}^{fb}\right)\right]+\right.$$
$$\left.+\sum_{j=0}^{\infty}\beta^j\left[\begin{array}{c}\sum_{s=0}^{\infty}\nu_2^{-s-1}E_t\left[\left(\widehat{\mathcal{T}}_{t+j+s+1}^{fb}-\widehat{\mathcal{T}}_{t+j+s}^{fb}\right)-\beta^{-1}\left(\widehat{\mathcal{T}}_{t+j+s}^{fb}-\widehat{\mathcal{T}}_{t+j+s-1}^{fb}\right)\right]\\-(1-\nu_1)\beta\left[\sum_{s=0}^{j}\nu_1^{j-s}\left(\sum_{h=0}^{\infty}\nu_2^{-h-1}E_t\left[\begin{array}{c}\left(\widehat{\mathcal{T}}_{t+h+s+1}^{fb}-\widehat{\mathcal{T}}_{t+h+s}^{fb}\right)\\-\beta^{-1}\left(\widehat{\mathcal{T}}_{t+h+s}^{fb}-\widehat{\mathcal{T}}_{t+h+s-1}^{fb}\right)\end{array}\right]\right)\right]\end{array}\right]\right\}.$$

Provided that $\frac{2a_{\mathrm{H}}(1-a_{\mathrm{H}})(\phi-1)}{1+2a_{\mathrm{H}}(\phi-1)\frac{(\beta\nu_2-1)}{\beta\nu_2(1-\beta\nu_1)}} > 0$ (which is the case for $\phi > 1$ and $\phi < 1-\frac{\beta\nu_2(1-\beta\nu_1)}{2a_{\mathrm{H}}(\beta\nu_2-1)} < 1$), the sign of $\widetilde{\mathcal{W}}_t$ depends on the sign of the expression in curly brackets on the right hand side. For instance, for $\phi > 1$ a sufficient condition to have both $\widetilde{\mathcal{W}}_t > 0$ and $\widehat{\mathcal{B}}_t < 0$ in the case of anticipated productivity shocks is for the expression in the curly bracket to be positive and also to satisfy the following inequality necessary to make $Z_t < 0$:

$$\left[\frac{1+2a_{\mathrm{H}}(\phi-1)\frac{(\beta\nu_2-1)}{\beta\nu_2(1-\beta\nu_1)}}{1+2a_{\mathrm{H}}(\phi-1)\frac{(\beta\nu_2-1)}{\beta\nu_2}}\right]\sum_{j=0}^{\infty}\nu_2^{-j-1}E_t\left[\left(\widehat{\mathcal{T}}_{t+j+1}^{fb}-\widehat{\mathcal{T}}_{t+j}^{fb}\right)-\beta^{-1}\left(\widehat{\mathcal{T}}_{t+j}^{fb}-\widehat{\mathcal{T}}_{t+j-1}^{fb}\right)\right] <$$
$$\left\{\beta\sum_{j=0}^{\infty}\beta^jE_t\left[\left(\widehat{\mathcal{T}}_{t+j+1}^{fb}-\widehat{\mathcal{T}}_{t+j}^{fb}\right)\right]+\right.$$
$$\left.\sum_{j=0}^{\infty}\beta^j\left[\begin{array}{c}\sum_{s=0}^{\infty}\nu_2^{-s-1}E_t\left[\left(\widehat{\mathcal{T}}_{t+j+s+1}^{fb}-\widehat{\mathcal{T}}_{t+j+s}^{fb}\right)-\beta^{-1}\left(\widehat{\mathcal{T}}_{t+j+s}^{fb}-\widehat{\mathcal{T}}_{t+j+s-1}^{fb}\right)\right]\\-(1-\nu_1)\beta\left[\sum_{s=0}^{j}\nu_1^{j-s}\left(\sum_{h=0}^{\infty}\nu_2^{-h-1}E_t\left[\begin{array}{c}\left(\widehat{\mathcal{T}}_{t+h+s+1}^{fb}-\widehat{\mathcal{T}}_{t+h+s}^{fb}\right)\\-\beta^{-1}\left(\widehat{\mathcal{T}}_{t+h+s}^{fb}-\widehat{\mathcal{T}}_{t+h+s-1}^{fb}\right)\end{array}\right]\right)\right]\end{array}\right]\right\},$$

where the coefficient in square brackets on the first line is positive since $\phi > 1$.

2.1.4 Constrained optimal allocation under LCP and Proof of Proposition 9

In order to derive the optimal allocation, consider again the difference of the sum of the within-country NKPC with $\sigma = 1$ and $\eta = 0$:

$$\begin{aligned} a_{\mathrm{H}}\pi_{H,t} + (1-a_{\mathrm{H}})\,\pi_{F,t} - \left(a_{\mathrm{H}}\pi^*_{F,t} + (1-a_{\mathrm{H}})\,\pi^*_{H,t}\right) - \\ \beta E_t\left(a_{\mathrm{H}}\pi_{H,t+1} + (1-a_{\mathrm{H}})\,\pi_{F,t+1}\right) - \beta E_t\left(a_{\mathrm{H}}\pi^*_{F,t+1} + (1-a_{\mathrm{H}})\,\pi^*_{H,t+1}\right) = \\ = \frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\left\{\widehat{\mathcal{Q}}_t - \widehat{\mathcal{Q}}^{fb}_t + (2a_{\mathrm{H}}-1)\left[\widehat{\mu}_t - \widehat{\mu}^*_t + \widehat{\mathcal{W}}_t\right]\right\}. \end{aligned}$$

We next substitute the relative target rule and derive a difference equation in the misalignment and demand gaps:

$$\begin{aligned} a_{\mathrm{H}}\pi_{H,t} + (1-a_{\mathrm{H}})\,\pi_{F,t} - \left(a_{\mathrm{H}}\pi^*_{F,t} + (1-a_{\mathrm{H}})\,\pi^*_{H,t}\right) - \beta E_t\left(a_{\mathrm{H}}\pi_{H,t+1} + (1-a_{\mathrm{H}})\,\pi_{F,t+1}\right) - \\ \beta E_t\left(a_{\mathrm{H}}\pi^*_{F,t+1} + (1-a_{\mathrm{H}})\,\pi^*_{H,t+1}\right) = \\ \theta^{-1}\Big\{\beta E_t\left[\left(\widehat{\mathcal{W}}_{t+1} - \widehat{\mathcal{W}}_t\right) + \left(\widehat{\mathcal{Q}}_{t+1} - \widehat{\mathcal{Q}}^{fb}_{t+1}\right) - \left(\widehat{\mathcal{Q}}_t - \widehat{\mathcal{Q}}^{fb}_t\right)\right] - \\ \left(\widehat{\mathcal{W}}_t - \widehat{\mathcal{W}}_{t-1}\right) - \left[\left(\widehat{\mathcal{Q}}_t - \widehat{\mathcal{Q}}^{fb}_t\right) - \left(\widehat{\mathcal{Q}}_{t-1} - \widehat{\mathcal{Q}}^{fb}_{t-1}\right)\right]\Big\} = \\ = \frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\left\{\widehat{\mathcal{Q}}_t - \widehat{\mathcal{Q}}^{fb}_t + (2a_{\mathrm{H}}-1)\,\widehat{\mathcal{W}}_t\right\}. \end{aligned}$$

The equation admits the following solution as a function of both current and future values of $\widehat{\mathcal{W}}_t$:

$$\begin{aligned} \left(\widehat{\mathcal{Q}}_t - \widehat{\mathcal{Q}}^{fb}_t\right) &= \varkappa_1\left(\widehat{\mathcal{Q}}_{t-1} - \widehat{\mathcal{Q}}^{fb}_{t-1}\right) - \frac{1}{\beta\varkappa_2}\sum_{j=0}^{\infty}\varkappa_2{}^{-j}E_t\left(\widehat{\mathcal{W}}_{t+j} - \widehat{\mathcal{W}}_{t+j-1}\right) + \\ &\quad - (2a_{\mathrm{H}}-1)\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\frac{\theta}{\beta\varkappa_2}\sum_{j=0}^{\infty}\varkappa_2^{-j}E_t\widehat{\mathcal{W}}_{t+j}. \end{aligned}$$

where $0 < \varkappa_1 < \beta < 1 < \beta^{-1} < \varkappa_2$ are the eigenvalues of the difference equation, solving the standard characteristic equation:

$$\beta\varkappa^2 - \left[1 + \beta + \frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\theta\right]\varkappa + 1 = 0,$$

namely

$$\varkappa_{1,2} = \frac{1 + \beta + \frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\theta \pm \sqrt{\left[1 + \beta + \frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\theta\right]^2 - 4\beta}}{2\beta}.$$

As in the PCP case, we can simplify further by using the law of motion for the wealth gap $\widehat{\mathcal{W}}_t$, $E_t\widehat{\mathcal{W}}_{t+j} = \widehat{\mathcal{W}}_t$:

$$\begin{aligned}\left(\widehat{\mathcal{Q}}_{t+j} - \widehat{\mathcal{Q}}^{fb}_{t+j}\right) &= \varkappa_1 \left(\widehat{\mathcal{Q}}_{t+j-1} - \widehat{\mathcal{Q}}^{fb}_{t+j-1}\right) - \frac{1}{\beta\varkappa_2}\left(\widehat{\mathcal{W}}_{t+j} - \widehat{\mathcal{W}}_{t+j-1}\right) + \\ &\quad - (2a_{\mathrm{H}} - 1)\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\frac{\theta}{\beta(\varkappa_2 - 1)}\widehat{\mathcal{W}}_t;\end{aligned}$$

we can also rewrite coefficients as follows

$$\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\theta = \frac{(\varkappa_2 - 1)(\beta\varkappa_2 - 1)}{\varkappa_2}.$$

The first term $\widehat{\mathcal{W}}_{t+j} - \widehat{\mathcal{W}}_{t+j-1} = 0$ for $j \geq 1$, while it is equal to $\widehat{\mathcal{W}}_t$ for $j = 0$; instead the last term represents a constant shifter proportional to $\widehat{\mathcal{W}}_t$ for any $j \geq 0$. Furthermore, recalling that,

$$\begin{gathered}\left[\widehat{\mathcal{W}}_t - \widehat{\mathcal{W}}_{t-1}\right] + \left[\left(\widehat{\mathcal{Q}}_t - \widehat{\mathcal{Q}}^{fb}_t\right) - \left(\widehat{\mathcal{Q}}_{t-1} - \widehat{\mathcal{Q}}^{fb}_{t-1}\right)\right] = \\ \left[\left(\widehat{C}_t - \widehat{C}^*_t\right) - \left(\widehat{C}^{fb}_t - \widehat{C}^{*fb}_t\right)\right] - \left[\left(\widehat{C}_{t-1} - \widehat{C}^*_{t-1}\right) - \left(\widehat{C}^{fb}_{t-1} - \widehat{C}^{*fb}_{t-1}\right)\right],\end{gathered}$$

we have that inefficient deviations in cross-country consumption differentials (and thus in CPI inflation) are given by:

$$\begin{gathered}\left[\left(\widehat{C}_t - \widehat{C}^*_t\right) - \left(\widehat{C}^{fb}_t - \widehat{C}^{*fb}_t\right)\right] - \left[\left(\widehat{C}_{t-1} - \widehat{C}^*_{t-1}\right) - \left(\widehat{C}^{fb}_{t-1} - \widehat{C}^{*fb}_{t-1}\right)\right] = \\ \left(\frac{\beta\varkappa_2 - 1}{\beta\varkappa_2}\right)\left(\widehat{\mathcal{W}}_t - \widehat{\mathcal{W}}_{t-1}\right) - (2a_{\mathrm{H}} - 1)\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\frac{\theta}{\beta(\varkappa_2 - 1)}\widehat{\mathcal{W}}_t \\ - (1 - \varkappa_1)\left(\widehat{\mathcal{Q}}_{t-1} - \widehat{\mathcal{Q}}^{fb}_{t-1}\right) = \\ -2\theta\left(a_{\mathrm{H}}\pi_{H,t} + (1 - a_{\mathrm{H}})\pi_{F,t}\right),\end{gathered}$$

which, interestingly, does not depend on the trade elasticity ϕ.

Thus, we also reach a solution for the deviations from the law of one price:

$$\widehat{\Delta}_t = \left(\widehat{\mathcal{Q}}_t - \widehat{\mathcal{Q}}^{fb}_t\right) - (2a_{\mathrm{H}} - 1)\left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}^{fb}_t + \widehat{\Delta}_t\right).$$

Finally, we can solve for the permanent response of $\widehat{\mathcal{W}}_t$ as a function only of exogenous shocks:

$$\begin{aligned}(1 - a_{\mathrm{H}})\widehat{\mathcal{W}}_t &= \beta^{-1}\left(\widehat{\mathcal{B}}_{t-1} - \beta\widehat{\mathcal{B}}_t\right) + \\ &\quad 2a_{\mathrm{H}}(1 - a_{\mathrm{H}})(\phi - 1)\left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}^{fb}_t + \widehat{\Delta}_t\right) + \\ &\quad (1 - a_{\mathrm{H}})\left[2a_{\mathrm{H}}(\phi - 1)\widehat{\mathcal{T}}^{fb}_t - \left(\widehat{\zeta}_{C,t} - \widehat{\zeta}^*_{C,t}\right)\right].\end{aligned}$$

Using again

$$\left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}_t^{fb} + \widehat{\Delta}_t\right) = \nu_1\left(\widehat{\mathcal{T}}_{t-1} - \widehat{\mathcal{T}}_{t-1}^{fb} + \widehat{\Delta}_{t-1}\right) - \frac{(\beta\nu_2 - 1)}{\beta\nu_2}\widehat{\mathcal{W}}_t + \sum_{s=0}^{\infty}\nu_2^{-s-1}E_t\left[\left(\widehat{\mathcal{T}}_{t+s+1}^{fb} - \widehat{\mathcal{T}}_{t+s}^{fb}\right) - \beta^{-1}\left(\widehat{\mathcal{T}}_{t+s}^{fb} - \widehat{\mathcal{T}}_{t+s-1}^{fb}\right)\right],$$

$$\begin{aligned}
&(1-a_{\mathrm{H}})\left[1 + 2a_{\mathrm{H}}(\phi-1)\frac{(\beta\nu_2-1)}{\beta\nu_2}\right]\widehat{\mathcal{W}}_t = \\
&\left(\beta^{-1}\widehat{\mathcal{B}}_{t-1} - \widehat{\mathcal{B}}_t\right) + \\
&2a_{\mathrm{H}}(1-a_{\mathrm{H}})(\phi-1)\sum_{s=0}^{\infty}\nu_2^{-s-1}E_t\left[\left(\widehat{\mathcal{T}}_{t+s+1}^{fb} - \widehat{\mathcal{T}}_{t+s}^{fb}\right) - \beta^{-1}\left(\widehat{\mathcal{T}}_{t+s}^{fb} - \widehat{\mathcal{T}}_{t+s-1}^{fb}\right)\right] - \\
&(1-a_{\mathrm{H}})\left[2a_{\mathrm{H}}(\phi-1)\widehat{\mathcal{T}}_t^{fb} - \left(\widehat{\zeta}_{C,t} - \widehat{\zeta}_{C,t}^{*}\right)\right].
\end{aligned}$$

Recalling the solution for capital flows

$$\begin{aligned}
&\widehat{\mathcal{B}}_t - \widehat{\mathcal{B}}_{t-1} = \\
&2a_{\mathrm{H}}(1-a_{\mathrm{H}})(\phi-1)(1-\nu_1)\left\{\beta\sum_{j=0}^{\infty}\beta^j\nu_1^{j+1}\left(\widehat{\mathcal{T}}_{t-1} - \widehat{\mathcal{T}}_{t-1}^{fb} + \widehat{\Delta}_{t-1}\right) + \right. \\
&\left.\beta\sum_{j=0}^{\infty}\beta^j\sum_{s=0}^{j}\nu_1^{j-s}\sum_{h=0}^{\infty}\nu_2^{-h-1}E_t\left[\begin{array}{c}\left(\widehat{\mathcal{T}}_{t+h+s+1}^{fb} - \widehat{\mathcal{T}}_{t+h+s}^{fb}\right) \\ -\beta^{-1}\left(\widehat{\mathcal{T}}_{t+h+s}^{fb} - \widehat{\mathcal{T}}_{t+h+s-1}^{fb}\right)\end{array}\right]\right\} + \\
&2a_{\mathrm{H}}(1-a_{\mathrm{H}})(\phi-1)\frac{(\beta\nu_2-1)\nu_1}{\nu_2(1-\beta\nu_1)}\widehat{\mathcal{W}}_t - \\
&2a_{\mathrm{H}}(1-a_{\mathrm{H}})(\phi-1)\beta\sum_{j=0}^{\infty}\beta^j\sum_{s=0}^{\infty}\nu_2^{-s-1}E_t\left[\begin{array}{c}\left(\widehat{\mathcal{T}}_{t+j+s+2}^{fb} - \widehat{\mathcal{T}}_{t+j+s+1}^{fb}\right) - \\ \beta^{-1}\left(\widehat{\mathcal{T}}_{t+j+s+1}^{fb} - \widehat{\mathcal{T}}_{t+j+s}^{fb}\right)\end{array}\right] - \\
&(1-a_{\mathrm{H}})\left[2a_{\mathrm{H}}(\phi-1)\right]\beta\sum_{j=0}^{\infty}\beta^jE_t\left[\left(\widehat{\mathcal{T}}_{t+j+1}^{fb} - \widehat{\mathcal{T}}_{t+j}^{fb}\right)\right] + \\
&(1-a_{\mathrm{H}})\beta\sum_{j=0}^{\infty}\beta^jE_t\left[\left(\widehat{\zeta}_{C,t+j+1} - \widehat{\zeta}_{C,t+j+1}^{*}\right) - \left(\widehat{\zeta}_{C,t+j} - \widehat{\zeta}_{C,t+j}^{*}\right)\right].
\end{aligned}$$

$$
\begin{aligned}
&(1-a_{\mathrm{H}})\left[1+2a_{\mathrm{H}}(\phi-1)\frac{(\beta\nu_2-1)}{\beta\nu_2(1-\beta\nu_1)}\right]\widehat{\mathcal{W}}_t = \\
&\beta^{-1}\widehat{\mathcal{B}}_{t-1}-\widehat{\mathcal{B}}_{t-1}+2a_{\mathrm{H}}(1-a_{\mathrm{H}})(\phi-1)\beta\sum_{j=0}^{\infty}\beta^j\cdot \\
&\left\{\begin{array}{c}
\sum_{s=0}^{\infty}\nu_2^{-s-1}E_t\Big[\left(\widehat{\mathcal{T}}^{fb}_{t+j+s+2}-\widehat{\mathcal{T}}^{fb}_{t+j+s+1}\right)- \\
\beta^{-1}\left(\widehat{\mathcal{T}}^{fb}_{t+j+s+1}-\widehat{\mathcal{T}}^{fb}_{t+j+s}\right)\Big]- \\
(1-\nu_1)\sum_{s=0}^{j}\nu_1^{j-s}\sum_{h=0}^{\infty}\nu_2^{-h-1}E_t\Big[\left(\widehat{\mathcal{T}}^{fb}_{t+h+s+1}-\widehat{\mathcal{T}}^{fb}_{t+h+s}\right)- \\
\beta^{-1}\left(\widehat{\mathcal{T}}^{fb}_{t+h+s}-\widehat{\mathcal{T}}^{fb}_{t+h+s-1}\right)\Big]
\end{array}\right\} \\
&+(1-a_{\mathrm{H}})\beta\sum_{j=0}^{\infty}\beta^j\left\{\begin{array}{c}
2a_{\mathrm{H}}(\phi-1)E_t\left[\left(\widehat{\mathcal{T}}^{fb}_{t+j+1}-\widehat{\mathcal{T}}^{fb}_{t+j}\right)\right]+ \\
-E_t\left[\left(\widehat{\zeta}_{C,t+j+1}-\widehat{\zeta}^{*}_{C,t+j+1}\right)-\left(\widehat{\zeta}_{C,t+j}-\widehat{\zeta}^{*}_{C,t+j}\right)\right]
\end{array}\right\} \\
&+2a_{\mathrm{H}}(1-a_{\mathrm{H}})(\phi-1)\sum_{s=0}^{\infty}\nu_2^{-s-1}E_t\left[\left(\widehat{\mathcal{T}}^{fb}_{t+s+1}-\widehat{\mathcal{T}}^{fb}_{t+s}\right)-\beta^{-1}\left(\widehat{\mathcal{T}}^{fb}_{t+s}-\widehat{\mathcal{T}}^{fb}_{t+s-1}\right)\right] \\
&+(1-a_{\mathrm{H}})\left[2a_{\mathrm{H}}(\phi-1)\widehat{\mathcal{T}}^{fb}_t-\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}^{*}_{C,t}\right)\right].
\end{aligned}
$$

$$
\begin{aligned}
&2a_{\mathrm{H}}(1-a_{\mathrm{H}})(\phi-1)\frac{(\beta\nu_2-1)\nu_1}{\nu_2(1-\beta\nu_1)}\widehat{\mathcal{W}}_t = \\
&\frac{2a_{\mathrm{H}}(\phi-1)\frac{(\beta\nu_2-1)\nu_1}{\nu_2(1-\beta\nu_1)}}{1+2a_{\mathrm{H}}(\phi-1)\frac{(\beta\nu_2-1)}{\beta\nu_2(1-\beta\nu_1)}}\Big\{\beta^{-1}\widehat{\mathcal{B}}_{t-1}-\widehat{\mathcal{B}}_{t-1}+2a_{\mathrm{H}}(1-a_{\mathrm{H}})(\phi-1)\beta\cdot \\
&\sum_{j=0}^{\infty}\beta^j\left[\begin{array}{c}
\sum_{s=0}^{\infty}\nu_2^{-s-1}E_t\left[\begin{array}{c}\left(\widehat{\mathcal{T}}^{fb}_{t+j+s+2}-\widehat{\mathcal{T}}^{fb}_{t+j+s+1}\right)- \\ \beta^{-1}\left(\widehat{\mathcal{T}}^{fb}_{t+j+s+1}-\widehat{\mathcal{T}}^{fb}_{t+j+s}\right)\end{array}\right]- \\
(1-\nu_1)\sum_{s=0}^{j}\nu_1^{j-s}\left(\sum_{h=0}^{\infty}\nu_2^{-h-1}E_t\left[\begin{array}{c}\left(\widehat{\mathcal{T}}^{fb}_{t+h+s+1}-\widehat{\mathcal{T}}^{fb}_{t+h+s}\right)- \\ \beta^{-1}\left(\widehat{\mathcal{T}}^{fb}_{t+h+s}-\widehat{\mathcal{T}}^{fb}_{t+h+s-1}\right)\end{array}\right]\right)
\end{array}\right]+ \\
&2a_{\mathrm{H}}(1-a_{\mathrm{H}})(\phi-1)\sum_{s=0}^{\infty}\nu_2^{-s-1}E_t\left[\begin{array}{c}\left(\widehat{\mathcal{T}}^{fb}_{t+s+1}-\widehat{\mathcal{T}}^{fb}_{t+s}\right)- \\ \beta^{-1}\left(\widehat{\mathcal{T}}^{fb}_{t+s}-\widehat{\mathcal{T}}^{fb}_{t+s-1}\right)\end{array}\right]+ \\
&(1-a_{\mathrm{H}})\beta\sum_{j=0}^{\infty}\beta^j\left[\begin{array}{c}2a_{\mathrm{H}}(\phi-1)E_t\left[\left(\widehat{\mathcal{T}}^{fb}_{t+j+1}-\widehat{\mathcal{T}}^{fb}_{t+j}\right)\right]- \\ E_t\left[\left(\widehat{\zeta}_{C,t+j+1}-\widehat{\zeta}^{*}_{C,t+j+1}\right)-\left(\widehat{\zeta}_{C,t+j}-\widehat{\zeta}^{*}_{C,t+j}\right)\right]\end{array}\right]+ \\
&(1-a_{\mathrm{H}})\left[2a_{\mathrm{H}}(\phi-1)\widehat{\mathcal{T}}^{fb}_t-\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}^{*}_{C,t}\right)\right]\Big\}.
\end{aligned}
$$

$$
\begin{aligned}
&\widehat{\mathcal{B}}_t - \widehat{\mathcal{B}}_{t-1} = \\
&2a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\phi-1\right)\left(1-\nu_1\right)\beta\sum_{j=0}^{\infty}\beta^j\nu_1^{j+1}\left(\widehat{\mathcal{T}}_{t-1}-\widehat{\mathcal{T}}_{t-1}^{fb}+\widehat{\Delta}_{t-1}\right)+ \\
&\frac{2a_{\mathrm{H}}\left(\phi-1\right)\frac{(\beta\nu_2-1)\nu_1}{\nu_2(1-\beta\nu_1)}}{1+2a_{\mathrm{H}}\left(\phi-1\right)\frac{(\beta\nu_2-1)}{\beta\nu_2(1-\beta\nu_1)}}\left(\beta^{-1}\widehat{\mathcal{B}}_{t-1}-\widehat{\mathcal{B}}_{t-1}\right)- \\
&\left[\frac{1+2a_{\mathrm{H}}\left(\phi-1\right)\frac{(\beta\nu_2-1)}{\beta\nu_2}}{1+2a_{\mathrm{H}}\left(\phi-1\right)\frac{(\beta\nu_2-1)}{\beta\nu_2(1-\beta\nu_1)}}\right]2a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\phi-1\right)\cdot \\
&\sum_{j=0}^{\infty}\beta^j\left\{\begin{array}{c}
\sum_{s=0}^{\infty}\nu_2^{-s-1}E_t\left[\begin{array}{c}\left(\widehat{\mathcal{T}}_{t+j+s+1}^{fb}-\widehat{\mathcal{T}}_{t+j+s}^{fb}\right)- \\ \beta^{-1}\left(\widehat{\mathcal{T}}_{t+j+s}^{fb}-\widehat{\mathcal{T}}_{t+j+s-1}^{fb}\right)\end{array}\right]- \\
\left(1-\nu_1\right)\beta\sum_{s=0}^{j}\nu_1^{j-s}\sum_{h=0}^{\infty}\nu_2^{-h-1}E_t\left[\begin{array}{c}\left(\widehat{\mathcal{T}}_{t+h+s+1}^{fb}-\widehat{\mathcal{T}}_{t+h+s}^{fb}\right)- \\ \beta^{-1}\left(\widehat{\mathcal{T}}_{t+h+s}^{fb}-\widehat{\mathcal{T}}_{t+h+s-1}^{fb}\right)\end{array}\right]
\end{array}\right\}- \\
&\left[\frac{1+2a_{\mathrm{H}}\left(\phi-1\right)\frac{(\beta\nu_2-1)}{\beta\nu_2}}{1+2a_{\mathrm{H}}\left(\phi-1\right)\frac{(\beta\nu_2-1)}{\beta\nu_2(1-\beta\nu_1)}}\right]\left(1-a_{\mathrm{H}}\right)\beta\cdot \\
&\sum_{j=0}^{\infty}\beta^j\left[\begin{array}{c}2a_{\mathrm{H}}\left(\phi-1\right)E_t\left[\left(\widehat{\mathcal{T}}_{t+j+1}^{fb}-\widehat{\mathcal{T}}_{t+j}^{fb}\right)\right]- \\ E_t\left[\left(\widehat{\zeta}_{C,t+j+1}-\widehat{\zeta}_{C,t+j+1}^{*}\right)-\left(\widehat{\zeta}_{C,t+j}-\widehat{\zeta}_{C,t+j}^{*}\right)\right]\end{array}\right]+ \\
&2a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\phi-1\right)\sum_{s=0}^{\infty}\nu_2^{-s-1}E_t\left[\begin{array}{c}\left(\widehat{\mathcal{T}}_{t+s+1}^{fb}-\widehat{\mathcal{T}}_{t+s}^{fb}\right)- \\ \beta^{-1}\left(\widehat{\mathcal{T}}_{t+s}^{fb}-\widehat{\mathcal{T}}_{t+s-1}^{fb}\right)\end{array}\right]+ \\
&\frac{2a_{\mathrm{H}}\left(\phi-1\right)\frac{(\beta\nu_2-1)\nu_1}{\nu_2(1-\beta\nu_1)}}{1+2a_{\mathrm{H}}\left(\phi-1\right)\frac{(\beta\nu_2-1)}{\beta\nu_2(1-\beta\nu_1)}}\left(1-a_{\mathrm{H}}\right)\left[2a_{\mathrm{H}}\left(\phi-1\right)\widehat{\mathcal{T}}_t^{fb}-\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}_{C,t}^{*}\right)\right].
\end{aligned}
$$

Furthermore,

$$(1-a_{\mathrm{H}})\left[1+2a_{\mathrm{H}}(\phi-1)\frac{(\beta\nu_2-1)}{\beta\nu_2(1-\beta\nu_1)}\right]\widehat{\mathcal{W}}_t=$$

$$\beta^{-1}\widehat{\mathcal{B}}_{t-1}-\widehat{\mathcal{B}}_{t-1}+2a_{\mathrm{H}}(1-a_{\mathrm{H}})(\phi-1)\sum_{j=0}^{\infty}\beta^j\cdot$$

$$\left\{\begin{array}{c}\sum_{s=0}^{\infty}\nu_2^{-s-1}E_t\left[\begin{array}{c}\left(\widehat{\mathcal{T}}_{t+j+s+1}^{fb}-\widehat{\mathcal{T}}_{t+j+s}^{fb}\right)-\\ \beta^{-1}\left(\widehat{\mathcal{T}}_{t+j+s}^{fb}-\widehat{\mathcal{T}}_{t+j+s-1}^{fb}\right)\end{array}\right]-\\ (1-\nu_1)\beta\sum_{s=0}^{j}\nu_1^{j-s}\sum_{h=0}^{\infty}\nu_2^{-h-1}E_t\left[\begin{array}{c}\left(\widehat{\mathcal{T}}_{t+h+s+1}^{fb}-\widehat{\mathcal{T}}_{t+h+s}^{fb}\right)-\\ \beta^{-1}\left(\widehat{\mathcal{T}}_{t+h+s}^{fb}-\widehat{\mathcal{T}}_{t+h+s-1}^{fb}\right)\end{array}\right]\end{array}\right\}$$

$$(1-a_{\mathrm{H}})\beta\sum_{j=0}^{\infty}\beta^j\left\{\begin{array}{c}2a_{\mathrm{H}}(\phi-1)E_t\left[\left(\widehat{\mathcal{T}}_{t+j+1}^{fb}-\widehat{\mathcal{T}}_{t+j}^{fb}\right)\right]-\\ E_t\left[\left(\widehat{\zeta}_{C,t+j+1}-\widehat{\zeta}_{C,t+j+1}^{*}\right)-\left(\widehat{\zeta}_{C,t+j}-\widehat{\zeta}_{C,t+j}^{*}\right)\right]\end{array}\right\}+$$

$$(1-a_{\mathrm{H}})\left[2a_{\mathrm{H}}(\phi-1)\widehat{\mathcal{T}}_t^{fb}-\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}_{C,t}^{*}\right)\right].$$

Lastly, we derive the link between the demand gap and capital flows shown in Section 5 in the main text:

$$(1-a_{\mathrm{H}})\left[1+2a_{\mathrm{H}}(\phi-1)\frac{(\beta\nu_2-1)}{\beta\nu_2}\right]\widehat{\mathcal{W}}_t=-\widehat{\mathcal{B}}_t+$$

$$(1-a_{\mathrm{H}})2a_{\mathrm{H}}(\phi-1)\sum_{s=0}^{\infty}\nu_2^{-s-1}E_t\left[\left(\widehat{\mathcal{T}}_{t+s+1}^{fb}-\widehat{\mathcal{T}}_{t+s}^{fb}\right)-\beta^{-1}\left(\widehat{\mathcal{T}}_{t+s}^{fb}-\widehat{\mathcal{T}}_{t+s-1}^{fb}\right)\right]+$$

$$\left[\frac{1+2a_{\mathrm{H}}(\phi-1)\frac{(\beta\nu_2-1)\nu_1}{\nu_2(1-\beta\nu_1)}}{1+2a_{\mathrm{H}}(\phi-1)\frac{(\beta\nu_2-1)}{\beta\nu_2(1-\beta\nu_1)}}\right](1-a_{\mathrm{H}})\left[2a_{\mathrm{H}}(\phi-1)\widehat{\mathcal{T}}_t^{fb}-\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}_{C,t}^{*}\right)\right].$$

Proof of Proposition 9: Derivation of the output gap.

We can finally derive the output gap under the constrained optimal allocation as follows:

$$\left(\widehat{Y}_{H,t}-\widehat{Y}_{H,t}^{fb}\right)-\left(\widehat{Y}_{F,t}-\widehat{Y}_{F,t}^{fb}\right)=2\left(\widehat{Y}_{H,t}-\widehat{Y}_{H,t}^{fb}\right)=$$

$$=\left[4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\phi-1)+1\right]\left(\widehat{\mathcal{T}}_t-\widehat{\mathcal{T}}_t^{fb}+\widehat{\Delta}_t\right)+(2a_{\mathrm{H}}-1)\left(\widehat{\mathcal{W}}_t+\widehat{\Delta}_t\right)$$

$$=4a_{\mathrm{H}}(1-a_{\mathrm{H}})\phi\left(\widehat{\mathcal{T}}_t-\widehat{\mathcal{T}}_t^{fb}+\widehat{\Delta}_t\right)+(2a_{\mathrm{H}}-1)\left(\widehat{\mathcal{W}}_t+\left(\widehat{\mathcal{Q}}_t-\widehat{\mathcal{Q}}_t^{fb}\right)\right),$$

namely:

$$2\left(\widehat{Y}_{H,t}-\widehat{Y}_{H,t}^{fb}\right)=$$

$$(2a_{\mathrm{H}}-1)\left(\widehat{\mathcal{W}}_t+\left(\widehat{\mathcal{Q}}_t-\widehat{\mathcal{Q}}_t^{fb}\right)\right)-4a_{\mathrm{H}}(1-a_{\mathrm{H}})\phi\frac{(\beta\nu_2-1)}{\beta\nu_2}\widehat{\mathcal{W}}_t\cdot$$

$$4a_{\mathrm{H}}(1-a_{\mathrm{H}})\phi\left[\begin{array}{c}\sum_{j=0}^{\infty}\nu_2^{-j-1}E_t\left[\left(\widehat{\mathcal{T}}_{t+j+1}^{fb}-\widehat{\mathcal{T}}_{t+j}^{fb}\right)-\beta^{-1}\left(\widehat{\mathcal{T}}_{t+j}^{fb}-\widehat{\mathcal{T}}_{t+j-1}^{fb}\right)\right]+\\ \nu_1\left(\widehat{\mathcal{T}}_{t-1}-\widehat{\mathcal{T}}_{t-1}^{fb}+\widehat{\Delta}_{t-1}\right)\end{array}\right]$$

This completes the derivation of the output gap in Proposition 9.

2.2 PCP economy

2.2.1 Proof of Proposition 3

The PCP loss function is given by (1) subject to $(\widehat{\Delta}_t) = 0$ and $\left[a_{\rm H}\pi^2_{H,t} + (1-a_{\rm H})\,\pi^{*2}_{H,t} + a_{\rm H}\pi^{*2}_{F,t} + (1-a_{\rm H})\,\pi^2_{F,t}\right] = \pi^2_{H,t} + \pi^{*2}_{F,t}$. Under PCP optimal monetary policy minimizes the loss function subject to:

1. NK Phillips curves determining inflation rates

$$\begin{aligned}
\pi_{H,t} &= \beta E_t \pi_{H,t+1} + \frac{(1-\alpha\beta)(1-\alpha)}{\alpha} \\
&\cdot \left\{ \begin{array}{c} (\eta+\sigma)\left(\widehat{Y}_{H,t} - \widehat{Y}^{fb}_{H,t}\right) + \widehat{\mu}_t + \\ -(1-a_{\rm H})\cdot\left[2a_{\rm H}(\sigma\phi-1)\left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}^{fb}_t\right) - \widetilde{\mathcal{W}}_t\right] \end{array} \right\}
\end{aligned}$$

$$\begin{aligned}
\pi^*_{F,t} &= \beta E_t \pi^*_{F,t+1} + \frac{(1-\alpha\beta)(1-\alpha)}{\alpha} \\
&\cdot \left\{ \begin{array}{c} (\eta+\sigma)\left(\widehat{Y}_{F,t} - \widehat{Y}^{fb}_{F,t}\right) + \widehat{\mu}^*_t + \\ +(1-a_{\rm H})\cdot\left[2a_{\rm H}(\sigma\phi-1)\left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}^{fb}_t\right) - \widetilde{\mathcal{W}}_t\right] \end{array} \right\},
\end{aligned}$$

where the equilibrium relations for first best outcomes $\widehat{Y}^{fb}_{H,t}$, $\widehat{Y}^{fb}_{F,t}$, $\widehat{\mathcal{T}}^{fb}_t$ in terms of fundamental shocks are as follows:

$$\begin{aligned}
&(\eta+\sigma)\,\widehat{Y}^{fb}_{H,t} = \\
&[2a_{\rm H}(1-a_{\rm H})(\sigma\phi-1)]\left(\widehat{\mathcal{T}}^{fb}_t\right) - (1-a_{\rm H})\left(\widehat{\zeta}_{C,t} - \widehat{\zeta}^*_{C,t}\right) + \widehat{\zeta}_{C,t} + (1+\eta)\,\widehat{\zeta}_{Y,t}, \\
&(\eta+\sigma)\,\widehat{Y}^{fb}_{F,t} = \\
&[2a_{\rm H}(1-a_{\rm H})(\sigma\phi-1)]\left(-\widehat{\mathcal{T}}^{fb}_t\right) + (1-a_{\rm H})\left(\widehat{\zeta}_{C,t} - \widehat{\zeta}^*_{C,t}\right) + \widehat{\zeta}^*_{C,t} + (1+\eta)\,\widehat{\zeta}^*_{Y,t},
\end{aligned}$$

whereas the terms of trade can in turn be written as a function of relative output and preference shocks

$$\left[4(1-a_{\rm H})\,a_{\rm H}\phi\sigma + (2a_{\rm H}-1)^2\right]\widehat{\mathcal{T}}^{fb}_t = \sigma\left(\widehat{Y}^{fb}_{H,t} - \widehat{Y}^{fb}_{F,t}\right) - (2a_{\rm H}-1)\left(\widehat{\zeta}_{C,t} - \widehat{\zeta}^*_{C,t}\right);$$

2. The equilibrium condition linking relative prices to output differentials and the wealth gap:

$$\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}^{fb}_t = \frac{\sigma\left[\left(\widehat{Y}_{H,t} - \widehat{Y}_{F,t}\right) - \left(\widehat{Y}^{fb}_{H,t} - \widehat{Y}^{fb}_{F,t}\right)\right] - (2a_{\rm H}-1)\,\widetilde{\mathcal{W}}_t}{4a_{\rm H}(1-a_{\rm H})(\sigma\phi-1)+1};$$

3. The definition of demand gap $\widetilde{\mathcal{W}}_t$ in terms of differences in budget constraints and real net wealth $\widehat{\mathcal{B}}_t$:

$$\begin{aligned}
\widetilde{\mathcal{W}}_t &= \widehat{\mathcal{W}}_t = \sigma\left[\left(\widehat{C}_t - \widehat{C}_t^*\right) - \left(\widehat{C}_t^{fb} - \widehat{C}_t^{*fb}\right)\right] - \left(\widehat{\mathcal{Q}}_t - \widehat{\mathcal{Q}}_t^{fb}\right) \\
&= \sigma\left[\begin{array}{c} \left(\widehat{Y}_{H,t} - \widehat{Y}_{F,t}\right) - \left(\widehat{Y}_{H,t}^{fb} - \widehat{Y}_{F,t}^{fb}\right) + \\ 2\beta^{-1}\left(\widehat{\mathcal{B}}_{t-1} - \beta\widehat{\mathcal{B}}_t + \sum_i \frac{\omega_i}{Y}\left(\widehat{R}_{i,t} - \left(\widehat{1+r_t}\right)\right)\right) \end{array}\right] + \\
&\quad - \left[2\left(1-a_{\mathrm{H}}\right)\sigma + \left(2a_{\mathrm{H}}-1\right)\right]\left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}_t^{fb}\right) + \\
&\quad 2\left(1-a_{\mathrm{H}}\right)\left[\left(2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1-\sigma\right)\widehat{\mathcal{T}}_t^{fb} - \left(\widehat{\zeta}_{C,t} - \widehat{\zeta}_{C,t}^*\right)\right],
\end{aligned}$$

4. the Euler equations characterizing the evolution of $\widetilde{\mathcal{W}}_t$ (and net wealth $\widehat{\mathcal{B}}_t$):

$$E_t\widetilde{\mathcal{W}}_{t+1} = \widetilde{\mathcal{W}}_t.$$

Bond economy

Observe that in the case of a bond economy, the program amounts to choosing $\widehat{Y}_{H,t}$, $\widehat{Y}_{F,t}$, $\widehat{\mathcal{D}}_t$, $\pi_{H,t}$, $\pi_{F,t}^*$ and $\widehat{\mathcal{B}}_t$ subject to the following expression for $\widetilde{\mathcal{W}}_t$ in terms of differences of budget constraints:

$$\begin{aligned}
\left(1-a_{\mathrm{H}}\right)\left[1+2a_{\mathrm{H}}\left(\phi-1\right)\right]\widetilde{\mathcal{W}}_t &= \left[4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1\right]\left(\beta^{-1}\widehat{\mathcal{B}}_{t-1} - \widehat{\mathcal{B}}_t\right) + \\
&\left(1-a_{\mathrm{H}}\right)\left[2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1-\sigma\right]\left[\left(\widehat{Y}_{H,t} - \widehat{Y}_{H,t}^{fb}\right) - \left(\widehat{Y}_{F,t} - \widehat{Y}_{F,t}^{fb}\right)\right] + \\
\left(1-a_{\mathrm{H}}\right)\left[4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1\right]\sigma^{-1}&\left[\left(2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1-\sigma\right)\widehat{\mathcal{T}}_t^{fb} - \left(\widehat{\zeta}_{C,t} - \widehat{\zeta}_{C,t}^*\right)\right];
\end{aligned}$$

The necessary FOC's with respect to inflation are given by:

$$\begin{aligned}
\pi_{H,t} &: \quad 0 = -\theta\frac{\alpha}{\left(1-\alpha\beta\right)\left(1-\alpha\right)}\pi_{H,t} - \gamma_{H,t} + \gamma_{H,t-1} \\
\pi_{F,t}^* &: \quad 0 = -\theta\frac{\alpha}{\left(1-\alpha\beta\right)\left(1-\alpha\right)}\pi_{F,t}^* - \gamma_{F,t}^* + \gamma_{F,t-1}^*,
\end{aligned}$$

implying

$$\begin{aligned}
-\frac{\left(1-\alpha\beta\right)\left(1-\alpha\right)}{\alpha}\left(\gamma_{H,t} - \gamma_{H,t-1}\right) &= \theta\pi_{H,t} = \theta\left(\widehat{p}_{H,t} - \widehat{p}_{H,t-1}\right) \\
-\frac{\left(1-\alpha\beta\right)\left(1-\alpha\right)}{\alpha}\left(\gamma_{F,t}^* - \gamma_{F,t-1}^*\right) &= \theta\left(\widehat{p}_{F,t}^* - \widehat{p}_{F,t-1}^*\right),
\end{aligned}$$

where $\gamma_{H,t}$ and $\gamma_{F,t}^*$ are the multipliers associated with the Phillips curves — whose lags appear reflecting the assumption of commitment; and with respect to output (where observe that we have switched to the gap notation, e.g. $\widetilde{Y}_{H,t} =$

$\widehat{Y}_{H,t}-\widehat{Y}^{fb}_{H,t}$):

$$\begin{aligned}
\widehat{Y}_{H,t} \quad : \quad & 0=(\sigma+\eta)\,\widetilde{Y}_{H,t}- \\
& \frac{2a_{\text{H}}\left(1-a_{\text{H}}\right)\left(\sigma\phi-1\right)\sigma}{4a_{\text{H}}\left(1-a_{\text{H}}\right)\left(\sigma\phi-1\right)+1}\left[\widetilde{Y}_{H,t}-\widetilde{Y}_{F,t}\right]+ \\
& \frac{2a_{\text{H}}\left(1-a_{\text{H}}\right)\phi}{4a_{\text{H}}\left(1-a_{\text{H}}\right)\left(\sigma\phi-1\right)+1}\frac{2a_{\text{H}}\left(\sigma\phi-1\right)+1-\sigma}{2a_{\text{H}}\left(\phi-1\right)+1}\widetilde{\mathcal{W}}_t+ \\
& \frac{2a_{\text{H}}\left(\sigma\phi-1\right)+1-\sigma}{2a_{\text{H}}\left(\phi-1\right)+1}\left(\lambda_t-\beta^{-1}\lambda_{t-1}\right)- \\
& \left[\sigma+\eta-\frac{\left(1-a_{\text{H}}\right)\left(\sigma-1\right)}{2a_{\text{H}}\left(\phi-1\right)+1}\right]\frac{\left(1-\alpha\beta\right)\left(1-\alpha\right)}{\alpha}\gamma_{H,t}+ \\
& \frac{\left(1-a_{\text{H}}\right)\left(\sigma-1\right)}{2a_{\text{H}}\left(\phi-1\right)+1}\frac{\left(1-\alpha\beta\right)\left(1-\alpha\right)}{\alpha}\gamma^*_{F,t};
\end{aligned}$$

$$\begin{aligned}
\widehat{Y}_{F,t} \quad : \quad & 0=(\sigma+\eta)\left(\widetilde{Y}_{F,t}\right)+ \\
& \frac{2a_{\text{H}}\left(1-a_{\text{H}}\right)\left(\sigma\phi-1\right)\sigma}{4a_{\text{H}}\left(1-a_{\text{H}}\right)\left(\sigma\phi-1\right)+1}\left[\widetilde{Y}_{H,t}-\widetilde{Y}_{F,t}\right]- \\
& \frac{2a_{\text{H}}\left(1-a_{\text{H}}\right)\phi}{4a_{\text{H}}\left(1-a_{\text{H}}\right)\left(\sigma\phi-1\right)+1}\frac{2a_{\text{H}}\left(\sigma\phi-1\right)+1-\sigma}{2a_{\text{H}}\left(\phi-1\right)+1}\widetilde{\mathcal{W}}_t- \\
& \frac{2a_{\text{H}}\left(\sigma\phi-1\right)+1-\sigma}{2a_{\text{H}}\left(\phi-1\right)+1}\left(\lambda_t-\beta^{-1}\lambda_{t-1}\right)- \\
& \left[\sigma+\eta+\frac{\left(1-a_{\text{H}}\right)\left(\sigma-1\right)}{2a_{\text{H}}\left(\phi-1\right)+1}\right]\frac{\left(1-\alpha\beta\right)\left(1-\alpha\right)}{\alpha}\gamma^*_{F,t}- \\
& \frac{\left(1-a_{\text{H}}\right)\left(\sigma-1\right)}{2a_{\text{H}}\left(\phi-1\right)+1}\frac{\left(1-\alpha\beta\right)\left(1-\alpha\right)}{\alpha}\gamma_{H,t};
\end{aligned}$$

Furthermore,

$$\begin{aligned}
\widehat{\mathcal{B}}_t \quad : \quad & 0=2a_{\text{H}}\left(1-a_{\text{H}}\right)\phi\left[E_t\widetilde{\mathcal{W}}_{t+1}-\widetilde{\mathcal{W}}_t\right]+ \\
& \left[4a_{\text{H}}\left(1-a_{\text{H}}\right)\left(\sigma\phi-1\right)+1\right]\left[\left(E_t\lambda_{t+1}-\lambda_t\right)-\beta^{-1}\left(\lambda_t-\lambda_{t-1}\right)\right]- \\
& \left(1-a_{\text{H}}\right)\left[2a_{\text{H}}\left(\sigma\phi-1\right)+1\right]\frac{\left(1-\alpha\beta\right)\left(1-\alpha\right)}{\alpha}\cdot \\
& \left[\left(\left(E_t\gamma_{H,t+1}-\gamma_{H,t}\right)\right)\left(\left(E_t\gamma^*_{F,t+1}-\gamma^*_{F,t}\right)\right)\right]
\end{aligned}$$

implying

$$\begin{aligned}
0 \quad = \quad & \left[\left(\beta E_t\lambda_{t+1}-\lambda_t\right)-\left(\beta\lambda_t-\lambda_{t-1}\right)\right]+ \\
& \left(1-a_{\text{H}}\right)\left(\frac{2a_{\text{H}}\left(\sigma\phi-1\right)+1}{4a_{\text{H}}\left(1-a_{\text{H}}\right)\left(\sigma\phi-1\right)+1}\right)\theta\left(\beta E_t\pi_{H,t+1}-\beta E_t\pi^*_{F,t+1}\right).
\end{aligned}$$

As stated in Proposition 3 and already shown above for the LCP case, the solution can be expressed in terms of a familiar sum rule for (the change in) world output gaps and inflation rates:

$$\begin{aligned} 0 &= \widetilde{Y}_{H,t} + \widetilde{Y}_{F,t} + \theta\left(\widehat{p}_{H,t} + \widehat{p}^{*}_{F,t}\right) \\ &= \left[\widetilde{Y}_{H,t} - \widetilde{Y}_{H,t-1}\right] + \left[\widetilde{Y}_{F,t} - \widetilde{Y}_{F,t-1}\right] + \\ &\quad \theta\left[\pi_{H,t} + \pi^{*}_{F,t}\right], \end{aligned}$$

and a difference rule.

Proof of Proposition 3: Difference rule. The difference rule under PCP can be obtained by subtracting the output FOC's to solve for λ_t :

$$\begin{gathered} -2\frac{2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma}{2a_{\mathrm{H}}(\phi-1)+1}\beta^{-1}(\beta\lambda_t-\lambda_{t-1}) = \\ \left[(\sigma+\eta)-\frac{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)\sigma}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\right]\left(\widetilde{Y}_{H,t}-\widetilde{Y}_{F,t}\right) + \\ \frac{4a_{\mathrm{H}}(1-a_{\mathrm{H}})\phi}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\frac{2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma}{2a_{\mathrm{H}}(\phi-1)+1}\widehat{\mathcal{W}}_t + \\ \left[\sigma+\eta-2\frac{(1-a_{\mathrm{H}})(\sigma-1)}{2a_{\mathrm{H}}(\phi-1)+1}\right]\theta\left(\widehat{p}_{H,t}-\widehat{p}^{*}_{F,t}\right). \end{gathered}$$

We can solve for $(\beta\lambda_t - \lambda_{t-1})$ from the first order condition for $\widehat{\mathcal{B}}_t$

$$\begin{gathered} 0 = \left[(\beta E_t\lambda_{t+1}-\lambda_t)-(\beta\lambda_t-\lambda_{t-1})\right] + \\ (1-a_{\mathrm{H}})\left(\frac{2a_{\mathrm{H}}(\sigma\phi-1)+1}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\right)\theta\beta\left(E_t\pi_{H,t+1}-E_t\pi^{*}_{F,t+1}\right), \end{gathered}$$

$$\begin{aligned} -\left[E_t(\beta\lambda_{t+1}-\lambda_t)-(\beta\lambda_t-\lambda_{t-1})\right] &= \\ & (1-a_{\mathrm{H}})\left(\frac{2a_{\mathrm{H}}(\sigma\phi-1)+1}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\right)\theta\beta E_t\left[\begin{array}{c}(\widehat{p}_{H,t+1}-\widehat{p}_{H,t}) \\ -(\widehat{p}^{*}_{F,t+1}-\widehat{p}^{*}_{F,t})\end{array}\right]. \end{aligned}$$

A solution to the above equation is given by the following:

$$-(\beta\lambda_t-\lambda_{t-1}) = (1-a_{\mathrm{H}})\left(\frac{2a_{\mathrm{H}}(\sigma\phi-1)+1}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\right)\theta\beta\left(\widehat{p}_{H,t}-\widehat{p}^{*}_{F,t}\right).$$

Effectively this assumes that the growth rate in the (quasi-change $(\beta\lambda_t - \lambda_{t\ \ 1})$ of the) Lagrange multiplier of relative wealth depends on contemporaneous shocks only via their effects on inflation differentials.

In turn, this implies the following difference rule:

$$\begin{aligned} 0 &= \left[(\sigma+\eta)-\frac{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)\sigma}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\right]\left(\widetilde{Y}_{H,t}-\widetilde{Y}_{F,t}\right) + \\ &\quad \frac{4a_{\mathrm{H}}(1-a_{\mathrm{H}})\phi}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\frac{2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma}{2a_{\mathrm{H}}(\phi-1)+1}\widehat{\mathcal{W}}_t + \\ &\quad \left[\sigma+\eta-\frac{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)\sigma}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\right]\theta\left(\widehat{p}_{H,t}-\widehat{p}^{*}_{F,t}\right). \end{aligned}$$

Therefore, in terms of inflation rates and growth rates the "difference" rule is the following:

$$0 = \left[(\sigma+\eta) - \frac{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)(\sigma\phi-1)\sigma}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)(\sigma\phi-1)+1}\right]\left\{\begin{array}{c}\left(\widetilde{Y}_{H,t}-\widetilde{Y}_{H,t-1}\right)-\left(\widetilde{Y}_{F,t}-\widetilde{Y}_{F,t-1}\right)+ \\ \theta\left(\pi_{H,t}-\pi^*_{F,t}\right)\end{array}\right\}+$$
$$\frac{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\phi}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)(\sigma\phi-1)+1}\frac{2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma}{2a_{\mathrm{H}}(\phi-1)+1}\left(\widehat{\mathcal{W}}_t-\widehat{\mathcal{W}}_{t-1}\right).$$

This complete the proof of Proposition 6.

2.2.2 Solving explicitly for the constrained optimal allocation under PCP

The targeting rule can thus be written:

$$0 = \left[\eta + \frac{\sigma}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)(\sigma\phi-1)+1}\right]\left[\left(\widetilde{Y}_{H,t}-\widetilde{Y}_{H,t-1}\right)+\theta\pi_{H,t}\right]+$$
$$\frac{2a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\phi}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)(\sigma\phi-1)+1}\frac{2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma}{2a_{\mathrm{H}}(\phi-1)+1}\left(\widehat{\mathcal{W}}_t-\widehat{\mathcal{W}}_{t-1}\right).$$

Using it to solve for inflation and substituting into the Phillips curve:

$$\theta\pi_{H,t} = -\left(\widetilde{Y}_{H,t}-\widetilde{Y}_{H,t-1}\right)+$$
$$-\frac{2a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\phi}{\eta\left[4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)(\sigma\phi-1)+1\right]+\sigma}\frac{2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma}{2a_{\mathrm{H}}(\phi-1)+1}\left(\widehat{\mathcal{W}}_t-\widehat{\mathcal{W}}_{t-1}\right),$$

and recalling the following relation for $\widehat{\mathcal{W}}_t$:

$$\left(1-a_{\mathrm{H}}\right)\left[\frac{2a_{\mathrm{H}}(\phi-1)+1}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)(\sigma\phi-1)+1}\right]\widehat{\mathcal{W}}_t = -\beta^{-1}\left(\beta\widehat{\mathcal{B}}_t-\widehat{\mathcal{B}}_{t-1}\right)+$$
$$\left(1-a_{\mathrm{H}}\right)\left[\frac{2a_{\mathrm{H}}(\sigma\phi-1)-(\sigma-1)}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)(\sigma\phi-1)+1}\right]\left[2\widetilde{Y}_{H,t}+\left(\widehat{Y}^{fb}_{H,t}-\widehat{Y}^{fb}_{F,t}\right)\right]-$$
$$\left(1-a_{\mathrm{H}}\right)\left[\frac{2a_{\mathrm{H}}(\phi-1)+1}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)(\sigma\phi-1)+1}\right]\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}^*_{C,t}\right)$$

we obtain the following system of difference equations in $\widetilde{Y}_{H,t}$ and $\widehat{\mathcal{B}}_t$:

$$\beta^{-1}\left[E_t\left(\beta\widehat{\mathcal{B}}_{t+1}-\widehat{\mathcal{B}}_t\right)-\left(\beta\widehat{\mathcal{B}}_t-\widehat{\mathcal{B}}_{t-1}\right)\right]-$$
$$2\left(1-a_{\mathrm{H}}\right)\left[\frac{2a_{\mathrm{H}}(\sigma\phi-1)-(\sigma-1)}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)(\sigma\phi-1)+1}\right]\left[E_t\left(\widetilde{Y}_{H,t+1}-\widetilde{Y}_{H,t}\right)\right]=$$
$$\left(1-a_{\mathrm{H}}\right)\left[\frac{2a_{\mathrm{H}}(\sigma\phi-1)-(\sigma-1)}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)(\sigma\phi-1)+1}\right]\left[E_t\left(\widehat{Y}^{fb}_{H,t+1}-\widehat{Y}^{fb}_{F,t+1}\right)-\left(\widehat{Y}^{fb}_{H,t}-\widehat{Y}^{fb}_{F,t}\right)\right]-$$
$$\left(1-a_{\mathrm{H}}\right)\left[\frac{2a_{\mathrm{H}}(\phi-1)+1}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)(\sigma\phi-1)+1}\right]\left[E_t\left(\widehat{\zeta}_{C,t+1}-\widehat{\zeta}^*_{C,t+1}\right)-\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}^*_{C,t}\right)\right],$$

and,

$$\left\{ \begin{array}{c} -\left[\eta+\frac{\sigma}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\right]\left[\widetilde{Y}_{H,t}-\widetilde{Y}_{H,t-1}\right]- \\ \frac{2a_{\mathrm{H}}(1-a_{\mathrm{H}})\phi}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\frac{2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma}{2a_{\mathrm{H}}(\phi-1)+1}\left(\widehat{\mathcal{W}}_t-\widehat{\mathcal{W}}_{t-1}\right) \end{array} \right\}$$

$$+\beta\left[\eta+\frac{\sigma}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\right]E_t\left[\widetilde{Y}_{H,t+1}-\widetilde{Y}_{H,t}\right]=$$

$$\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\theta\left[\eta+\frac{\sigma}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\right]^2\widetilde{Y}_{H,t}$$

$$+\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\left(1-a_{\mathrm{H}}\right)\theta\left[\frac{2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\right]\cdot$$

$$\left[\eta+\frac{\sigma}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\right]\widehat{\mathcal{W}}_t.$$

We use the method of undetermined coefficients to solve this system, exploiting the martingale nature of the variable $\widehat{\mathcal{W}}_t$, namely $E_t\widehat{\mathcal{W}}_{t+j}=\widehat{\mathcal{W}}_t$.

Rearranging the last difference equation for the output gap as follows:

$$\beta E_t\left[\widetilde{Y}_{H,t+1}-\widetilde{Y}_{H,t}\right]-\left[\widetilde{Y}_{H,t}-\widetilde{Y}_{H,t-1}\right]-$$

$$\left[\eta+\frac{\sigma}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\right]\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\theta\widetilde{Y}_{H,t}$$

$$=\frac{2a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\phi}{\eta\left[4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1\right]+\sigma}\frac{2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1-\sigma}{2a_{\mathrm{H}}\left(\phi-1\right)+1}\left(\widehat{\mathcal{W}}_t-\widehat{\mathcal{W}}_{t-1}\right)+$$

$$\left(1-a_{\mathrm{H}}\right)\frac{2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\theta\widehat{\mathcal{W}}_t,$$

we can solve for $\widetilde{Y}_{H,t}$ as function of current and future values of $\widehat{\mathcal{W}}_t$:

$$\widetilde{Y}_{H,t}-\delta_1\widetilde{Y}_{H,t-1} = -\left[\begin{array}{c} \left(1-a_{\mathrm{H}}\right)\frac{2a_{\mathrm{H}}\phi}{\eta[4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1]+\sigma}\frac{2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma}{2a_{\mathrm{H}}(\phi-1)+1}\frac{1}{\beta\delta_2}\cdot \\ \sum_{j=0}^{\infty}\delta_2^{-j}E_t\left(\widehat{\mathcal{W}}_{t+j}-\widehat{\mathcal{W}}_{t+j-1}\right) \end{array} \right]$$

$$-\left(1-a_{\mathrm{H}}\right)\frac{2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\frac{\theta}{\beta\delta_2}\sum_{j=0}^{\infty}\delta_2^{-j}E_t\widehat{\mathcal{W}}_{t+j}.$$

where $0<\delta_1<1<\beta^{-1}<\delta_2$ are the eigenvalues of the difference equation, solving the standard characteristic equation:

$$\beta\delta^2-\left\{1+\beta+\left[\eta+\frac{\sigma}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\right]\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\theta\right\}\delta+1=0,$$

namely,

$$\delta = \frac{1}{2\beta}\left(1+\beta+\left[\eta+\frac{\sigma}{4a_{\rm H}\left(1-a_{\rm H}\right)\left(\sigma\phi-1\right)+1}\right]\frac{\left(1-\alpha\beta\right)\left(1-\alpha\right)}{\alpha}\theta\right) \pm$$

$$\frac{1}{2\beta}\sqrt{\left\{1+\beta+\left[\eta+\frac{\sigma}{4a_{\rm H}\left(1-a_{\rm H}\right)\left(\sigma\phi-1\right)+1}\right]\frac{\left(1-\alpha\beta\right)\left(1-\alpha\right)}{\alpha}\theta\right\}^2-4\beta}.$$

Observe that for $\sigma=\phi=1$ and $\eta=0$ these eigenvalues are the same derived above under LCP in Section 3.1.4 and denoted with $\varkappa_{1,2}$.

We can simplify the above solution which is solely a function of $\widehat{\mathcal{W}}_t$, as $E_t\widehat{\mathcal{W}}_{t+j}=\widehat{\mathcal{W}}_t$:

$$\left(\widehat{Y}_{H,t+j}-\widehat{Y}^{fb}_{H,t+j}\right) =$$

$$\delta_1\left(\widehat{Y}_{H,t+j-1}-\widehat{Y}^{fb}_{H,t+j-1}\right) -$$

$$\left(1-a_{\rm H}\right)\frac{2a_{\rm H}\phi}{\eta\left[4a_{\rm H}\left(1-a_{\rm H}\right)\left(\sigma\phi-1\right)+1\right]+\sigma}\frac{2a_{\rm H}\left(\sigma\phi-1\right)+1-\sigma}{2a_{\rm H}\left(\phi-1\right)+1}\frac{1}{\beta\delta_2}E_t\left(\widehat{\mathcal{W}}_{t+j}-\widehat{\mathcal{W}}_{t+j-1}\right) -$$

$$\left(1-a_{\rm H}\right)\frac{2a_{\rm H}\left(\sigma\phi-1\right)+1}{4a_{\rm H}\left(1-a_{\rm H}\right)\left(\sigma\phi-1\right)+1}\frac{\left(1-\alpha\beta\right)\left(1-\alpha\right)}{\alpha}\frac{\theta}{\beta\left(\delta_2-1\right)}\widehat{\mathcal{W}}_t;$$

and we have that

$$\frac{\left(1-\alpha\beta\right)\left(1-\alpha\right)}{\alpha}\theta\left[\eta+\frac{\sigma}{4a_{\rm H}\left(1-a_{\rm H}\right)\left(\sigma\phi-1\right)+1}\right]=\frac{\left(\delta_2-1\right)\left(\beta\delta_2-1\right)}{\delta_2}$$

Furthermore,

$$E_t\widetilde{Y}_{H,t+s} =$$

$$\delta_1\left[\left(\widehat{Y}_{H,t+s-1}-\widehat{Y}^{fb}_{H,t+s-1}\right)\right]-\left(1-a_{\rm H}\right)\frac{2a_{\rm H}\left(\sigma\phi-1\right)+1}{\eta\left[4a_{\rm H}\left(1-a_{\rm H}\right)\left(\sigma\phi-1\right)+1\right]+\sigma}\frac{\left(\beta\delta_2-1\right)}{\beta\delta_2}\widehat{\mathcal{W}}_t +$$

$$-\left(1-a_{\rm H}\right)\frac{2a_{\rm H}\phi}{\eta\left[4a_{\rm H}\left(1-a_{\rm H}\right)\left(\sigma\phi-1\right)+1\right]+\sigma}\frac{2a_{\rm H}\left(\sigma\phi-1\right)+1-\sigma}{2a_{\rm H}\left(\phi-1\right)+1}\frac{1}{\beta\delta_2}E_t\left(\widehat{\mathcal{W}}_{t+s}-\widehat{\mathcal{W}}_{t+s-1}\right)$$

Notice that the second term $E_t\left(\widehat{\mathcal{W}}_{t+j}-\widehat{\mathcal{W}}_{t+j-1}\right)=0$ for $j\geq 1$, while it is equal to $\widehat{\mathcal{W}}_t$ for $j=0$. The last term represents a constant shifter proportional to $\widehat{\mathcal{W}}_t$ for any $j\geq 0$.

We can compare the above with the allocation under $\pi_{H,t}=\pi_{F,t}=0$, characterized as follows:

$$\left(\widehat{\mathcal{T}}_t-\widehat{\mathcal{T}}^{fb}_t\right) = -\frac{\sigma+\left(2a_{\rm H}-1\right)\eta}{\eta\left[4a_{\rm H}\left(1-a_{\rm H}\right)\left(\sigma\phi-1\right)+1\right]+\sigma}\widehat{\mathcal{W}}_t$$

$$\left(\widehat{Y}_{H,t}-\widehat{Y}^{fb}_{H,t}\right) = -\left(1-a_{\rm H}\right)\frac{1+2a_{\rm H}\left(\sigma\phi-1\right)}{\eta\left[4a_{\rm H}\left(1-a_{\rm H}\right)\left(\sigma\phi-1\right)+1\right]+\sigma}\widehat{\mathcal{W}}_t.$$

We can also solve for inflation using the targeting rule:

$$\theta\pi_{H,t} = -\left[\left(\widehat{Y}_{H,t} - \widehat{Y}^{fb}_{H,t}\right) - \left(\widehat{Y}_{H,t-1} - \widehat{Y}^{fb}_{H,t-1}\right)\right] +$$

$$-\frac{2a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\phi}{\eta\left[4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1\right]+\sigma}\frac{2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1-\sigma}{2a_{\mathrm{H}}\left(\phi-1\right)+1}\left(\widehat{\mathcal{W}}_t - \widehat{\mathcal{W}}_{t-1}\right),$$

which implies:

$$\theta E_t\pi_{H,t+j} = \left(1-\delta_1\right)\left(\widehat{Y}_{H,t+j-1} - \widehat{Y}^{fb}_{H,t+j-1}\right) +$$

$$\left(1-a_{\mathrm{H}}\right)\frac{2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\frac{\left(1-\alpha\beta\right)\left(1-\alpha\right)}{\alpha}\frac{\theta}{\beta\left(\delta_2-1\right)}\widehat{\mathcal{W}}_t +$$

$$-\frac{2a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\phi}{\eta\left[4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1\right]+\sigma}\frac{2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1-\sigma}{2a_{\mathrm{H}}\left(\phi-1\right)+1}\left(\frac{\beta\delta_2-1}{\beta\delta_2}\right)E_t\left(\widehat{\mathcal{W}}_{t+j} - \widehat{\mathcal{W}}_{t+j-1}\right).$$

Likewise, armed with the above solution for $\left(\widehat{Y}_{H,t+j} - \widehat{Y}^{fb}_{H,t+j}\right)$, we can solve the following difference equation for $\widehat{\mathcal{B}}_t$:

$$\beta E_t\left(\widehat{\mathcal{B}}_{t+1} - \widehat{\mathcal{B}}_t\right) - \left(\widehat{\mathcal{B}}_t - \widehat{\mathcal{B}}_{t-1}\right) =$$

$$2\left(1-a_{\mathrm{H}}\right)\left[\frac{2a_{\mathrm{H}}\left(\sigma\phi-1\right)-\left(\sigma-1\right)}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\right]\beta\left[E_t\widetilde{Y}_{H,t+1} - \widetilde{Y}_{H,t}\right] +$$

$$\left(1-a_{\mathrm{H}}\right)\left[\frac{2a_{\mathrm{H}}\left(\sigma\phi-1\right)-\left(\sigma-1\right)}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\right]\beta\left[E_t\left(\widehat{Y}^{fb}_{H,t+1} - \widehat{Y}^{fb}_{F,t+1}\right) - \left(\widehat{Y}^{fb}_{H,t} - \widehat{Y}^{fb}_{F,t}\right)\right] +$$

$$-\left(1-a_{\mathrm{H}}\right)\left[\frac{2a_{\mathrm{H}}\left(\phi-1\right)+1}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\right]\beta\left[E_t\left(\widehat{\zeta}_{C,t+1} - \widehat{\zeta}^*_{C,t+1}\right) - \left(\widehat{\zeta}_{C,t} - \widehat{\zeta}^*_{C,t}\right)\right].$$

The eigenvalues of this difference equation are 1 and $1/\beta$, yielding the following standard solution:

$$\widehat{\mathcal{B}}_t = \left[\begin{array}{c}\widehat{\mathcal{B}}_{t-1} - 2\left(1-a_{\mathrm{H}}\right)\left[\frac{2a_{\mathrm{H}}\left(\sigma\phi-1\right)-\left(\sigma-1\right)}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\right]\beta \\ \sum_{j=0}^{\infty}\beta^j E_t\left[\left(\widehat{Y}_{H,t+j+1} - \widehat{Y}^{fb}_{H,t+j+1}\right) - \left(\widehat{Y}_{H,t+j} - \widehat{Y}^{fb}_{H,t+j}\right)\right]\end{array}\right] -$$

$$\left(1-a_{\mathrm{H}}\right)\left[\frac{2a_{\mathrm{H}}\left(\sigma\phi-1\right)-\left(\sigma-1\right)}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\right]\beta\sum_{j=0}^{\infty}\beta^j E_t\left[\left(\widehat{Y}^{fb}_{H,t+j+1} - \widehat{Y}^{fb}_{F,t+j+1}\right) - \left(\widehat{Y}^{fb}_{H,t+j} - \widehat{Y}^{fb}_{F,t+j}\right)\right] -$$

$$\left(1-a_{\mathrm{H}}\right)\left[\frac{2a_{\mathrm{H}}\left(\phi-1\right)+1}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\right]\beta\sum_{j=0}^{\infty}\beta^j E_t\left[\left(\widehat{\zeta}_{C,t+j+1} - \widehat{\zeta}^*_{C,t+j+1}\right) - \left(\widehat{\zeta}_{C,t+j} - \widehat{\zeta}^*_{C,t+j}\right)\right].$$

Using the above solution for the output gap, we have that for $j \geq 0$:

$$E_t\left[\left(\widehat{Y}_{H,t+j+1} - \widehat{Y}^{fb}_{H,t+j+1}\right) - \left(\widehat{Y}_{H,t+j} - \widehat{Y}^{fb}_{H,t+j}\right)\right] =$$

$$-\left(1-\delta_1\right)E_t\left(\widehat{Y}_{H,t+j} - \widehat{Y}^{fb}_{H,t+j}\right) +$$

$$-\left(1-a_{\mathrm{H}}\right)\frac{2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\frac{\left(1-\alpha\beta\right)\left(1-\alpha\right)}{\alpha}\frac{\theta}{\beta\left(\delta_2-1\right)}\widehat{\mathcal{W}}_t,$$

where

$$E_t\left(\widehat{Y}_{H,t+j}-\widehat{Y}^{fb}_{H,t+j}\right)=$$

$$\delta_1^j\left[\delta_1\left(\widehat{Y}_{H,t-1}-\widehat{Y}^{fb}_{H,t-1}\right)-\frac{2a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\phi}{\eta\left[4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1\right]+\sigma}\frac{2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1-\sigma}{2a_{\mathrm{H}}\left(\phi-1\right)+1}\frac{1}{\beta\delta_2}\widehat{\mathcal{W}}_t\right]-$$

$$\left(1-a_{\mathrm{H}}\right)\frac{2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\frac{\left(1-\alpha\beta\right)\left(1-\alpha\right)}{\alpha}\frac{\theta}{\beta\left(\delta_2-1\right)}\sum_{s=0}^{j}\delta_1^s\widehat{\mathcal{W}}_t,$$

which also implies that:

$$E_t\left[\left(\widehat{Y}_{H,t+j+1}-\widehat{Y}^{fb}_{H,t+j+1}\right)-\left(\widehat{Y}_{H,t+j}-\widehat{Y}^{fb}_{H,t+j}\right)\right]=$$

$$-\left(1-\delta_1\right)\delta_1^j\left\{\delta_1\left(\widehat{Y}_{H,t-1}-\widehat{Y}^{fb}_{H,t-1}\right)\right\}$$

$$-\left(1-\delta_1\right)\delta_1^j\left\{\frac{2a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\phi}{\eta\left[4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1\right]+\sigma}\frac{2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1-\sigma}{2a_{\mathrm{H}}\left(\phi-1\right)+1}\frac{1}{\beta\delta_2}\widehat{\mathcal{W}}_t\right\}+$$

$$\left(1-a_{\mathrm{H}}\right)\frac{2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\frac{\left(1-\alpha\beta\right)\left(1-\alpha\right)}{\alpha}\frac{\theta}{\beta\left(\delta_2-1\right)}\left[\left(1-\delta_1\right)\frac{1-\delta_1^{j+1}}{1-\delta_1}-1\right]\widehat{\mathcal{W}}_t.$$

As a result we have that:

$$\beta\sum_{j=0}^{\infty}\beta^jE_t\left[\left(\widehat{Y}_{H,t+j+1}-\widehat{Y}^{fb}_{H,t+j+1}\right)-\left(\widehat{Y}_{H,t+j}-\widehat{Y}^{fb}_{H,t+j}\right)\right]=$$

$$-\left(1-\delta_1\right)\beta\sum_{j=0}^{\infty}\beta^j\delta_1^j\left[\begin{array}{c}\delta_1\left(\widehat{Y}_{H,t-1}-\widehat{Y}^{fb}_{H,t-1}\right)-\\ \frac{2a_{\mathrm{H}}(1-a_{\mathrm{H}})\phi}{\eta[4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1]+\sigma}\frac{2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma}{2a_{\mathrm{H}}(\phi-1)+1}\frac{1}{\beta\delta_2}\widehat{\mathcal{W}}_t\end{array}\right]-$$

$$\left(1-a_{\mathrm{H}}\right)\frac{2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\frac{\left(1-\alpha\beta\right)\left(1-\alpha\right)}{\alpha}\frac{\theta}{\beta\left(\delta_2-1\right)}\sum_{j=0}^{\infty}\beta^{j+1}\delta_1^{j+1}\widehat{\mathcal{W}}_t$$

$$=-\frac{\left(1-\delta_1\right)\beta}{1-\beta\delta_1}\left[\begin{array}{c}\delta_1\left(\widehat{Y}_{H,t-1}-\widehat{Y}^{fb}_{H,t-1}\right)-\\ \frac{2a_{\mathrm{H}}(1-a_{\mathrm{H}})\phi}{\eta[4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1]+\sigma}\frac{2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma}{2a_{\mathrm{H}}(\phi-1)+1}\frac{1}{\beta\delta_2}\widehat{\mathcal{W}}_t\end{array}\right]-$$

$$\left(1-a_{\mathrm{H}}\right)\frac{2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1}{\eta\left[4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1\right]+\sigma}\frac{\beta\delta_2-1}{\beta\delta_2}\frac{\beta\delta_1}{1-\beta\delta_1}\widehat{\mathcal{W}}_t$$

Therefore the solution for NFA is the following:

$$\widehat{\mathcal{B}}_t = \widehat{\mathcal{B}}_{t-1} + 2\left(1-a_{\rm H}\right)\left[\frac{2a_{\rm H}\left(\sigma\phi-1\right)-\left(\sigma-1\right)}{4a_{\rm H}\left(1-a_{\rm H}\right)\left(\sigma\phi-1\right)+1}\right]\frac{\beta}{1-\beta\delta_1}\cdot$$

$$\left\{\begin{array}{c}\left(1-\delta_1\right)\delta_1\left(\widehat{Y}_{H,t-1}-\widehat{Y}^{fb}_{H,t-1}\right)-\left(1-a_{\rm H}\right)\frac{2a_{\rm H}\phi}{\eta[4a_{\rm H}(1-a_{\rm H})(\sigma\phi-1)+1]+\sigma}\frac{2a_{\rm H}(\sigma\phi-1)+1-\sigma}{2a_{\rm H}(\phi-1)+1}\frac{(1-\delta_1)}{\beta\delta_2}\widehat{\mathcal{W}}_t+\\ \left(1-a_{\rm H}\right)\delta_1\frac{2a_{\rm H}(\sigma\phi-1)+1}{\eta[4a_{\rm H}(1-a_{\rm H})(\sigma\phi-1)+1]+\sigma}\frac{\beta\delta_2-1}{\beta\delta_2}\widehat{\mathcal{W}}_t\end{array}\right\}-$$

$$\left(1-a_{\rm H}\right)\left[\frac{2a_{\rm H}\left(\sigma\phi-1\right)-\left(\sigma-1\right)}{4a_{\rm H}\left(1-a_{\rm H}\right)\left(\sigma\phi-1\right)+1}\right]\beta\sum_{j=0}^{\infty}\beta^j E_t\left[\left(\widehat{Y}^{fb}_{H,t+j+1}-\widehat{Y}^{fb}_{F,t+j+1}\right)-\left(\widehat{Y}^{fb}_{H,t+j}-\widehat{Y}^{fb}_{F,t+j}\right)\right]+$$

$$\left(1-a_{\rm H}\right)\left[\frac{2a_{\rm H}\left(\phi-1\right)+1}{4a_{\rm H}\left(1-a_{\rm H}\right)\left(\sigma\phi-1\right)+1}\right]\beta\sum_{j=0}^{\infty}\beta^j E_t\left[\left(\widehat{\zeta}_{C,t+j+1}-\widehat{\zeta}^*_{C,t+j+1}\right)-\left(\widehat{\zeta}_{C,t+j}-\widehat{\zeta}^*_{C,t+j}\right)\right].$$

Finally, recalling the following relation for $\widehat{\mathcal{W}}_t$:

$$\left(1-a_{\rm H}\right)\left[\frac{2a_{\rm H}\left(\phi-1\right)+1}{4a_{\rm H}\left(1-a_{\rm H}\right)\left(\sigma\phi-1\right)+1}\right]\widehat{\mathcal{W}}_t = -\beta^{-1}\left(\beta\widehat{\mathcal{B}}_t-\widehat{\mathcal{B}}_{t-1}\right)+$$

$$\left(1-a_{\rm H}\right)\left[\frac{2a_{\rm H}\left(\sigma\phi-1\right)-\left(\sigma-1\right)}{4a_{\rm H}\left(1-a_{\rm H}\right)\left(\sigma\phi-1\right)+1}\right]\left[2\left(\widehat{Y}_{H,t}-\widehat{Y}^{fb}_{H,t}\right)+\left(\widehat{Y}^{fb}_{H,t}-\widehat{Y}^{fb}_{F,t}\right)\right]-$$

$$\left(1-a_{\rm H}\right)\left[\frac{2a_{\rm H}\left(\phi-1\right)+1}{4a_{\rm H}\left(1-a_{\rm H}\right)\left(\sigma\phi-1\right)+1}\right]\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}^*_{C,t}\right),$$

we can solve for the impact response on $\widetilde{\mathcal{W}}_t$ for $j=0$ as a function only of exogenous shocks. The permanent response of the wealth gap under the optimal policy is given by:

$$\widetilde{\mathcal{W}}_t\left[2a_{\rm H}\left(\phi-1\right)+1+\frac{2\left(1-a_{\rm H}\right)\left[2a_{\rm H}\left(\sigma\phi-1\right)-\left(\sigma-1\right)\right]}{\eta\left[4a_{\rm H}\left(1-a_{\rm H}\right)\left(\sigma\phi-1\right)+1\right]+\sigma}\frac{1}{\beta\delta_2\left(1-\beta\delta_1\right)}\cdot\left(\begin{array}{c}2a_{\rm H}\phi\frac{2a_{\rm H}(\sigma\phi-1)+1-\sigma}{2a_{\rm H}(\phi-1)+1}\left(1-\beta\right)+\\ \left[2a_{\rm H}\left(\sigma\phi-1\right)+1\right]\left(\beta\delta_2-1\right)\end{array}\right)\right]=$$

$$\left[2a_{\rm H}\left(\sigma\phi-1\right)+1-\sigma\right]\beta\sum_{j=0}^{\infty}\beta^j E_t\left[\left(\widehat{Y}^{fb}_{H,t+j+1}-\widehat{Y}^{fb}_{F,t+j+1}\right)-\left(\widehat{Y}^{fb}_{H,t+j}-\widehat{Y}^{fb}_{F,t+j}\right)\right]+$$

$$-\left[2a_{\rm H}\left(\sigma\phi-1\right)+1\right]\beta\sum_{j=0}^{\infty}\beta^j E_t\left[\left(\widehat{\zeta}_{C,t+j+1}-\widehat{\zeta}^*_{C,t+j+1}\right)-\left(\widehat{\zeta}_{C,t+j}-\widehat{\zeta}^*_{C,t+j}\right)\right]+$$

$$\left[2a_{\rm H}\left(\sigma\phi-1\right)+1-\sigma\right]\left(\widehat{Y}^{fb}_{H,t}-\widehat{Y}^{fb}_{F,t}\right)-\left[2a_{\rm H}\left(\sigma\phi-1\right)+1\right]\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}^*_{C,t}\right).$$

Similarly, we can derive the response of NFAs as a function of exogenous shocks.

2.2.3 Comparison with strict PPI price stability and full characterization of the optimal allocation for $\phi \geq 0$

Under PPI price stability the output gap generally obeys the following relation,

$$\left(\widehat{Y}_{H,t}-\widehat{Y}^{fb}_{H,t}\right)=-\left(1-a_{\rm H}\right)\frac{1+2a_{\rm H}\left(\sigma\phi-1\right)}{\eta\left[4a_{\rm H}\left(1-a_{\rm H}\right)\left(\sigma\phi-1\right)+1\right]+\sigma}\widehat{\mathcal{W}}_t,$$

and capital flows are given by:

$$\begin{aligned}\widehat{\mathcal{B}}_t &= \widehat{\mathcal{B}}_{t-1} - \frac{(1-a_{\mathrm{H}})}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\beta\sum_{j=0}^{\infty}\beta^j \\ &\left\{\begin{array}{l}(2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma)E_t\left(\left(\widehat{Y}^{fb}_{H,t+j+1}-\widehat{Y}^{fb}_{F,t+j+1}\right)-\left(\widehat{Y}^{fb}_{H,t+j}-\widehat{Y}^{fb}_{F,t+j}\right)\right)+ \\ \quad -(2a_{\mathrm{H}}(\phi-1)+1)E_t\left(\left(\widehat{\zeta}_{C,t+1+j}-\widehat{\zeta}^*_{C,t+1+j}\right)-\left(\widehat{\zeta}_{C,t+j}-\widehat{\zeta}^*_{C,t+j}\right)\right)\end{array}\right\}.\end{aligned}$$

As a result, the wealth gap is given by

$$\left[[2a_{\mathrm{H}}(\phi-1)+1]+2(1-a_{\mathrm{H}})\frac{(2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma)([2a_{\mathrm{H}}(\sigma\phi-1)+1])}{\eta[4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1]+\sigma}\right]\widehat{\mathcal{W}}_t =$$

$$\beta\sum_{j=0}^{\infty}\beta^j\left[\begin{array}{l}(2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma)E_t\left(\left(\widehat{Y}^{fb}_{H,t+j+1}-\widehat{Y}^{fb}_{F,t+j+1}\right)-\left(\widehat{Y}^{fb}_{H,t+j}-\widehat{Y}^{fb}_{F,t+j}\right)\right)+ \\ \quad -(2a_{\mathrm{H}}(\phi-1)+1)E_t\left(\left(\widehat{\zeta}_{C,t+1+j}-\widehat{\zeta}^*_{C,t+1+j}\right)-\left(\widehat{\zeta}_{C,t+j}-\widehat{\zeta}^*_{C,t+j}\right)\right)\end{array}\right]$$

$$[2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma]\left(\widehat{Y}^{fb}_{H,t}-\widehat{Y}^{fb}_{F,t}\right)-[2a_{\mathrm{H}}(\sigma\phi-1)+1]\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}^*_{C,t}\right).$$

For comparability with the LCP case, from now on we will continue to impose $\eta=0$ and $\sigma=1$, although these restrictions are not necessary to obtain tractable expressions as shown above. Similarly, we also focus on "news shocks" only.

The following Lemma characterizes how capital flows responds to shocks under the optimal policy in comparison with the natural allocation. A remarkable result is that the optimal policy will reduce the relative size of these flows for elasticities above unity; but amplify the relative size for elasticities below unity.

Lemma A1. *For $\sigma=1,\eta=0,\phi\geq 0$, capital inflows in the constrained-efficient allocation are given by the following expression:*

$$\begin{aligned}&\widehat{\mathcal{B}}_t=\widehat{\mathcal{B}}_{t-1}+ \\ &-\tfrac{(1-a_{\mathrm{H}})}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\phi-1)+1}\cdot\mathbf{B}\beta\textstyle\sum_{j=0}^{\infty}\beta^j\left[\begin{array}{l}2a_{\mathrm{H}}(\phi-1)E_t\left(\left(\widehat{Y}^{fb}_{H,t+j+1}-\widehat{Y}^{fb}_{F,t+j+1}\right)-\left(\widehat{Y}^{fb}_{H,t+j}-\widehat{Y}^{fb}_{F,t+j}\right)\right)+ \\ -(2a_{\mathrm{H}}(\phi-1)+1)E_t\left(\left(\widehat{\zeta}_{C,t+1+j}-\widehat{\zeta}^*_{C,t+1+j}\right)-\left(\widehat{\zeta}_{C,t+j}-\widehat{\zeta}^*_{C,t+j}\right)\right)\end{array}\right]+ \\ &\qquad 2(1-a_{\mathrm{H}})\left[\tfrac{2a_{\mathrm{H}}(\phi-1)}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\phi-1)+1}\right]\tfrac{1-\delta_1}{1-\beta\delta_1}\beta\delta_1\widetilde{Y}_{H,t-1}.\end{aligned}$$

where

$$\mathbf{B}=\left[1-\tfrac{1-\delta_1}{\delta_2-1}\tfrac{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\phi-1)}{[2a_{\mathrm{H}}(\phi-1)+1]^2}\tfrac{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\phi-1)+1}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\phi-1)+1+4a_{\mathrm{H}}(1-a_{\mathrm{H}})\phi\frac{4a_{\mathrm{H}}^2(\phi-1)^2}{[2a_{\mathrm{H}}(\phi-1)+1]^2}\frac{(1-\beta)}{\beta(\delta_2-1)}}\right]\geq 0,$$

The sign of capital flows is the same in the constrained-efficient allocation as in the natural rate allocation; however, capital flows are smaller in the constrained-efficient allocation for $\phi>1$, and greater for $1>\phi\geq 0$.

Proof. Constrained-efficient capital flows on impact are obtained in the above expression by setting $\widehat{\mathcal{B}}_{t-1}=\widetilde{Y}_{H,t-1}=0$, noting that $0<\mathbf{B}<1$ for $\phi>1$.

The lemma follows from the fact that the impact response of capital flows in the natural rate allocation is given by the same expression in the proposition but for setting **B**=1 for the case $\eta = 0$ and $\sigma = 1$:

$$\widehat{\mathcal{B}}_t = \widehat{\mathcal{B}}_{t-1} - \frac{(1-a_{\mathrm{H}})}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\phi-1)+1}\beta\sum_{j=0}^{\infty}\beta^j$$

$$\left\{\begin{array}{c} (2a_{\mathrm{H}}(\phi-1))\,E_t\left(\left(\widehat{Y}^{fb}_{H,t+j+1}-\widehat{Y}^{fb}_{F,t+j+1}\right)-\left(\widehat{Y}^{fb}_{H,t+j}-\widehat{Y}^{fb}_{F,t+j}\right)\right)+ \\ -(2a_{\mathrm{H}}(\phi-1)+1)\,E_t\left(\left(\widehat{\zeta}_{C,t+1+j}-\widehat{\zeta}^{*}_{C,t+1+j}\right)-\left(\widehat{\zeta}_{C,t+j}-\widehat{\zeta}^{*}_{C,t+j}\right)\right) \end{array}\right\}.$$

Relative to PPI price stability, expected shocks are now multiplied by the term

$$-\frac{(1-a_{\mathrm{H}})}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\phi-1)+1}\left[\left[\,1-\frac{1}{(1-\beta\delta_1)\,\delta_2}\cdot\right.\right.$$

$$\left.\frac{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\phi-1)\left\{4a_{\mathrm{H}}^2(\phi-1)\left[(\phi-1)(\beta\delta_2\delta_1-1)-(1-\delta_1)\right]+(1+4a_{\mathrm{H}}(\phi-1))(\beta\delta_2-1)\,\delta_1\right\}}{\left[2a_{\mathrm{H}}(\phi-1)+1\right]^2\left[4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\phi-1)+1+4a_{\mathrm{H}}(1-a_{\mathrm{H}})\,\phi\frac{4a_{\mathrm{H}}^2(\phi-1)^2}{[2a_{\mathrm{H}}(\phi-1)+1]^2}\frac{(1-\beta)}{\beta\delta_2(1-\beta\delta_1)}\right]}\right];$$

since $\beta\delta_2\delta_1 = 1$ the above further simplifies:

$$-\frac{(1-a_{\mathrm{H}})}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\phi-1)+1}\left[1-\frac{1-\delta_1}{\delta_2-1}\frac{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\phi-1)}{\left[2a_{\mathrm{H}}(\phi-1)+1\right]^2}\cdot\right.$$

$$\left.\frac{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\phi-1)+1}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\phi-1)+1+4a_{\mathrm{H}}(1-a_{\mathrm{H}})\,\phi\frac{4a_{\mathrm{H}}^2(\phi-1)^2}{[2a_{\mathrm{H}}(\phi-1)+1]^2}\frac{(1-\beta)}{\beta(\delta_2-1)}}\right].$$

The second term in brackets is positive for $\phi > 1$ and always less than 1 in absolute value, since $\delta_2 - 1 > 1 - \delta_1$:

$$\begin{aligned}
\delta_2 - 1 &= \frac{1}{2\beta}\left(1-\beta+\left[\eta+\frac{\sigma}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\right]\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\theta\right)+ \\
&\quad \frac{1}{2\beta}\sqrt{\left\{1+\beta+\left[\eta+\frac{\sigma}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\right]\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\theta\right\}^2-4\beta}, \\
1-\delta_1 &= \frac{1}{2\beta}\left(\beta-1-\left[\eta+\frac{\sigma}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\right]\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\theta\right)+ \\
&\quad \frac{1}{2\beta}\sqrt{\left\{1+\beta+\left[\eta+\frac{\sigma}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\right]\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\theta\right\}^2-4\beta},
\end{aligned}$$

which implies that,

$$\begin{aligned}
\delta_2 - 1 &\geq 1-\delta_1 <=> \\
1+\left[\eta+\frac{\sigma}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\right]\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\theta &> \beta.
\end{aligned}$$

Therefore, optimal policy dampens capital flows for $\phi > 1$ and makes them larger in absolute value for $\phi < 1$.■

The full allocation under the optimal policy in PCP economies for the case $\eta = 0$ and $\sigma = 1$ is shown in Table A3, once again abstracting from contemporaneous shocks.

Table A1
Constrained-efficient allocation under PCP with news shocks, for $\phi \geq 0$

$\widetilde{\mathcal{W}}_t = \mathbf{A} \cdot \beta \sum_{j=0}^{\infty} \beta^j \left[\begin{array}{c} 2a_{\rm H}(\phi-1) E_t \left(\left(\widehat{Y}^{fb}_{H,t+j+1} - \widehat{Y}^{fb}_{F,t+j+1} \right) - \left(\widehat{Y}^{fb}_{H,t+j} - \widehat{Y}^{fb}_{F,t+j} \right) \right) + \\ -\left(2a_{\rm H}(\phi-1)+1\right) E_t \left(\left(\widehat{\zeta}_{C,t+1+j} - \widehat{\zeta}^*_{C,t+1+j} \right) - \left(\widehat{\zeta}_{C,t+j} - \widehat{\zeta}^*_{C,t+j} \right) \right) \end{array} \right]$
$\widetilde{Y}_{H,t} = \varkappa_1 \widetilde{Y}_{H,t-1} - (1-a_{\rm H}) \left\{ \begin{array}{c} \left[2a_{\rm H}(\phi-1)+1\right] \frac{(\beta\delta_2-1)}{\beta\delta_2} \widetilde{\mathcal{W}}_t + \\ 2a_{\rm H}\phi \frac{2a_{\rm H}(\phi-1)}{2a_{\rm H}(\phi-1)+1} \frac{1}{\beta\delta_2} \left(\widetilde{\mathcal{W}}_t - \widetilde{\mathcal{W}}_{t-1} \right) \end{array} \right\}$
$\theta\pi_{H,t} = (1-\varkappa_1) \widetilde{Y}_{H,t-1} + (1-a_{\rm H}) \frac{(\beta\delta_2-1)}{\beta\delta_2} \left\{ \begin{array}{c} \left[2a_{\rm H}(\phi-1)+1\right] \widetilde{\mathcal{W}}_t + \\ -2a_{\rm H}\phi \frac{2a_{\rm H}(\phi-1)}{2a_{\rm H}(\phi-1)+1} \left(\widetilde{\mathcal{W}}_t - \widetilde{\mathcal{W}}_{t-1} \right) \end{array} \right\}$
$\widetilde{\mathcal{Q}}_t = (2a_{\rm H}-1) \frac{2\widetilde{Y}_{H,t} - (2a_{\rm H}-1)\widetilde{\mathcal{W}}_t}{4a_{\rm H}(1-a_{\rm H})(\phi-1)+1},$

where

$$\mathbf{A} = \frac{\left[2a_{\rm H}(\phi-1)+1\right]^{-1}}{\left[4a_{\rm H}(1-a_{\rm H})(\phi-1)+1+4a_{\rm H}(1-a_{\rm H})\phi \frac{4a_{\rm H}^2(\phi-1)^2}{\left[2a_{\rm H}(\phi-1)+1\right]^2} \frac{(1-\beta)}{\beta\delta_2(1-\beta\delta_1)}\right]}.$$

Observe that the sign of the coefficient A depends on whether ϕ is above or below the threshold (36) in the text under the natural allocation, since the denominator is always positive for any $\phi \geq 0$; therefore, given a capital inflow $\widehat{\mathcal{B}}_t < 0$, $\widetilde{\mathcal{W}}_t$ will be positive if $\phi > \frac{2a_{\rm H}-1}{2a_{\rm H}}$, negative otherwise. As a result, on impact monetary policy is always expansionary and inflation positive for any sign of $\widetilde{\mathcal{W}}_t$, as the term $2a_{\rm H}(\phi-1)+1 < 0$ when $\widetilde{\mathcal{W}}_t < 0$. The full implications of different values of the elasticities for the dynamic responses are illustrated in Figure 2 in the main text.

The following proposition (which is the counterpart of Proposition 6 in the text) states the properties of this constrained efficient allocation, showing that the results for the CO economy generalize to any value of the trade elasticity, but for the output gap and misalignment. For these two variables to behave the same way as in the CO economy, a sufficient condition is that the trade elasticity be greater or equal to unity. The proposition also stresses a key new finding. Namely, the optimal policy now stabilizes the wealth gap, making it less volatile than under strict price stability.

Proposition. *For $\sigma = 1$, $\eta = 0$, and $\phi \geq 0$, under PCP, in response to news shocks generating inefficient capital flows, the GDP deflator is more volatile under the optimal policy than in a regime pursuing strict inflation stability, while the wealth gap is less volatile. Misalignment and the output gap are less volatile on impact for $\phi \geq 1$.*

Proof: The result from inflation follows from Table A1. The rest of the proof proceeds in two steps. First, assuming that $\widetilde{\mathcal{W}}_t$ is always less volatile than $\widetilde{\mathcal{W}}_t^{na}$, the result for the output gap follows by setting $\widetilde{Y}_{H,t\;\;1} = \widetilde{\mathcal{W}}_{t\;\;1} = 0$ in Table A3, and comparing the impact response of the constrained-efficient output gap, $\widetilde{Y}_{H,t_0}$, with $\widetilde{Y}_{H,t_0}^{na}$:

$$
\begin{aligned}
\widetilde{Y}_{H,t_0} &= -\left(1-a_{\mathrm{H}}\right)\left[2a_{\mathrm{H}}\left(\phi-1\right)+1\right]\left\{1-\frac{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\phi-1\right)+1}{\left[2a_{\mathrm{H}}\left(\phi-1\right)+1\right]^2\beta\varkappa_2}\right\}\widetilde{\mathcal{W}}_{t_0} \\
\widetilde{Y}_{H,t_0}^{na} &= -\left(1-a_{\mathrm{H}}\right)\left[2a_{\mathrm{H}}\left(\phi-1\right)+1\right]\widetilde{\mathcal{W}}_{t_0}^{na},
\end{aligned}
$$

whereas the coefficient of $\widetilde{\mathcal{W}}_{t_0}$ in $\widetilde{Y}_{H,t_0}$ is smaller in absolute value that of $\widetilde{\mathcal{W}}_{t_0}^{na}$ in $\widetilde{Y}_{H,t_0}^{na}$ for any $\phi \geq 1$ (since the term $0 < 1-\frac{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\phi-1)+1}{[2a_{\mathrm{H}}(\phi-1)+1]^2\beta\varkappa_2} < 1$ for $\phi \geq 1$). The result for misalignment (the real exchange rate) follows from noting that its expression in Table A3 for the constrained-efficient allocation also holds in the natural allocation, and using the fact that $\widetilde{\mathcal{W}}_{t_0}$ is always less volatile than $\widetilde{\mathcal{W}}_{t_0}^{na}$, while $\widetilde{Y}_{H,t_0}$ is less volatile than $\widetilde{Y}_{H,t_0}^{na}$ for $\phi \geq 1$.

Finally, we need to show that $\left|\widetilde{\mathcal{W}}_t^{na}\right| > \left|\widetilde{\mathcal{W}}_t\right|$ .Compare the coefficient multiplying the wealth gap under PPI price stability and the optimal policy for the case $\eta = 0$ and $\sigma = 1$:

$$
\begin{aligned}
PPI\ coefficient &= \left[2a_{\mathrm{H}}\left(\phi-1\right)+1\right]\left[4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\phi-1\right)+1\right] \\
Optimal\ coefficient &= \left[2a_{\mathrm{H}}\left(\phi-1\right)+1\right] \\
&\quad \left[\begin{array}{c} 4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\phi-1\right)+1+ \\ 4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\phi\frac{4a_{\mathrm{H}}^2(\phi-1)^2}{[2a_{\mathrm{H}}(\phi-1)+1]^2}\frac{(1-\beta)}{\beta\delta_2(1-\beta\delta_1)} \end{array}\right],
\end{aligned}
$$

where we also used the fact that:

$$
\begin{aligned}
& 1-\beta^2\delta_2\delta_1 \\
= \;& 1-\frac{1}{4}\left(1+\beta+\left[\frac{1}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\phi-1\right)+1}\right]\frac{\left(1-\alpha\beta\right)\left(1-\alpha\right)}{\alpha}\theta\right)^2- \\
& \frac{1}{4}\left\{1+\beta+\left[\eta+\frac{\sigma}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\right]\frac{\left(1-\alpha\beta\right)\left(1-\alpha\right)}{\alpha}\theta\right\}^2+\beta \\
= \;& 1-\beta > 0.
\end{aligned}
$$

The first term in square bracket $\left[2a_{\mathrm{H}}\left(\phi-1\right)+1\right]$ is positive for $\phi > 1-1/2a_{\mathrm{H}}$, while the term in the second square bracket is always positive for both the PPI and the optimal policy coefficients, but larger under the optimal policy for $\phi = 1$. Hence, for given shocks, the wealth gap has always the same sign under both policies. Moreover, as its coefficient is larger when positive and smaller when negative, the wealth gap is always smaller in absolute value under the optimal policy.■

3 The transmission of monetary policy with imperfect capital markets: Proof of Proposition 8

Here we analyze the effects of monetary policy on the wealth gap and capital flows, and offer the proof of Proposition 8 in Section 5 in the paper. As is well known, there are notable differences in the transmission of monetary decisions across LCP and PCP economies. Specifically, a monetary expansion causing nominal depreciation weakens the terms of trade under PCP but tends to strengthen the terms of trade under LCP. Here, our specific interest is to understand how monetary transmission is affected by incomplete markets.

3.1 LCP model

Starting with the LCP model, consider for simplicity a Home monetary shock such that CPI inflation follows an autoregressive process, $a_{\mathrm{H}}\pi_{Ht+s}+(1-a_{\mathrm{H}})\,\pi_{Ft+s} = \rho^s\pi > 0$, $s \geq 0$—assuming that the Foreign monetary authority responds by keeping CPI price stability, i.e., $a_{\mathrm{H}}\pi^*_{Ft+s} + (1-a_{\mathrm{H}})\,\pi^*_{Ht+s} = 0, s \geq 0$. For the reasons explained in the text, we focus on the case $\eta = 0$, when the LCP model is relatively straightforward to solve. With $\eta = 0$, the responses of key variables to the above monetary policy shock are given in Table A2, which can be obtained as in Section 2.1 of this Appendix, by substituting the above process of inflation into the Phillips curves. Specifically, the eigenvalues ν_1 and ν_2 are the same as those derived above. In the table, since an expansionary Home monetary policy shock is obviously inefficient (all first-best deviations are equal to zero), the responses of welfare-relevant gaps coincide with the response of actual variables.

Table A2: The effect of a monetary policy shock under LCP

$$\widetilde{\mathcal{W}}_{t+s} = \widetilde{\mathcal{W}}_t = \frac{(\sigma-1)}{2(1-a_{\mathrm{H}})+\sigma\left[2a_{\mathrm{H}}\left((\phi-1)\frac{1-\nu_1}{\nu_2-1}+1\right)-1\right]}\frac{1-\beta}{\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}}\pi$$

$$\widetilde{\mathcal{B}}_t = (1-a_{\mathrm{H}})\left\{2a_{\mathrm{H}}\,(\phi-1)\,\frac{(\beta\nu_2-1)}{\beta\nu_2}\frac{1}{1-\beta\nu_1}\widetilde{\mathcal{W}}_t + \frac{(\sigma-1)}{\sigma}\frac{(1-\rho)}{\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}}\beta\pi\right\}$$

$$\widetilde{\mathcal{T}}_{t+s} + \widetilde{\Delta}_{t+s} = -\frac{1-\nu_1^{s+1}}{1-\nu_1}\frac{(\beta\nu_2-1)}{\beta\nu_2}\widetilde{\mathcal{W}}_{t+s}$$

$$\widetilde{\Delta}_{t+s} = \frac{(1-\rho\beta)}{\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}}\rho^s\pi - (2a_{\mathrm{H}}-1)\left[1-\frac{1-\nu_1^{s+1}}{1-\nu_1}\frac{(\beta\nu_2-1)}{\beta\nu_2}\right]\widetilde{\mathcal{W}}_{t+s}$$

$$\widetilde{\mathcal{Q}}_{t+s} = \frac{(1-\rho\beta)}{\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}}\rho^s\pi - (2a_{\mathrm{H}}-1)\,\widetilde{\mathcal{W}}_{t+s}$$

$$\sigma\widetilde{Y}_{H,t+s} = a_{\mathrm{H}}\frac{(1-\rho\beta)}{\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}}\rho^s\pi - (1-a_{\mathrm{H}})\left[1+2a_{\mathrm{H}}\left(\sigma\phi\frac{1-\nu_1^{s+1}}{1-\nu_1}\frac{(\beta\nu_2-1)}{\beta\nu_2}-1\right)\right]\widetilde{\mathcal{W}}_{t+s}$$

$$\sigma\widetilde{Y}_{F,t+s} = (1-a_{\mathrm{H}})\,\frac{(1-\rho\beta)}{\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}}\rho^s\pi + (1-a_{\mathrm{H}})\left[1+2a_{\mathrm{H}}\left(\sigma\phi\frac{1-\nu_1^{s+1}}{1-\nu_1}\frac{(\beta\nu_2-1)}{\beta\nu_2}-1\right)\right]\widetilde{\mathcal{W}}_{t+s}$$

$$\sigma\widetilde{\mathcal{D}}_{t+s} = \frac{(1-\rho\beta)}{\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}}\rho^s\pi + 2\,(1-a_{\mathrm{H}})\,\widetilde{\mathcal{W}}_{t+s}.$$

When markets are incomplete, a monetary shock generally causes the wealth gap $\widetilde{\mathcal{W}}_t$ to deviate from zero (recall that in the bond economy $E_t\widetilde{\mathcal{W}}_{t+1} = \widetilde{\mathcal{W}}_t$)—implying that the effects of a monetary policy shock under incomplete markets are generally different than those under complete markets. A monetary expansion can open a wealth gap in different directions, depending on elasticities, as stated in Proposition 8. By the same token, a monetary expansion can lead to either an external surplus or an external deficit. In turn, a positive $\widetilde{\mathcal{W}}_t$ would attenuate (or amplify) the effects of monetary policy on domestic output and the real exchange rate (domestic consumption and foreign output).

In a few notable special cases, however, the effects of monetary policy are the same as in economies with complete markets. One such case is $\sigma = 1$ (log consumption utility), where $\widetilde{\mathcal{W}}_t = 0$, and neither capital flows $\widetilde{\mathcal{B}}_t$, nor the relative price misalignment, $\widetilde{\mathcal{T}}_t + \widetilde{\Delta}_t$, are affected by monetary policy, as shown in Proposition 5 of the paper. In this case, a monetary easing unambiguously results in positive domestic and foreign output gaps, a positive real exchange rate gap, and a higher relative demand gap. Relative to this benchmark, if the gap $\widetilde{\mathcal{W}}_t$ is positive the effects of monetary policy on the domestic output gap and the real exchange rate misalignment are smaller, while the foreign output and the relative demand gaps react more. These differences reflect the fact that the misalignment $\widetilde{\mathcal{T}}_t + \widetilde{\Delta}_t$ is negative when $\widetilde{\mathcal{W}}_t > 0$, implying some "expenditure switching" in favor of Foreign exports. The opposite is true if the wedge is negative: the domestic output and real exchange rate gaps react by more, while the transmission abroad is muted.

Proof of Proposition 8 under LCP. From the first equation in Table A2 it is clear that monetary easing brings about a positive wealth gap $\widetilde{\mathcal{W}}_t > 0$ if $\sigma > 1$ and $\phi \geq 1$, since both the numerator and denominator are positive under home bias ($a_{\mathrm{H}} \geq 1/2$). Under the same conditions an expansion leads also to an (inefficient) capital outflow $\widetilde{\mathcal{B}}_t = \widehat{\mathcal{B}}_t > 0$, since both terms in the second equation in the Table A2 are positive.

3.2 PCP model

The transmission of monetary policy under PCP is shown in Table A3, where we also set $\eta = 0$, and can be derived following the same steps as in Section 2.2 in this Appendix. Relative to the previous table, monetary easing is now modelled as an increase in domestic PPI inflation $\pi_{Ht+s} = \rho^s \pi > 0$, $s \geq 0$, under the assumption that the Foreign monetary authority responds by keeping PPI price stability, i.e., $\pi^*_{Ft+s} = 0, s \geq 0$.

Table A3: The effect of a monetary policy shock under PCP

$$\widetilde{\mathcal{W}}_{t+s} = \widetilde{\mathcal{W}}_t = \frac{(2a_{\mathrm{H}}\phi - 1)\sigma - (2a_{\mathrm{H}} - 1)}{1 + (2a_{\mathrm{H}}\phi - 1)\sigma - (2a_{\mathrm{H}} - 1)} \frac{(1-\beta)}{\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}} \pi$$

$$\widetilde{\mathcal{B}}_t = (1 - a_{\mathrm{H}}) \frac{(2a_{\mathrm{H}}\phi - 1)\sigma - (2a_{\mathrm{H}} - 1)}{\sigma} \frac{1}{\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}} \beta\pi$$

$$\widetilde{\mathcal{Q}}_{t+s} = (2a_{\mathrm{H}} - 1)\, \widetilde{\mathcal{T}}_{t+s} = (2a_{\mathrm{H}} - 1) \left[\frac{1}{\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}} \rho^s \pi - \widetilde{\mathcal{W}}_{t+s} \right]$$

$$\sigma \widetilde{Y}_{H,t+s} = \left[1 + 2a_{\mathrm{H}}(1 - a_{\mathrm{H}})(\sigma\phi - 1)\right] \frac{1}{\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}} \rho^s \pi - (1 - a_{\mathrm{H}}) \frac{2a_{\mathrm{H}}(\sigma\phi - 1) + 1}{\sigma} \widetilde{\mathcal{W}}_{t+s}$$

$$\sigma \widetilde{Y}_{F,t+s} = -2a_{\mathrm{H}}(1 - a_{\mathrm{H}})(\sigma\phi - 1) \frac{1}{\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}} \rho^s \pi + (1 - a_{\mathrm{H}}) \frac{2a_{\mathrm{H}}(\sigma\phi - 1) + 1}{\sigma} \widetilde{\mathcal{W}}_{t+s}$$

$$\sigma \widetilde{\mathcal{D}}_{t+s} = \frac{(2a_{\mathrm{H}} - 1)}{\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}} \rho^s \pi + 2(1 - a_{\mathrm{H}}) \widetilde{\mathcal{W}}_{t+s}.$$

An expansionary Home monetary policy shock also causes the gap $\widetilde{\mathcal{W}}_t$ to deviate from zero under PCP: under incomplete markets, the effects of a monetary policy shock do not coincide with those under complete markets. Again there are a few notable exceptions: under PCP, the special case in which monetary policy affects neither $\widetilde{\mathcal{W}}_t$ (= 0) nor capital flows arises when $\phi = \frac{1+\frac{2a_{\mathrm{H}}-1}{\sigma}}{2a_{\mathrm{H}}}$; if $\sigma = 1$, then, this requires $\phi = 1$—a Cobb-Douglas consumption aggregator. In this special case, just like under complete markets, a monetary easing unambiguously results in a higher domestic output and relative demand, and a positive real exchange rate gap. However, foreign output is affected only when $\sigma\phi \neq 1$, and increases if $\sigma\phi < 1$, namely, when goods are Edgeworth-complement. Relative to the benchmark with $\phi = \frac{1+\frac{2a_{\mathrm{H}}-1}{\sigma}}{2a_{\mathrm{H}}}$, similar to LCP, a positive (negative) wealth gap means that the effects of monetary policy on domestic output and the real exchange rate are smaller (larger) than under complete markets, while domestic consumption and foreign output react more (less). These effects reflect the fact that the response of the terms of trade, $\widetilde{\mathcal{T}}_t$, is also smaller (larger), implying a weaker (stronger) expenditure switching in favor of Home goods. Therefore, also under PCP a positive $\widetilde{\mathcal{W}}_t > 0$ may be associated with either outflows or inflows of capital, in turn attenuating or amplifying the effects of monetary policy on domestic output and the real exchange rate (domestic consumption and foreign output).

Proof of Proposition 8 under PCP. From the first equation in Table A3 the wealth gap is positive if the following condition holds:

$$\phi > \frac{1+\frac{2a_{\mathrm{H}}-1}{\sigma}}{2a_{\mathrm{H}}}.$$

From the second equation in the table, it is apparent that a monetary easing leads to an inefficient capital outflow on impact, $\widetilde{\mathcal{B}}_t > 0$, if it is also the case that $\phi > \frac{1+\frac{2a_{\mathrm{H}}-1}{\sigma}}{2a_{\mathrm{H}}}$.

4 Costly intermediation and stationarity of net foreign assets

Our results so far have been derived in a specification of the model in which both $\widehat{\mathcal{B}}_t$ and $\widehat{\mathcal{W}}_t$ are not stationary. In this subsection, we show that nonstationarity does not play any substantive role. In the literature, a standard approach to ensure that $\widehat{\mathcal{B}}_t$ is stationary in bond economies is to assume that its changes are subject to some (portfolio) adjustment costs; Gabaix and Maggiori [2015] have recently shown that this sluggish adjustment can result from costly intermediation of cross-border flows when financial intermediaries operate under borrowing constraints. In our framework, a simple way to capture the same idea is to posit deviations from the uncovered interest rate parity condition that are proportional to net foreign assets:

$$E_t\widehat{\mathcal{W}}_{t+1} - \widehat{\mathcal{W}}_t = -\delta\widehat{\mathcal{B}}_t.$$

With this modification, the solutions for $\widehat{\mathcal{B}}_t$ and $\widehat{\mathcal{W}}_t$ in the CO economy become:

$$\widehat{\mathcal{B}}_t = \gamma_1\widehat{\mathcal{B}}_t + (1-a_{\mathrm{H}})\sum_{j=0}^{\infty}\gamma_2^{-j-1}E_t\left[\left(\widehat{\zeta}_{C,t+1+j} - \widehat{\zeta}^*_{C,t+1+j}\right) - \left(\widehat{\zeta}_{C,t+j} - \widehat{\zeta}^*_{C,t+j}\right)\right],$$

$$\begin{aligned}\widehat{\mathcal{W}}_t &= \frac{\widehat{\mathcal{B}}_{t-1} - \beta\widehat{\mathcal{B}}_t}{(1-a_{\mathrm{H}})\,\beta}\Bigg) - \left(\widehat{\zeta}_{C,t} - \widehat{\zeta}^*_{C,t}\right)\\ &= -\left[\left(\widehat{\zeta}_{C,t} - \widehat{\zeta}^*_{C,t}\right) + \sum_{j=0}^{\infty}\gamma_2^{-j-1}E_t\left[\left(\widehat{\zeta}_{C,t+1+j} - \widehat{\zeta}^*_{C,t+1+j}\right) - \left(\widehat{\zeta}_{C,t+j} - \widehat{\zeta}^*_{C,t+j}\right)\right] - \frac{\gamma_1-\beta}{(1-a_{\mathrm{H}})\,\beta}\widehat{\mathcal{B}}_{t-1}\right].\end{aligned}$$

where $\beta < \gamma_1 < 1 < \gamma_2$ are the roots of the characteristic equation associated with the above second-order difference equation:

$$\beta\gamma^2 - (1+\beta+\beta\delta)\,\gamma + 1 = 0.$$

Both $\widehat{\mathcal{W}}_t$ and $\widehat{\mathcal{B}}_t$ are now stationary, but still functions of exogenous shocks only, so the optimal targeting rules are the same as those derived above under both LCP and PCP. Therefore, optimal monetary policy will react in the same way to a capital inflow, by tightening under LCP and easing under PCP (although of course with a different strength). Clearly, setting $\delta = 0$ in the last expression leads to $\gamma_1 = 1$ and $\gamma_2 = 1/\beta$, which yields expressions (31) and (32) in the subsection 4.1.1 of the main text.

4.1 Stationary wealth distribution in the CO Economy

4.1.1 Net foreign assets dynamics and the natural rate allocation

In this appendix, we first show that the exogeneity of $\widehat{\mathcal{B}}_t$ and $\widetilde{\mathcal{W}}_t$ remains unaffected in the CO economy if cross-border flows are subject to costly intermediation in the vein of Gabaix and Maggiori [2015], resulting in their stationarity

— a result emphasized by Cavallino [2019]. Secondly, we show that even under stationarity, our results for the optimal policy in the CO economy still hold for a non-trivial range of values of the parameter Γ determining stationarity.

A simple way to capture costly intermediation in our framework is to posit deviations from the uncovered interest rate parity condition that are proportional to net foreign assets:

$$E_t\widetilde{\mathcal{W}}_{t+1} - \widetilde{\mathcal{W}}_t = -\Gamma\widehat{\mathcal{B}}_t.$$

With this modification, the solutions for $\widehat{\mathcal{B}}_t$ and $\widetilde{\mathcal{W}}_t$ become:

$$\widehat{\mathcal{B}}_t = \gamma_1\widehat{\mathcal{B}}_{t-1}+(1-a_{\mathrm{H}})\sum_{j=0}^{\infty}\gamma_2^{-j-1}E_t\left[\left(\widehat{\zeta}_{C,t+1+j} - \widehat{\zeta}^*_{C,t+1+j}\right) - \left(\widehat{\zeta}_{C,t+j} - \widehat{\zeta}^*_{C,t+j}\right)\right],$$

$$\widetilde{\mathcal{W}}_t = -\left[\left(\widehat{\zeta}_{C,t} - \widehat{\zeta}^*_{C,t}\right) + \sum_{j=0}^{\infty}\gamma_2^{-j-1}E_t\left[\left(\widehat{\zeta}_{C,t+1+j} - \widehat{\zeta}^*_{C,t+1+j}\right) - \left(\widehat{\zeta}_{C,t+j} - \widehat{\zeta}^*_{C,t+j}\right)\right] - \frac{\gamma_1-\beta}{(1-a_{\mathrm{H}})\,\beta}\widehat{\mathcal{B}}_{t-1}\right],$$

where $0 < \gamma_1 < 1 < \beta^{-1} < \gamma_2$, for $\Gamma > 0$. Specifically, γ_1 and γ_2 are the roots of the characteristic equation associated with the second-order difference equation:

$$\begin{aligned}\widetilde{\mathcal{W}}_t &= \left.\frac{\widehat{\mathcal{B}}_{t-1} - \beta\widehat{\mathcal{B}}_t}{(1-a_{\mathrm{H}})\,\beta}\right) - \left(\widehat{\zeta}_{C,t} - \widehat{\zeta}^*_{C,t}\right)\\ E_t\widetilde{\mathcal{W}}_{t+1} - \widetilde{\mathcal{W}}_t &= -\Gamma\widehat{\mathcal{B}}_t,\end{aligned}$$

namely:

$$\beta\gamma^2 - (1+\beta+\beta\Gamma)\,\gamma + 1 = 0$$

$$\gamma_{1,2} = \frac{(1+(1+\Gamma)\,\beta) \pm \sqrt{(1+(1+\Gamma)\,\beta)^2 - 4\beta}}{2\beta}$$

Clearly, setting $\Gamma = 0$ in the last expression leads to $\gamma_1 = 1$ and $\gamma_2 = 1/\beta$, which yields expressions in Section 4 in the main text.

The response of both $\widehat{\mathcal{B}}_t$ and $\widetilde{\mathcal{W}}_t$ to shocks are smaller in absolute value when they are stationary, since future expected shocks are discounted by the factor γ_2, which is larger than $1/\beta$ for $\Gamma > 0$. But for the typically small values of Γ used in the literature, this discrepancy will be small. Moreover, the sign of their response remains the same. Specifically, anticipated shocks bringing about a persistent Home capital inflow, still result in a persistently positive wealth gap; iterating forward the above solution for $\widehat{\mathcal{B}}_t$ we have the following expression $\widehat{\mathcal{B}}_{t_0+s}$ conditional on shocks known as of t_0:

$$E_{t_0}\widehat{\mathcal{B}}_{t_0+s} = (1-a_{\mathrm{H}})\left\{\sum_{k=0}^{s}\gamma_1^{s-k}\sum_{j=k}^{\infty}\gamma_2^{k-j-1}E_{t_0}\left[\left(\widehat{\zeta}_{C,t_0+1+j} - \widehat{\zeta}^*_{C,t_0+1+j}\right) - \left(\widehat{\zeta}_{C,t_0+j} - \widehat{\zeta}^*_{C,t_0+j}\right)\right]\right\}, s \geq 0$$

or alternatively for $s \geq 1$:

$$E_{t_0}\widehat{\mathcal{B}}_{t_0+s} = \gamma_1^s\widehat{\mathcal{B}}_{t_0}+(1-a_{\mathrm{H}})\left\{\sum_{k=1}^{s}\gamma_1^{s-k}\sum_{j=k}^{\infty}\gamma_2^{k-j-1}E_{t_0}\left[\left(\widehat{\zeta}_{C,t_0+1+j}-\widehat{\zeta}^*_{C,t_0+1+j}\right)-\left(\widehat{\zeta}_{C,t_0+j}-\widehat{\zeta}^*_{C,t_0+j}\right)\right]\right\};$$

the latter expression shows that the degree of persistence in net foreign assets is determined by γ_1 ($\to 1$ for $\Gamma \to 0$). Observe that $E_{t_0}\widehat{\mathcal{B}}_{t_0+s} \to 0$ for $s \to \infty$ if shocks do not diverge. A transparent way to see this is to consider the case in which anticipated taste shocks follow a random walk, i.e.:

$$E_{t_0}\left[\left(\widehat{\zeta}_{C,t_0+1+j}-\widehat{\zeta}^*_{C,t_0+1+j}\right)-\left(\widehat{\zeta}_{C,t_0+j}-\widehat{\zeta}^*_{C,t_0+j}\right)\right] = \left\{\begin{array}{c}\widehat{\zeta}_h, j = h \geq 0\\ 0, j = h\end{array}\right.;$$

then even in this case $limE_{t_0}\widehat{\mathcal{B}}_{t_0+s} = 0$ since only one element of $E_{t_0}\left[\left(\widehat{\zeta}_{C,t_0+1+j}-\widehat{\zeta}^*_{C,t_0+1+j}\right)-\left(\widehat{\zeta}_{C,t_0+j}-\widehat{\zeta}^*_{C,t_0+j}\right.\right.$ 0 for the period $t_0 + 1 + h$ when the shock is expected to occur; in turn this element will be discounted by the factor γ_2.

By the same token, $E_{t_0}\widetilde{\mathcal{W}}_{t_0+s}$ will also converge to zero by the following expression:

$$E_{t_0}\widetilde{\mathcal{W}}_{t_0+s+1} = E_{t_0}\left(\frac{\widehat{\mathcal{B}}_{t_0+s}-\beta\widehat{\mathcal{B}}_{t_0+s+1}}{(1-a_{\mathrm{H}})\,\beta}\right)-E_{t_0}\left(\widehat{\zeta}_{C,t_0+s+1}-\widehat{\zeta}^*_{C,t_0+s+1}\right), s \geq 0,$$

where all terms on the right-hand side converge to zero if shocks do not diverge — e.g., when shocks follow a random walk as assumed above then $E_{t_0}\widetilde{\mathcal{W}}_{t_0+s+1} \to \widehat{\zeta}_h, s \to \infty$.

For future reference it also useful to express the expected wealth gap as follows:

$$E_{t_0}\widetilde{\mathcal{W}}_{t_0+s} = \widetilde{\mathcal{W}}_{t_0} - \Gamma\sum_{j=0}^{s}E_{t_0}\widehat{\mathcal{B}}_{t_0+j-1}, s \geq 1.$$

Finally, the natural rate allocation as a function of $\widetilde{\mathcal{W}}_t$ is independent of whether there is a unit root in $\widehat{\mathcal{B}}_t, \widetilde{\mathcal{W}}_t$; only its equilibrium dynamics is affected. Specifically, $\widetilde{\mathcal{W}}_{t_0} > 0$ still implies an overvaluation with negative Home output gap. Setting $\pi_{H,t_0+s} = 0$ in the Home Phillips curve still yields the same expression for the output gap (and the other variables) as in Table 3:

$$\widetilde{Y}_{H,t_0+s} = -(1-a_{\mathrm{H}})\,\widetilde{\mathcal{W}}_{t_0+s}.$$

The key difference is that now $\widetilde{\mathcal{W}}_{t_0+s}$ will be generally different from its impact value $\widetilde{\mathcal{W}}_{t_0}$.

4.1.2 Optimal response to capital inflows under PCP

Since both $\widetilde{\mathcal{W}}_t$ and $\widehat{\mathcal{B}}_t$ are still functions of exogenous shocks only, the optimal targeting rules are the same as those derived under both LCP and PCP for the

CO economy in Section 3. However, the allocation will depend on the dynamics of $\widetilde{\mathcal{W}}_t$ and not just its impact value. Consider the key relations between the output gap, domestic inflation $\pi_{H,t}$ and $\widetilde{\mathcal{W}}_t$ derived substituting the PCP target rule into the Home Phillips curve:

$$\begin{aligned}
\theta\pi_{H,t} &= -\left(\widetilde{Y}_{H,t} - \widetilde{Y}_{H,t-1}\right) \\
\widetilde{Y}_{H,t} &= -(1-a_{\mathrm{H}})\frac{(\beta\varkappa_2 - 1)}{\beta\varkappa_2}\frac{(\varkappa_2 - 1)}{\varkappa_2}\sum_{j=0}^{\infty}\varkappa_2^{-j}E_t\widetilde{\mathcal{W}}_{t+j} + \varkappa_1\widetilde{Y}_{H,t-1};
\end{aligned}$$

using the expression for $E_{t_0}\widetilde{\mathcal{W}}_{t_0+j}$ above, we have the following impact response of the output gap under the optimal policy:

$$\begin{aligned}
\widetilde{Y}_{H,t_0} &= -(1-a_{\mathrm{H}})\frac{(\beta\varkappa_2 - 1)}{\beta\varkappa_2}\frac{(\varkappa_2 - 1)}{\varkappa_2}\sum_{j=0}^{\infty}\varkappa_2^{-j}\left[\widetilde{\mathcal{W}}_{t_0} - \Gamma\sum_{h=0}^{j}E_{t_0}\widehat{\mathcal{B}}_{t_0+h-1}\right] \\
&= -(1-a_{\mathrm{H}})\frac{(\beta\varkappa_2 - 1)}{\beta\varkappa_2}\left[\widetilde{\mathcal{W}}_{t_0} - \Gamma\frac{(\varkappa_2 - 1)}{\varkappa_2}\sum_{j=0}^{\infty}\varkappa_2^{-j}\sum_{h=0}^{j}E_{t_0}\widehat{\mathcal{B}}_{t_0+h-1}\right] \\
&= -(1-a_{\mathrm{H}})\frac{(\beta\varkappa_2 - 1)}{\beta\varkappa_2}\left[\widetilde{\mathcal{W}}_{t_0} - \Gamma\sum_{j=0}^{\infty}\varkappa_2^{-j-1}E_{t_0}\widehat{\mathcal{B}}_{t_0+j}.\right] \\
&= -\frac{(\beta\varkappa_2 - 1)}{\beta\varkappa_2}\left[-\widehat{\mathcal{B}}_{t_0} - (1-a_{\mathrm{H}})\Gamma\sum_{j=0}^{\infty}\varkappa_2^{-j-1}E_{t_0}\widehat{\mathcal{B}}_{t_0+j}.\right]
\end{aligned}$$

Two remarks are in order. First, in order for a capital inflow $\widehat{\mathcal{B}}_{t_0} < 0$ with a positive wealth gap $\widetilde{\mathcal{W}}_{t_0} > 0$ on impact to result in a negative output gap and positive inflation, the following condition has to be satisfied when $\Gamma > 0$:

$$\begin{aligned}
0 &> \frac{(\beta\varkappa_2 - 1)}{\beta\varkappa_2}\left[\widehat{\mathcal{B}}_{t_0} + (1-a_{\mathrm{H}})\Gamma\sum_{j=0}^{\infty}\varkappa_2^{-j-1}E_{t_0}\widehat{\mathcal{B}}_{t_0+j}.\right] \\
-\widehat{\mathcal{B}}_{t_0} &> (1-a_{\mathrm{H}})\Gamma\sum_{j=0}^{\infty}\varkappa_2^{-j-1}E_{t_0}\widehat{\mathcal{B}}_{t_0+j}.
\end{aligned}$$

Clearly this condition is satisfied for a sufficiently small value of Γ, which by continuity will always exist. Moreover, since the process of $\widehat{\mathcal{B}}_t$ is more persistent the closer Γ to zero, for a small Γ NFAs would also revert slowly to their steady state value, remaining negative for some time after an impact inflow, also helping the condition to be satisfied.

Second, in contrast with the unit root case ($\Gamma = 0$), the impact output gap under the optimal allocation can be more negative and thus respond more to the capital inflow than under the natural allocation. In this case, the real exchange rate will also be less stabilized. A sufficient condition for the results in Proposition 6 to carry over is given by the following condition:

$$-\widehat{\mathcal{B}}_{t_0} > (1-a_{\mathrm{H}})\,\Gamma\sum_{j=0}^{\infty}\varkappa_2^{-j-1}E_{t_0}\widehat{\mathcal{B}}_{t_0+j} > 0.$$

The inequality on the right hand side now imposes restrictions not only on the value of Γ but also on other parameters affecting the value of $\varkappa_2$ and thus the model dynamics, and also on the shock process. This can be seen most transparently by considering again the case of random walk shocks, for which the impact NFA and wealth gap is given by:[1]

$$(1-a_{\mathrm{H}})\,\widetilde{\mathcal{W}}_{t_0} = -\widehat{\mathcal{B}}_{t_0} = (1-a_{\mathrm{H}})\,\widehat{\zeta}_h\gamma_2^{-h-1}.$$

In order to determine the impact response of $\widetilde{Y}_{H,t_0}$ we can solve for $\sum_{j=0}^{\infty}\varkappa_2^{-j-1}E_{t_0}\widehat{\mathcal{B}}_{t_0+j}$ as follows using the above expression for the process of NFAs:

$$\begin{aligned}
\sum_{j=0}^{\infty}\varkappa_2^{-j-1}E_{t_0}\widehat{\mathcal{B}}_{t_0+j} &= \varkappa_2^{-1}\sum_{j=0}^{\infty}\frac{\gamma_1^j}{\varkappa_2^j}\widehat{\mathcal{B}}_{t_0} + \underbrace{(1-a_{\mathrm{H}})\,\widehat{\zeta}_h\gamma_2^{-h-1}}_{=-\widehat{\mathcal{B}}_{t_0}}\sum_{j=1}^{h-1}\gamma_1^j\sum_{k=1}^{j}\left(\frac{\gamma_2}{\gamma_1}\right)^k \\
&= \left[\varkappa_2^{-1}\sum_{j=0}^{\infty}\frac{\gamma_1^j}{\varkappa_2^j} - \sum_{j=1}^{h-1}\gamma_1^j\sum_{k=1}^{j}\left(\frac{\gamma_2}{\gamma_1}\right)^k\right]\widehat{\mathcal{B}}_{t_0} \\
&= \left[\frac{1}{\varkappa_2-\gamma_1} - \frac{\frac{\gamma_2}{\gamma_1}}{\gamma_2-\gamma_1}\left(\gamma_2\frac{\gamma_2^h-1}{\gamma_2-1} - \gamma_1\frac{1-\gamma_1^h}{1-\gamma_1}\right)\right]\widehat{\mathcal{B}}_{t_0}.
\end{aligned}$$

In turn, also using the relation between $\widetilde{\mathcal{W}}_{t_0}$ and $\widehat{\mathcal{B}}_{t_0}$, this yields the following expression for the impact output gap:

$$\begin{aligned}
\widetilde{Y}_{H,t_0} &= -(1-a_{\mathrm{H}})\,\frac{(\beta\varkappa_2-1)}{\beta\varkappa_2}\left[\widetilde{\mathcal{W}}_{t_0} - \Gamma\sum_{j=0}^{\infty}\varkappa_2^{-j-1}E_{t_0}\widehat{\mathcal{B}}_{t_0+j}.\right] \\
&= -(1-a_{\mathrm{H}})\,\frac{(\beta\varkappa_2-1)}{\beta\varkappa_2}\left\{\widetilde{\mathcal{W}}_{t_0} - \Gamma\left[\frac{1}{\varkappa_2-\gamma_1} - \frac{\frac{\gamma_2}{\gamma_1}}{\gamma_2-\gamma_1}\left(\gamma_2\frac{\gamma_2^h-1}{\gamma_2-1} - \gamma_1\frac{1-\gamma_1^h}{1-\gamma_1}\right)\right]\widehat{\mathcal{B}}_{t_0}\right\} \\
&= -(1-a_{\mathrm{H}})\,\frac{(\beta\varkappa_2-1)}{\beta\varkappa_2}\left\{1-(1-a_{\mathrm{H}})\,\Gamma\left[\frac{\frac{\gamma_2}{\gamma_1}}{\gamma_2-\gamma_1}\left(\gamma_2\frac{\gamma_2^h-1}{\gamma_2-1} - \gamma_1\frac{1-\gamma_1^h}{1-\gamma_1}\right) - \frac{1}{\varkappa_2-\gamma_1}\right]\right\}\widetilde{\mathcal{W}}_{t_0}.
\end{aligned}$$

[1] In the case of a purely transitory anticipated shocks to preferences occurring as of $t+h, h \geq 1$:

$$E_{t_0}\left[\left(\widehat{\zeta}_{C,t_0+1+j} - \widehat{\zeta}^*_{C,t_0+1+j}\right) - \left(\widehat{\zeta}_{C,t_0+j} - \widehat{\zeta}^*_{C,t_0+j}\right)\right] = \begin{cases} \widehat{\zeta}_h, j = h \geq 0 \\ -\widehat{\zeta}_h, j = h+1 \\ 0, j \neq h, h+1 \end{cases};$$

the impact NFA and wealth gap are given by:

$$(1-a_{\mathrm{H}})\,\widetilde{\mathcal{W}}_{t_0} = -\widehat{\mathcal{B}}_{t_0} = (1-a_{\mathrm{H}})\,\widehat{\zeta}_h\,(\gamma_2-1)\,\gamma_2^{-h-2}.$$

Therefore the gist of the argument below would still hold.

Sufficient conditions for the output gap to be negative but more stabilized than in the natural allocation, so that the results in Proposition 6 continue to hold, is thus given by the following:

$$1 > (1 - a_{\mathrm{H}})\,\Gamma \left[\frac{\frac{\gamma_2}{\gamma_1}}{\gamma_2 - \gamma_1} \left(\gamma_2 \frac{\gamma_2^h - 1}{\gamma_2 - 1} - \gamma_1 \frac{1 - \gamma_1^h}{1 - \gamma_1} \right) - \frac{1}{\varkappa_2 - \gamma_1} \right] \geq 0,$$

which can be rewritten as follows:

$$\varkappa_2 > \gamma_1 + \frac{-(1 - a_{\mathrm{H}})\,\Gamma}{1 - (1 - a_{\mathrm{H}})\,\Gamma \left[\frac{\frac{\gamma_2}{\gamma_1}}{\gamma_2 - \gamma_1} \left(\gamma_2 \frac{\gamma_2^h - 1}{\gamma_2 - 1} - \gamma_1 \frac{1 - \gamma_1^h}{1 - \gamma_1} \right) \right]}$$

$$\varkappa_2 \geq \gamma_1 + \left[\frac{\frac{\gamma_2}{\gamma_1}}{\gamma_2 - \gamma_1} \left(\gamma_2 \frac{\gamma_2^h - 1}{\gamma_2 - 1} - \gamma_1 \frac{1 - \gamma_1^h}{1 - \gamma_1} \right) \right]^{-1};$$

these conditions are a function of Γ directly and through the associated eigenvalues γ_1, γ_2, but also of the horizon h at which the shock is expected to materialize and of $\varkappa_2$, which depends on nominal rigidities. Nevertheless, since $\varkappa_2 > 1 > \gamma_1$, it is clear that a relatively small value of Γ will ensure that these conditions hold.

4.1.3 Optimal response under LCP

Since the dynamics of $\widetilde{\mathcal{W}}_t$ and $\widehat{\mathcal{B}}_t$ is independent of ERPT, to characterize the optimal constrained allocation under LCP we only need to consider the key relations between the demand gap, CPI inflation π_t and $\widetilde{\mathcal{W}}_t$, derived as follows by substituting the LCP target rule into the Home Phillips curves:

$$\begin{aligned} \theta \pi_t &= -\left(\widetilde{D}_t - \widetilde{D}_{t-1} \right) \\ \widetilde{D}_t &= 2\,(1 - a_{\mathrm{H}})\, \frac{(\beta \varkappa_2 - 1)}{\beta \varkappa_2} \frac{(\varkappa_2 - 1)}{\varkappa_2} \sum_{j=0}^{\infty} \varkappa_2^{-j} E_t \widetilde{\mathcal{W}}_{t+j} + \varkappa_1 \widetilde{D}_{t-1}. \end{aligned}$$

Using the expression for $E_{t_0} \widetilde{\mathcal{W}}_{t_0+j}$ above, we have the following impact response of the demand gap under the optimal policy:

$$\begin{aligned} \widetilde{D}_{t_0} &= 2\,(1 - a_{\mathrm{H}})\, \frac{(\beta \varkappa_2 - 1)}{\beta \varkappa_2} \frac{(\varkappa_2 - 1)}{\varkappa_2} \sum_{j=0}^{\infty} \varkappa_2^{-j} \left[\widetilde{\mathcal{W}}_{t_0} - \Gamma \sum_{h=0}^{j} E_{t_0} \widehat{\mathcal{B}}_{t_0+h-1} \right] \\ &= 2 \frac{(\beta \varkappa_2 - 1)}{\beta \varkappa_2} \left[-\widehat{\mathcal{B}}_{t_0} - (1 - a_{\mathrm{H}})\, \Gamma \sum_{j=0}^{\infty} \varkappa_2^{-j-1} E_{t_0} \widehat{\mathcal{B}}_{t_0+j}. \right] \end{aligned}$$

Similar considerations as those emerging under PCP apply; specifically observe that the last expression for $\widetilde{D}_{t_0}$ under LCP is equal to the negative of twice the output gap under PCP ($\widetilde{D}_{t_0}^{LCP} = -2\widetilde{Y}_{H,t_0}^{PCP}$), as derived in the previous section. As a result, the same sufficient conditions under PCP for an expansionary monetary policy response to an inflow, with positive GDP inflation and output gap more stable than in the natural allocation, will also result under LCP in a monetary contraction with negative CPI inflation, and a more stable demand gap and less stable real exchange rate than under CPI stability.

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