The Optimal Composition of Public Spending in a Deep Recession[*]

Hafedh Bouakez[†]   Michel Guillard[‡]   Jordan Roulleau-Pasdeloup[§]

July 2016

Abstract

We study optimal monetary and fiscal policy under commitment in an economy where monetary policy is constrained by the zero lower bound on the nominal interest rate, and where the government can allocate spending to public consumption and public investment. We show that the optimal response to an adverse shock that precipitates the economy into a liquidity trap entails a small and short-lived increase in public consumption but a large and persistent increase in public investment, which lasts well after the natural rate of interest has ceased to be negative. During this period, the optimal composition of public spending is therefore heavily skewed towards public investment. Contrary to the literature that abstracts from public investment, we find that the optimal increase in total public spending in a deep recession is sizable. However, we show that this fiscal expansion has little to do with a stabilization motive and is instead warranted by the intertemporal allocation of resources that efficiency dictates even in the absence of an output gap.

JEL classification: E4, E52, E62, H54
Key words: Public spending, Public investment, Time to build, Ramsey policies, Zero lower bound.

[†]Department of Applied Economics and CIRPÉE, HEC Montréal, 3000 chemin de la Côte-Sainte-Catherine, Montréal, Québec, Canada H3T 2A7. Tel.: 1-514-340-7003; Fax: 1-514-340-6469; E-mail: `hafedh.bouakez@hec.ca`.

[‡]EPEE, Université d'Evry Val d'Essonne, bd F. Mitterrand, 91025 Evry Cedex, France; E-mail: `michel.guillard@univ-evry.fr`.

[§]DEEP, HEC Lausanne – Quartier UNIL-Dorigny, Bâtiment Internef, Lausanne, Switzerland. Email: `jordan.Roulleau-Pasdeloup@unil.ch`.

1   Introduction

What is the optimal composition of a fiscal expansion in a depressed economy? Despite the widespread interest in the stimulative effects of fiscal policy generated by the unprecedentedly large stimulus plans enacted in most industrialized economies at the onset of the Great Recession, this question has remained largely overlooked by the literature. Existing studies indeed mostly focus on the size of the spending multiplier when the economy is plunged in a liquidity trap, that is, when nominal interest rates are at their zero lower bound (ZLB), and on the desirability of public spending from a welfare standpoint in such circumstances.

Two main conclusions emerge from that literature. First, the spending multiplier can be substantially large when the ZLB binds.[1] Second, it is optimal to temporarily increase public spending while the economy is in a liquidity trap.[2] The intuition for why public spending improves welfare is the following. When monetary policy is unconstrained — so that it can replicate the flexible-price allocation — and to the extent that government spending provides utility to households, optimality requires that the marginal utilities of private and public goods be equated, a condition commonly referred to as the Samuelson rule. The latter thus implies that government spending co-moves with consumption: if an adverse shock causes a fall in consumption, it will also lead to a fall in public spending. When the ZLB binds, however, nominal interest rates cannot be used to stabilize the economy, and an undesired (negative) output gap emerges, creating another motive for varying public spending. In response to an adverse shock, public spending rises to help close the output gap.

Bilbiie et al. (2014), however, argue that the optimal increase in public spending in response to a typical recession is tiny — less than 0.5 percent of steady-state output. Essentially, this result reflects the fact that optimal public spending needs to strike a balance between stabilizing the output gap and meeting the Samuelson condition. Under empirically plausible scenarios about the size of the adverse shock, optimal public spending rises but only by a small amount in order not to deviate too far from the Samuelson condition. Bilbiie et al. (2014) show that to obtain large optimal levels of public spending, one needs to assume implausibly severe recessions that take the economy close to the starvation point (the point where private consumption is zero). Furthermore, Sims & Wolff (2013) argue that the welfare multiplier of public spending, defined as the change in aggregate welfare for a one unit change in government spending, albeit positive, is procyclical. That is, it tends to be low during recessions and high during expansions. Together, these findings cast doubt on the usefulness of public spending from a normative perspective.

A common characteristic of all of these studies is the assumption that public spending consists

---

[1]See Christiano et al. (2011), Eggertsson (2011), and Woodford (2011), among many others.

[2]See, for instance, Woodford (2011), Werning (2011), Schmidt (2013), Nakata (2013), Bilbiie et al. (2014), and Nakata (2015) in the context of a closed economy, and Bhattarai & Egorov (2016) in the context of a small open economy.

exclusively of purchases of consumption goods, so that there is no scope for public investment. This assumption is unlikely to be innocuous when it comes to determining the optimal level of public spending and its welfare consequences. But perhaps more importantly, it precludes the analysis of the optimal composition of a fiscal expansion, an issue that was at the center of policy debates during the Great Recession. The various fiscal plans that have been implemented worldwide in 2008-2009 assigned a significant fraction of the additional spending to public investment in infrastructure,[3] but to our knowledge, there has not been any formal attempt to determine whether this allocation scheme was warranted from a welfare standpoint.

The objective of this paper is to study optimal monetary and fiscal policy in an economy where monetary policy is constrained by the ZLB on the nominal interest rate, and where the government can allocate spending to public consumption and public investment. More specifically, we study the optimal policy response under commitment to an adverse shock that precipitates the economy into a deep recession characterized by a liquidity trap. As is common in the literature cited above, we assume that the ZLB binds as a result of a large preference shock that raises agents' discount factor. The main novelty of our model with respect to those studied in the literature is the possibility for the government to accumulate public capital, which is an external input in the firms' production technology.[4] As in Leeper et al. (2010), Leduc & Wilson (2013), and Bouakez et al. (2015), we assume that the accumulation of public capital is subject to lengthy time-to-build delays, a distinctive feature of public infrastructure projects.[5] As a benchmark, we compute the first-best (efficient) allocation, i.e., the welfare-maximizing allocation chosen by a benevolent central planner.

We find that the optimal policy response to an adverse shock that makes the ZLB bind is to initially raise public consumption and public investment above their steady-state levels. The increase in public consumption is negligible and short-lived, followed by a prolonged cut that persists even after the natural rate of interest has ceased to be negative. In contrast, the increase in public investment is relatively large — reaching 1.7 percent of steady-state output at the peak — and persistent, lasting well after the natural rate of interest has become positive again. These patterns are subsequently reversed, as the optimal plan eventually entails raising public consumption and decreasing public investment relative to their steady-state levels.

Our findings have two salient implications. First, the optimal size of a fiscal expansion in a severe economic downturn is sizable. The cumulative increase in total public spending amounts to roughly 12 percent of steady-state output. This result stands in sharp contrast with the conclusion

---

[3]For instance, the American Recovery and Reinvestment Act and the European Economic Recovery Plan allocated, respectively, 40 and 71 percent of the additional public spending to investment in infrastructure.

[4]Earlier papers that also study optimal public consumption and investment include those by Lansing (1998), Ambler & Paquet (1996), and Ambler & Cardia (1997). Unlike our paper, however, these authors consider a real-business-cycle framework and abstract from monetary policy and the ZLB.

[5]Leeper et al. (2010) and Leduc & Wilson (2013) do not consider the case of a binding ZLB, and none of the three papers studies optimal policy.

based on optimal fiscal plans in which only public consumption is adjusted, as is the case in existing studies. When we exclude public investment from the set of policy instruments that are available to the policymaker, the optimal plan is such that the cumulative increase in public spending is virtually zero. Second, the optimal plan features a change in the composition of public spending in a way that assigns a larger weight to public investment relative to public consumption for a prolonged period of time after the shock. At the peak, the fraction of public investment in total public spending exceeds its steady-state value by 6 percentage points in our baseline simulation.

We then ask: how much of this fiscal expansion is due to the fact that the ZLB is binding, or, equivalently, to the fact that the economy is producing below its potential? We refer to this spending component as *stimulus spending* and we compute it as the difference between the spending level obtained under the Ramsey allocation and that obtained under fully flexible prices. The latter, which we label *neoclassical spending*, would occur if monetary policy were able to fully eliminate any output gap. We find that the stimulus component of public consumption and public investment is front-loaded and negligible, summing to less than 0.1 percent of steady-state output at the time of the shock and cumulating to approximately −0.1 percent of steady-state output over time. This result highlights an important point: the desirability of a fiscal expansion and the larger weight assigned to public investment in a response to an adverse shock have very little to do with a stabilization motive. Instead, they mostly reflect the role of public capital in enabling the intertemporal allocation of resources that efficiency dictates even in the absence of an output gap.

We also evaluate the welfare gain associated with the optimally designed fiscal plan relative to a scenario in which only the nominal interest rate is chosen optimally. When both public consumption and public investment are chosen optimally, this gain is two orders of magnitude larger than the gain achieved by only adjusting public consumption while keeping public investment constant.

In the last part of the paper, we study the robustness of our results along three dimensions. First, we use a fully non-linear solution method to compute the optimal allocation; second, we consider a larger shock — one that generates a recession of the size of the Great Depression; and finally, we extend the model to allow for the accumulation of private capital. We find our main conclusions to be robust in all three cases. In particular, the optimal policy response to the adverse shock is to raise significantly public spending and to shift its composition towards public investment for a prolonged period of time. The stimulus component of this increase is (initially) somewhat larger under the Great-Depression scenario and when we allow for private capital than in the baseline model, but it remains small in absolute term (less than 1 percent of steady-state output) and is more than fully offset by a spending cut during the subsequent periods.

The rest of this paper is organized as follows. Section 2 presents the baseline model with public capital. Section 2 characterizes the first-best response to the preference shock. Section 3 studies optimal monetary and fiscal policy. Section 4 performs a robustness analysis. Section 5 concludes.

2 A New Keynesian Model with Public Capital

We consider a simple new-Keynesian economy without private capital as in Bouakez et al. (2015). The economy is composed of infinitely lived households, firms, a government, and a monetary authority. The key feature of the model is that a fraction of government spending can be invested in public capital subject to a time-to-build requirement. The remaining fraction, i.e., government consumption, directly affects households' utility. The breakdown of public spending into investment and consumption expenditures is chosen optimally by the government. The stock of public capital enters as an external input in the production of intermediate goods, which are used to produce an homogenous final good. The latter is used for consumption and investment purposes. There is a continuum of monopolistically competitive intermediate-good producers, indexed by $z \in (0,1)$, which set prices subject to a Rotemberg (1982)-type adjustment cost. Final-good producers are perfectly competitive. The nominal interest rate is subject to a non-negativity constraint.

2.1 Households

The economy is populated by a large number of identical households who have the following lifetime utility function:

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \xi_{t+s} \left[U\left(C_{t+s}, N_{t+s}\right) + V\left(G_{t+s}^c\right) \right], \tag{1}$$

where β is the discount factor, C_t is consumption, N_t denotes hours worked, G_t^c denotes government consumption. We assume that $U\left(.\right)$ is increasing and concave in C_t and decreasing and convex in N_t; and $V\left(.\right)$ is increasing and concave in G_t^c. The total time endowment of the representative household is normalized to 1, so that leisure time is equal to $1 - N_t$. Both consumption and leisure are assumed to be normal goods. The term ξ_t is a preference shock that evolves according to the following process:

$$\log(\xi_t) = \rho \log(\xi_{t-1}) + \epsilon_t, \quad \epsilon_t \sim \mathcal{N}(0,1),$$

where $\rho \in (0,1)$. A negative shock to ξ_t thus raises the effective discount factor in the subsequent periods, which increases households' incentive to save, *ceteris paribus.*

The representative household enters period t with $\mathcal{B}_{t-1}$ units of one-period riskless nominal bonds. During the period, it receives a nominal wage payment, $W_t N_t$, and dividends, $\mathcal{D}_t = \int_0^1 \mathcal{D}_t\left(z\right) dz$, from monopolistically competitive firms. This income is used to pay a lump-sum tax, $\mathcal{T}_t$, to the government, to consumption, and to the purchase of new bonds. The household's budget constraint is therefore

$$P_t C_t + \mathcal{T}_t + \frac{\mathcal{B}_t}{1 + R_t} \leq W_t N_t + \mathcal{D}_t + \mathcal{B}_{t-1}, \tag{2}$$

where P_t is the price of the final good, W_t is nominal wage rate, and $\frac{1}{1+R_t}$ is the price of a nominal

bond purchased at time t, R_t being the nominal interest rate. The household maximizes (1) subject to (2) and to a no-Ponzi-game condition. The first-order conditions for this problem are given by

$$w_t = -\frac{U_{N,t}}{U_{C,t}}, \tag{3}$$

$$\frac{1}{1+R_t} = \beta \mathbb{E}_t \left(\frac{\xi_{t+1}}{\xi_t} \frac{U_{C,t+1}}{U_{C,t}} \frac{P_t}{P_{t+1}} \right), \tag{4}$$

where $w_t = \frac{W_t}{P_t}$ is the real wage rate and $U_{X,t} = \partial U (C_t, N_t) / \partial X_t$.

2.2 Firms

The final good is produced by perfectly competitive firms using the following constant-elasticity-of-substitution technology:

$$Y_t = \left(\int_0^1 Y_t (z) \, dz \right)^{\frac{\theta}{\theta-1}}, \tag{5}$$

where $Y_t (z)$ is the quantity of intermediate good z and $\theta \geq 1$ is the elasticity of substitution between intermediate goods. Denoting by $P_t (z)$ the price of intermediate good z, demand for z is given by

$$Y_t (z) = \left(\frac{P_t (z)}{P_t} \right)^{-\theta} Y_t. \tag{6}$$

Firms in the intermediate-good sector are monopolistically competitive, each producing a differentiated good using labor as a direct input and public capital as an external input

$$Y_t (z) \leq F (N_t (z), K_{G,t}), \tag{7}$$

where $F(.)$ is increasing and concave in both of its arguments. This specification implies that public capital acts as a positive externality that improves the marginal productivity of private inputs. This assumption is consistent with the empirical evidence reported by Aschauer (1989), Fernald (1999), and Leduc & Wilson (2013), among many others.[6] Aschauer (1989) estimates an aggregate production function for the U.S. economy and finds that non-military infrastructure has a strong positive effect on total factor productivity. Fernald (1999) finds a causal effects of growth in roads (the largest component of U.S. public infrastructure) on the productivity of U.S. vehicle-intensive industries. Leduc & Wilson (2013) find evidence that federal highway spending works like an anticipated productivity shock, raising output several years in the future.

We assume that firms set prices subject to a Rotemberg (1982)-type adjustment cost. That is, in each period, a given firm pays a quadratic adjustment cost to reset its nominal price, $P_t (z)$,

---

[6]For a survey of empirical studies on this subject, see Bom & Ligthart (2013).

measured in terms of the final good and given by

$$\Xi_t(z) = \frac{\psi}{2}\left(\frac{P_t(z)}{P_{t-1}(z)} - 1\right)^2 Y_t, \tag{8}$$

where $\psi \geq 0$ governs the magnitude of the price-adjustment cost.

Dividends paid by firm z are given by

$$\mathcal{D}_t(z) = (1+\tau)P_t(z)Y_t(z) - W_t N_t(z) - P_t \Xi_t(z), \tag{9}$$

where $\tau = 1/(\theta - 1)$ is a subsidy that corrects the steady-state distortion stemming from monopolistic competition in the goods market. Firm z chooses $P_t(z)$ for all t to maximize its total real market value

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \frac{\xi_{t+s}}{\xi_t} \frac{U_{C,t+s}}{U_{C,t}} \frac{\mathcal{D}_{t+s}(z)}{P_{t+s}}, \tag{10}$$

subject to the production technology (7) and the Hicksian demand function (6).

Since all the firms face an identical problem, the optimal price will satisfy the following condition:

$$0 = \theta(mc_t - 1) - \psi\left[(1 + \pi_t)\pi_t - \beta\mathbb{E}_t \frac{\xi_{t+1}}{\xi_t} \frac{U_{C,t+1}}{U_{C,t}} \frac{Y_{t+1}}{Y_t}(1 + \pi_{t+1})\pi_{t+1}\right], \tag{11}$$

where $\pi_t = \frac{P_t}{P_{t-1}} - 1$ is the inflation rate and mc_t is the real marginal cost of production, defined by

$$mc_t = \frac{w_t}{F_N(N_t, K_{G,t})}. \tag{12}$$

2.3 Fiscal and monetary authorities

The government levies lump-sum taxes to finance its expenditures and the subsidy given to firms in the intermediate-good sector. Its budget constraint is given by

$$G_t + \tau Y_t = \frac{\mathcal{T}_t}{P_t}, \tag{13}$$

where G_t is total government spending, which is composed of two parts, public consumption and public investment

$$G_t = G_t^c + G_t^i. \tag{14}$$

Public investment increases the stock of public capital according to the following accumulation equation:

$$K_{G,t+T} = (1-\delta)K_{G,t+T-1} + \left(1 - S\left(\frac{G_t^i}{G_{t-1}^i}\right)\right)G_t^i, \tag{15}$$

where $T \geq 0$ and the function $S(\cdot)$ satisfies $S'(\cdot) \geq 0$, $S''(\cdot) \geq 0$, $S(1) = 0$, and $S'(1) = 0$. Equation (15) allows for the possibility that several periods may be required to build new productive capital, i.e., time to build (see Kydland & Prescott (1982)). This feature reflects the implementation delays

typically associated with the different stages of public investment projects (planning, bidding, contracting, construction, etc.). The function $S(\cdot)$ captures adjustment costs of public investment, as an additional unit of investment at time t increases the stock of capital at time $t + T$ by less than one unit.

2.4 Market clearing and private-sector equilibrium

Since households are identical and there is no private capital, the net supply of bonds must be zero in equilibrium ($\mathcal{B}_t = 0$). Substituting the definition of dividends in the representative household's budget constraint and using (14), one obtains the resource constraint of this economy

$$\Delta_t F(N_t, K_{G,t}) = \Delta_t Y_t = C_t + G_t^c + G_t^i, \tag{16}$$

where $\Delta_t = \left(1 - \frac{\psi}{2}\pi_t^2\right)$.

A private-sector equilibrium for this economy is a sequence of quantities and prices $\{N_t, C_t, K_{G,t}, \pi_t, w_t, mc_t\}_{t=0}^{\infty}$ such that, for a given sequence of policy variables $\{R_t, G_t^c, G_{t-T}^i\}_{t=0}^{\infty}$ and exogenous variables $\{\xi_t\}_{t=0}^{\infty}$, and given an initial stock of public capital, $K_{G,-T}$, and the definitions of the utility, production and adjustment-cost functions, equations (3), (4), (11), (12), (15), and (16) hold.

A Ramsey planner maximizes utility by choosing (and committing to) a path for R_t, G_t^c, and G_t^i subject to the constraints implied by the optimizing behavior of households and firms. In other words, the Ramsey planner selects among all the implementable private-sector equilibria described above the one that maximizes social welfare.

3 The First-Best Allocation

Before studying the Ramsey (second-best) equilibrium, it is useful to have as a benchmark the efficient (or first-best) allocation of resources, i.e., the allocation that would be chosen by a benevolent central planner.

3.1 Maximization program and solution

The planner's problem is to choose the sequence of allocations that maximize households' lifetime utility given the sequence of the economy's resource constraints and accumulation equations for

capital. Formally,

$$
\max_{Z_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big\{ \xi_t \left[U\left(C_t, N_t\right) + V\left(G_t^c\right) \right]
$$
$$
+ \lambda_{1,t} \left[F(N_t, K_{G,t}) - C_t - G_t^c - G_t^i \right]
$$
$$
+ \lambda_{2,t} \left[(1-\delta) K_{G,t+T-1} + \left(1 - S\left(\frac{G_t^i}{G_{t-1}^i} \right) \right) G_t^i - K_{G,t+T} \right] \Big\},
$$

where $Z_t = \left[C_t, N_t, G_t^c, G_t^i, K_{G,t+T} \right]$ is the vector of choice variables and the λ's are Lagrange multipliers. Note that, due to the time-to-build delay, the planner does not have any control over K_{t+i} (for $i < T$) at time t, so that the true choice variables are K_{t+T+i} for $i \geq 0$. The efficient allocation is the solution to the following set of equations:

$$
0 = \xi_t U_{C,t} - \lambda_{1,t}, \tag{17}
$$
$$
0 = \frac{U_{N,t}}{U_{C,t}} + F_N(N_t, K_{G,t}), \tag{18}
$$
$$
0 = \xi_t V_{G,t} - \lambda_{1,t}, \tag{19}
$$
$$
0 = \lambda_{1,t} - \left[1 - S\left(\frac{G_t^i}{G_{t-1}^i} \right) - \frac{G_t^i}{G_{t-1}^i} S'\left(\frac{G_t^i}{G_{t-1}^i} \right) \right] \lambda_{2,t} - \beta \mathbb{E}_t \lambda_{2,t+1} \left(\frac{G_{t+1}^i}{G_t^i} \right)^2 S'\left(\frac{G_{t+1}^i}{G_t^i} \right), \tag{20}
$$
$$
0 = \lambda_{2,t} - \beta(1-\delta) \mathbb{E}_t \lambda_{2,t+1} - \beta^T \mathbb{E}_t \lambda_{1,t+T} F_{K_G}\left(N_{t+T}, K_{G,t+T} \right), \tag{21}
$$
$$
0 = F(N_t, K_{G,t}) - C_t + G_t^C + G_t^i, \tag{22}
$$
$$
0 = K_{G,t+T} - (1-\delta) K_{G,t+T-1} - \left[1 - S\left(\frac{G_t^i}{G_{t-1}^i} \right) \right] G_t^i. \tag{23}
$$

Equation (17) defines the Lagrangian multiplier λ_1 as the marginal utility of consumption. Equation (18) equates the marginal rate of substitution between consumption and labor to their marginal rate of transformation, which is equal to the marginal product of labor. Equations (17) and (19) imply the so-called Samuelson condition:

$$
V_{G,t} = U_{C,t}, \tag{24}
$$

which equates the marginal utilities of private and public consumption. This condition states that since final output can be transformed into public as well as private consumption goods, the marginal rate of substitution between C_t and G_t^c must be equal to their marginal rate of transformation, which is 1. Equation (20) is the first-order condition with respect to public investment, relating the shadow value of private consumption to that of public capital. Without investment adjustment cost, we have $\lambda_{1,t} = \lambda_{2,t}$. The Euler equation (21) equates the costs and benefits of an additional unit of capital in period t. The cost is equal to the shadow value of capital in utils ($\lambda_{2,t}$). The two additional terms in the right-hand side of the equation are interpreted as follows. At time

$t + 1$, the invested unit of capital is worth $\beta(1 - \delta)\mathbb{E}_t\lambda_{2,t+1}$ in utils as of time t. Furthermore, investing in public capital today will generate higher output T periods in the future by a factor of $F_{K_G,t+T}$, which is valued at $\beta^T\mathbb{E}_t\lambda_{1,t+T}$ in utils as of time t. Finally, equations (22) and (23) are, respectively, the resource constraint and the accumulation equation for public capital.

3.2 Functional forms and calibration

In order to study the way in which the efficient allocation changes in response to a negative shock to ξ_t, we need to specify functional forms for the utility and production functions, and to assign values to the model parameters. We assume that preferences take the form

$$
\begin{aligned}
U\left(C_t, N_t\right) + V\left(G_t^c\right) \quad &= \frac{\left(C_t^\gamma(1 - N_t)^{1-\gamma}\right)^{1-\sigma}}{1 - \sigma} + \chi\frac{(G_t^c)^{1-\sigma}}{1 - \sigma} \qquad \text{if } \sigma \neq 1 \\
&= \gamma \ln C_t + (1 - \gamma)\ln(1 - N_t) + \chi \ln\left(G_t^c\right) \qquad \text{if } \sigma = 1,
\end{aligned}
\tag{25}
$$

where $\sigma > 0$ and $0 < \gamma \leq 1$, and that the production function is given by

$$
F\left(N_t, K_{G,t}\right) = N_t^a K_{G,t}^b,
\tag{26}
$$

where $0 \leq a, b \leq 1$. This specification nests the linear technology assumed by Christiano et al. (2011) and Woodford (2011) as a special case in which $a = 1$ and $b = 0$. We also assume that the adjustment-cost function, S, is given by

$$
S\left(\frac{G_t^i}{G_{t-1}^i}\right) = \frac{\varpi}{2}\left(1 - \frac{G_t^i}{G_{t-1}^i}\right)^2,
$$

where $\varpi > 0$.

Our calibration closely follows Bouakez et al. (2015), and is summarized in Table 1. The values of β and σ are based on Christiano et al. (2011). The value of b is based on the meta-regression results of Bom & Ligthart (2013) and is very close to the values considered by Baxter & King (1993) and Leeper et al. (2010). The elasticity of substitution between domestic goods, θ, is chosen so as to yield a steady-state markup of 20 percent. The price-cost-adjustment parameter, ψ, is set such that, conditional on the chosen value of θ, it implies a slope of the (linearized Phillips curve) equal to 0.03. Consistent with the evidence discussed in Leeper et al. (2010), Leduc & Wilson (2013), and Bouakez et al. (2015) regarding the delays associated with the completion of public investment projects, we set $T = 16$. We also follow Leeper et al. (2010) and set the depreciation rate, δ, to 0.02. The investment-adjustment-cost parameter, ϖ, is more difficult to pin down, as empirical estimates are only available for private investment. We set $\varpi = 2.5$, which is very close to the macro estimate of 2.48 obtained by Christiano et al. (2005) and to the micro estimate of 1.86 obtained by Eberly et al. (2012) for private investment. Finally, we calibrate γ and χ such that, given the values of

the remaining parameters, the fraction of time devoted to work in steady state, N, is equal to $1/3$, and the steady-state ratio of total government spending to output, $g \equiv \frac{G^c + G^i}{Y}$, is equal to 0.2.[7] Given our calibration, the implied share of public investment in total public spending, $\alpha \equiv \frac{G^i}{G^c + G^i}$, is equal to 0.2286, which is very close to the historical average of 0.23 that we observe in U.S. data.

Table 1: Parameter values.

Discount factor	$\beta = 0.99$
Preference parameter	$\sigma = 2$
Preference parameter	$\gamma = 0.29$
Preference parameter	$\chi = 0.054$
Elasticity of output w.r.t public capital	$b = 0.08$
Elasticity of output w.r.t hours worked	$a = 1$
Elasticity of substitution between intermediate goods	$\theta = 6$
Time-to-build delay	$T = 16$
Price-adjustment-cost parameter	$\psi = 200$
Depreciation rate of public capital	$\delta = 0.02$
Investment-adjustment-cost parameter	$\varpi = 2.5$
Steady-state ratio of public spending to output	$g = 0.2$
Autocorrelation of the preference shock	$\rho = 0.9$

3.3 The efficient response to a negative preference shock

Assume that the economy is initially at the steady state when a negative preference shock hits. The shock is assumed to be persistent, with an autocorrelation coefficient of 0.9.[8] Using a linearized version of the equilibrium conditions (17)–(21) around the steady state, we compute the economy's response to the shock. The results are depicted in Figure 1. All the responses, except that of hours worked, are expressed as percentage deviations from steady-state output.

The negative preference shock increases households' desire to save. Because the accumulation of physical (public) capital allows the intertemporal substitution of consumption while raising future production capacities, current consumption falls and public investment rises in response to the shock. Due to the time-to-build delays, public investment increases the stock of capital — and thus the marginal productivity of labor — 16 quarters later. Eventually, public investment falls below its steady-state level, before converging to it from below. Note that due to investment-adjustment costs, the response of public investment is hump shaped. In the absence of these costs, the maximum increase in investment would take place at the time of the shock, and the response would be less persistent. The figure also shows that the optimal path of government consumption follows closely that of private consumption, in accordance with the Samuelson condition.

Hours worked respond in an opposite way to consumption during the first 15 quarters after

---

[7]See the Appendix for a detailed description of the steady state.

[8]The size of the shock will be discussed in section 4.2.
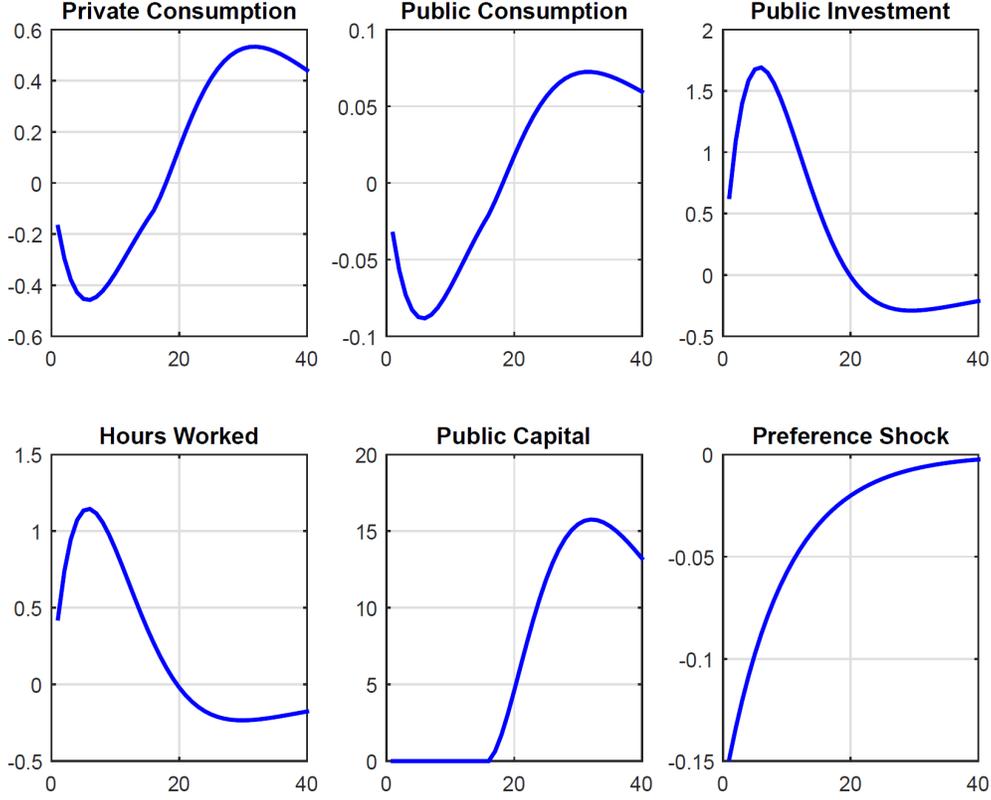
Figure 1: First best allocation after a negative preference shock.

Notes: The figure shows the economy's efficient response to a negative preference shock. The response of private consumption, public consumption, public investment, and public capital are expressed as percentage deviations from steady-state output.

the shock. This follows from (18): since public capital is predetermined for $t < 16$, and given our assumptions that the production function is concave in labor, that the utility is concave in consumption and leisure, and that the latter are assumed to be normal goods, an increase in the marginal utility of consumption $(U_{C,t})$ requires an increase in the marginal disutility of labor $(-U_{N,t})$ to restore the equilibrium. This in turn implies that hours worked must increase following the shock.[9]

[9]To see this, log-linearize equation (18) around the steady state. This yields

$$\Upsilon \hat{C}_t + \Theta \hat{N}_t = \frac{N F_{NN}}{F_N} \hat{N}_t + \frac{K_G F_{NK_G}}{F_N} \hat{K}_{G,t}, \tag{27}$$

where $\hat{X}_t \equiv \frac{X_t - X}{X}$, variables without a time subscript denote steady-state values, and

$$\Upsilon \equiv \frac{C U_{CN}}{U_N} - \frac{C U_{CC}}{U_C},$$

$$\Theta \equiv \frac{N U_{NN}}{U_N} - \frac{N U_{CN}}{U_C}.$$

Because public capital is predetermined for $t < 16$, the last term in the right-hand side of equation (27) is equal to

The main take-away from these results is that the efficient response of public investment to a (negative) preference shock differs drastically from that of public consumption. At least during the first quarters following the shock, the optimal allocation of resources calls for a simultaneous increase in public investment and a cut in public consumption. In other words, the efficient response to the shock requires a substantial change in the composition of public spending, assigning a substantially larger weight on public investment compared with the steady-state allocation.

4 Optimal Monetary and Fiscal Policy

Consider now the Ramsey problem. The policy maker has three tools : i) the nominal interest rate, R_t, ii) government consumption, G_t^c, and iii) government investment, G_t^i. Assuming that the policy maker can commit to future paths of these variables, the Ramsey problem consists in maximizing household's lifetime utility subject to the private-sector equilibrium defined in Section 2.4 and the non-negativity constraint on the nominal interest rate. The Lagrangian of this problem is given by

$$
\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \bigg\{ \xi_t \left[U\left(C_t, N_t\right) + V\left(G_t^c\right) \right]
$$

$$
+ \phi_{1,t} \left[\beta \frac{\xi_{t+1}}{\xi_t} \frac{U_{C,t+1}}{1+\pi_{t+1}} \left(1+R_t\right) - U_{C,t} \right]
$$

$$
+ \phi_{2,t} \left[\theta\left(mc_t - 1\right) - \psi\left(d\left(\pi_t\right) - \Lambda_{t+1} \frac{F\left(N_{t+1}, K_{G,t+1}\right)}{F\left(N_t, K_{G,t}\right)} d\left(\pi_{t+1}\right) \right) \right]
$$

$$
+ \phi_{3,t} \left[F(N_t, K_{G,t}) \left(1 - \frac{\psi}{2}\pi_t^2\right) - C_t - G_t^c - G_t^i \right]
$$

$$
+ \phi_{4,t} \left[\left(1-\delta\right) K_{G,t+T-1} + \left(1 - S\left(\frac{G_t^i}{G_{t-1}^i}\right)\right) G_t^i - K_{G,t+T} \right]
$$

$$
+ \phi_{5,t} \left[R_t - 0 \right] \bigg\},
$$

where $mc_t = -\frac{U_{N,t}}{U_{C,t}F_{N,t}}$, and where we have defined $d(\pi_t) \equiv \pi_t(1+\pi_t)$ and $\Lambda_{t+1} \equiv \beta \frac{\xi_{t+1}}{\xi_t} \frac{U_{C,t+1}}{U_{C,t}}$.

After deriving the (non-linear) first-order conditions (see the Appendix for details) with respect to the control variables, which are grouped in the vector $Z_t' = [Z_t, R_t, \pi_t]$, the system of equilibrium

zero. The condition for consumption and leisure to be both normal goods is

$$
\frac{\Upsilon}{\Theta} > 0.
$$

Concavity of the utility function with respect to consumption and leisure in turn implies that $\Upsilon, \Theta > 0$. Rearranging equation (27) yields

$$
\left(\frac{NF_{NN}}{F_N} - \Theta\right) \hat{N}_t = \Upsilon \hat{C}_t.
$$

Finally, noting that $\frac{NF_{NN}}{F_N} \leq 0$, one can easily see that hours and consumption move in opposite directions.

conditions is solved up to a first-order approximation around the steady state. In what follows, we study the Ramsey-optimal policy under two distinct cases: one in which the preference shock is relatively small, such that the economy never hits the ZLB, and one in which the preference shock is large enough to make the ZLB bind, sending the economy into a liquidity trap. We use the piecewise linear perturbation algorithm developed by Guerrieri & Iacoviello (2014) to deal with the ZLB constraint.[10] In what follows, we will refer to the level of public spending that would be optimal under fully flexible prices as *neoclassical spending*, and label as *stimulus spending* the difference between the spending level obtained under the Ramsey plan and neoclassical spending. Stimulus spending would therefore be solely due to the distortions stemming from price stickiness (or, equivalently, the presence of a non-zero output gap), which the monetary authority cannot fully eliminate when the nominal interest rate hits the ZLB.[11]

4.1 Non-binding ZLB

Consider first the case in which monetary policy is not constrained by the ZLB. Since the only distortion in this economy stems from price rigidity in the goods market,[12] monetary policy can replicate the flexible-price allocation by equating the nominal interest rate to the (efficient) natural rate of interest. This policy fully stabilizes inflation and the output gap in every period, a result that has come to be known as the divine coincidence (Blanchard & Galí (2007)). The optimal levels of government consumption and investment are therefore obvious: they must coincide with those obtained under the efficient allocation, discussed in Section 3. In other words, to the extent that monetary policy can maneuver freely without hitting the ZLB, the Ramsey allocation replicates the first best. In this case, stimulus spending is nil by construction.

4.2 Binding ZLB

We now consider the scenario in which the preference shock is large enough to precipitate the economy into a liquidity trap.[13] Below, we discuss three different sets of policy responses: In the

---

[10]Guerrieri & Iacoviello (2014) show that their algorithm approximates reasonably well the global solution in models with an occasionally binding ZLB constraint. In section 4.4.3, we study the robustness of our results to the use of a non-linear solution method.

[11]Werning (2011) proposes an alternative decomposition whereby *stimulus* spending is defined as deviations from *opportunistic* spending; the latter being defined as "the level of government purchases that is optimal from a static, cost-benefit standpoint, taking into account that, due to slack resources, shadow costs may be lower during a slump."

[12]Recall that the monopolistic competition distortion is corrected using per-unit subsidy given to monopolistically competitive producers.

[13]To calibrate the size of this shock in a realistic manner, we proceed as follows. Consider a version of our economy in which monetary policy is set according to the following Taylor rule

$$R_t = \max\left\{0; \phi_\pi \pi_t - \ln(\beta)\right\},$$

where $\phi_\pi > 1$. We select the size of the shock such that the resulting decline in output when $\phi_\pi = 1.5$ and in the absence of any fiscal-policy response matches the observed decline in U.S. GDP from peak to trough during the Great Recession, which amounted to 5.65%.

first, only the nominal interest rate is used as an instrument; in the second, both the nominal interest rate and public consumption are used; and in the third, government investment is added. Studying the optimal choice of public investment in an economy plunged in a liquidity trap is the main novelty of this paper with respect to the existing literature, which has focused exclusively on optimal public consumption.

4.2.1 Optimal monetary policy alone

We start with the case in which the Ramsey planner chooses the nominal interest rate optimally while keeping public consumption and investment constant at their (optimal) steady-state levels, a scenario that we will refer to as scenario A. Figure 2 shows that, in response to the negative preference shock, the nominal interest rate falls until it reaches the ZLB floor where it remains for 6 quarters before gradually reverting to its steady-state level. In line with the results obtained by Werning (2011), Nakata (2013), and Nakata (2015), consumption and inflation fall initially but rise subsequently during a few quarters before falling again below their steady-state levels, to which they ultimately converge from below. The Ramsey allocation deviates from the flexible-price allocation.[14] The reason is that the natural rate of interest., i.e., the nominal rate that implements the flexible-price allocation is negative during the first 4 periods. As is well known by now, the optimal policy in this case is to commit to keeping the nominal rate at zero even after the natural rate has become positive.[15] By doing so, the Ramsey planner commits to generating a boom in consumption at some point in the future, which raises inflation expectations. Although the negative output gap is not fully eliminated (-0.6 percent), it is much smaller than that obtained under a standard Taylor rule discussed (-5.65 percent).[16]

4.2.2 Optimal monetary and fiscal policy without public investment

Next, consider the case where both the nominal interest rate and public consumption are chosen optimally, while public investment is kept constant at its steady-state level, a plan that we will refer to as scenario B. Under this scenario, depicted in Figure 3, the nominal interest rate, inflation, and consumption exhibit very similar responses — both qualitatively and quantitatively — to those obtained under scenario A. Optimal public consumption exhibits an opposite response to that of private consumption, rising during the first three quarters after the shock, then falling during the subsequent 4 quarters before returning to its steady-state value. In other words, the optimal fiscal plan is front-loaded, and eventually entails a spending cut, a pattern also shown by Werning (2011), Nakata (2013), and Nakata (2015). Note that, under flexible prices, optimal public consumption

---

[14]In the absence of capital accumulation, the flexible-price allocation remains equal to its steady-state level even if the preference shock creates an incentive for households to cut their current consumption, since there is no other mechanism that enables intertemporal substitution.

[15]See Eggertsson & Woodford (2003), Jung et al. (2005), and Adam & Billi (2006).

[16]See Footnote 13.
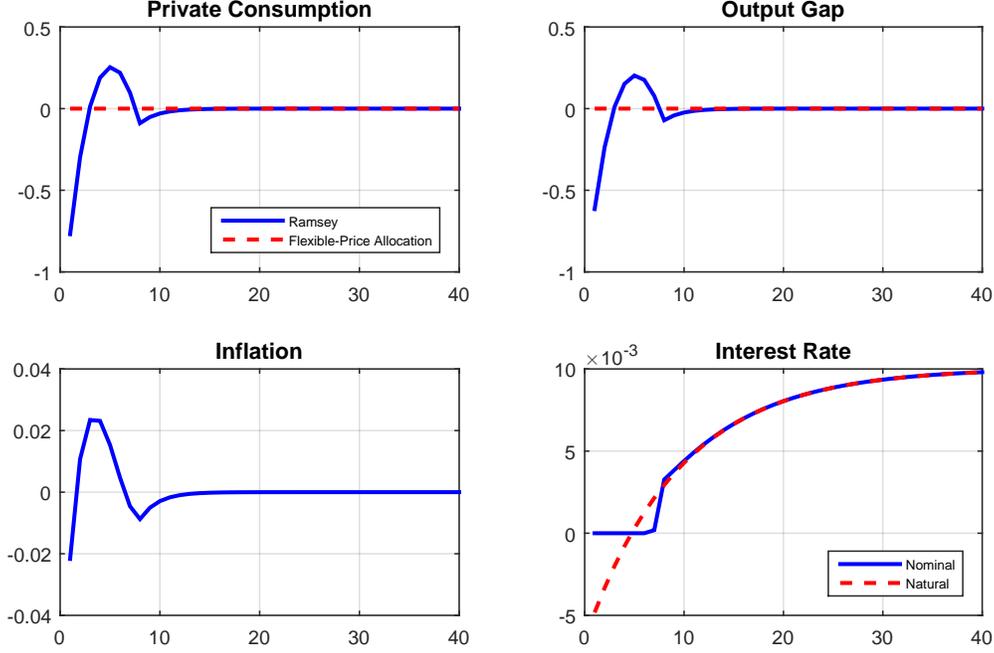
Figure 2: Ramsey optimal monetary policy after a negative preference shock.

Notes: The figure shows the economy's response to a negative preference shock when only the nominal interest rate is chosen optimally by a Ramsey planner, while public consumption and public investment are kept constant at their steady-state levels. The response of private consumption is expressed as a percentage deviation from steady-state output.

remains equal to its steady-state level, which means that public spending under the Ramsey plan is all stimulus spending.

In order to understand the intuition behind the optimal response of public consumption under the Ramsey plan, differentiate the households' lifetime utility with respect to government consumption and evaluate it at the maximum. This yields

$$U_{C,t}\frac{dC_t}{dG_t^c} + U_{N,t}\frac{dN_t}{dG_t^c} + V_{G,t} + \mathbb{E}_t \sum_{s=t+1}^{\infty} \beta^{s-t}\xi_s \left\{ \frac{dU(C_s, N_s)}{dG_t^c} + \frac{dV(G_s^c)}{dG_t^c} \right\} = 0.$$

For simplicity, let us abstract from the effects of G_t^c on future variables. Since $\frac{dC_t}{dG_t^c} = \Delta_t \left(\frac{dY_t}{dG_t^c} + \frac{Y_t}{\Delta_t}\frac{d\Delta_t}{dG_t^c} \right) - 1$ and $\frac{dN_t}{dG_t^c} = \frac{dY_t}{dG_t^c}F_{N,t}^{-1}$, the condition above can be rewritten as

$$\begin{aligned} \frac{V_{G,t}}{U_{C,t}} &= 1 - \left(\Delta_t + \frac{U_{N,t}}{U_{C,t}F_{N,t}} \right) \frac{dY_t}{dG_t^c} - Y_t \frac{d\Delta_t}{dG_t^c} \\ &= 1 - (1 - mc_t)\frac{dY_t}{dG_t^c} + \frac{\psi}{2}\pi_t^2 \frac{dY_t}{dG_t^c} - Y_t \frac{d\Delta_t}{dG_t^c}. \end{aligned} \quad (28)$$

Under flexible prices, $\Delta_t = 1$, $d\Delta_t = 0$ and $mc_t = 1$, so that the Samuelson condition holds: $V_{G,t} = U_{C,t}$. This condition would also hold when prices are sticky but the monetary authority can
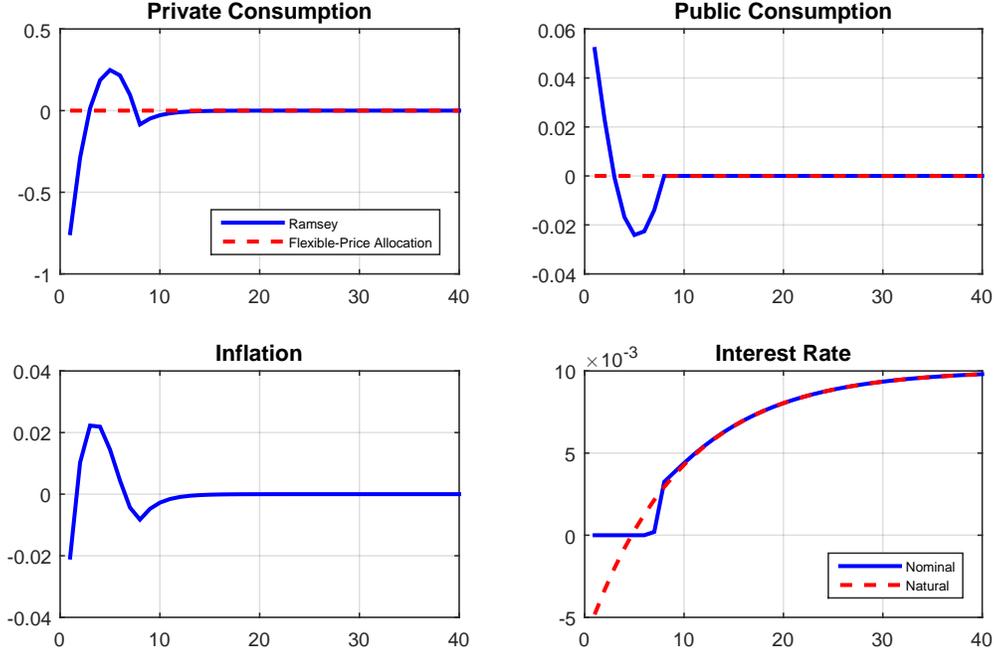
Figure 3: Ramsey optimal monetary and fiscal policy after a negative preference shock (without public investment).

Notes: The figure shows the economy's response to a negative preference shock when the nominal interest rate and public consumption are chosen optimally by a Ramsey planner, while public investment is kept constant at its steady-state level. The response of private and public consumption are expressed as percentage deviations from steady-state output.

fully stabilize inflation and real marginal cost (thus eliminating the output gap).

In contrast, when monetary policy is constrained by the ZLB – so that full stabilization is unattainable – the optimal choice of G_t^c deviates from the Samuelson condition. In particular, when the economy is hit by a preference shock that makes the ZLB bind, real marginal cost falls, which implies that the term $(1 - mc_t) \frac{dY_t}{dG_t^c}$ is positive.[17] Equation (28) therefore implies that the marginal rate of substitution between public and private consumption, $\frac{V_{G,t}}{U_{C,t}}$, must be smaller than 1.[18] Intuitively, this condition reflects the trade-off that the Ramsey planner faces in the presence of a negative output gap: the Samuelson condition calls for lowering G_t^c but a lower G_t^c further widens the output gap. In this case, it is optimal to increase G_t^c but only by a small extent in order not to deviate too much from the Samuelson condition.

This intuition is confirmed by Figure 3, which shows that the optimal increase in public con-

---

[17]Following the shock, mc_t falls below its steady-state of 1. Thus, $1 - mc_t$ is positive. In addition, a well established result is that the output multiplier associated with public consumption, $\frac{dY_t}{dG_t^c}$, is positive and even exceeds 1 when the ZLB binds. The inflationary effect of higher public spending lowers the real interest rate (since the nominal rate is constant) and raises consumption.

[18]The last two terms in the right-hand side of this equation are of second order and will therefore disappear in a log-linear version of the equilibrium conditions.

Table 2: Welfare Implications of Ramsey Policies in a Deep Recession.

Scenario	Instrument	Welfare gain relative to A
B	R, G^c	2×10^{-4}
C	R, G^c, G^i	0.082

Notes: Entries are expressed in percent. Scenario A corresponds to the Ramsey plan in which only the nominal interest rate is adjusted optimally while public spending is kept constant. Welfare gains are measured by the compensating variation in consumption, i.e., the percentage change in consumption that would make households as well off under under scenario A as under the alternative scenario.

sumption is very small: the largest response, which occurs immediately after the shock, is roughly 0.05 percent of steady-state output. The total size of the fiscal expansion is measured by the cumulative variation of public spending from its steady-state level (expressed as a percentage of steady-state output), $\sum_{t=0}^{\infty}(G_t - G)/Y$. Since government investment is constant in this scenario, the numerator only captures changes in public consumption. As expected from the path of public consumption, the total size of the stimulus is virtually nil: -0.003 percent of steady-state output.[19] Though in a different setting, this result echoes Bilbiie et al. (2014)'s observation that the optimal level of public spending is tiny when the ZLB binds.

The close resemblance of the Ramsey allocations obtained under scenarios A and B in turn suggests that the welfare gain from optimally adjusting G_t^c is quite small. To verify this conjecture, we compute the compensating variation in consumption, i.e., the percentage in consumption that would make households as well off under scenario A as under scenario B. The results, reported in Table 2, indicate that this quantity is indeed negligible (2×10^{-4} percent).

4.2.3 Public investment as an additional tool

Finally, consider the case in which the set of policy instruments is enlarged to include public investment. The economy's optimal response to a negative preference shock in this case — which we label scenario C — is shown in Figure 4. The response of private consumption is characterized by a protracted decline followed by a persistent increase above its steady-state level. The initial decline in consumption is larger than that occurring under the efficient allocation of resources, giving rise to a negative output gap. Public consumption increases immediately after the shock by 0.05 percent of steady-state output but falls persistently in the subsequent periods and remains below its steady-state level even after the natural rate of interest has ceased to be negative. Public investment initially increases by 0.6 percent of steady-state output and reaches its maximum response of 1.7 percent of steady-state output 3 quarters after the shock. It remains above average for more than 4 years — i.e., well after the natural interest rate has become positive again — before eventually falling below its steady-state level. Once the ZLB has ceased to bind, the Ramsey allocation tracks

---

[19]We truncate the summation at horizon 1000.

the first best almost exactly.
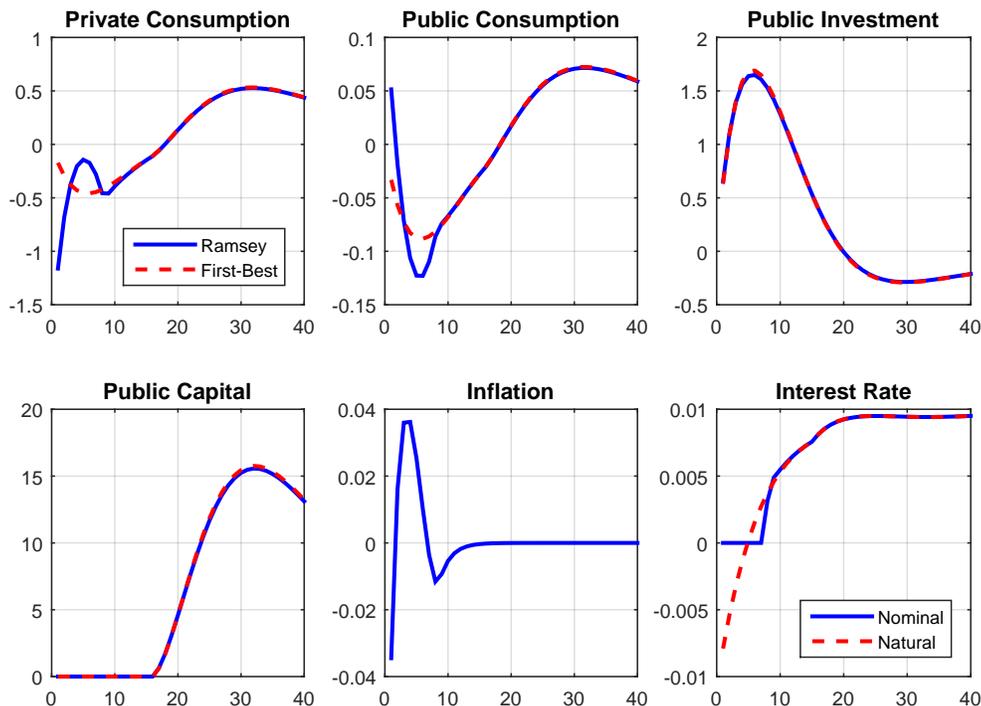
Figure 4: Ramsey optimal monetary and fiscal policy after a negative preference shock.

Notes: The figure shows the economy's response to a negative preference shock when the nominal interest rate, public consumption, and public investment are chosen optimally by a Ramsey planner. The response of private consumption, public consumption, public investment, and public capital are expressed as percentage deviations from steady-state output.

Three important observations about optimal fiscal policy are worth emphasizing. First, as is the case with the first-best response to the preference shock, the Ramsey plan entails a change in the composition of public spending in a way that assigns a larger weight to public investment relative to public consumption. Figure 5 depicts the (time-varying) optimal share of public investment in total public spending. At the steady state, this fraction is equal to 22.8 percent. Immediately after the shock, it surges to 25.2 percent, reaches a peak of roughly 29 percent, before falling steadily until it reaches a trough of roughly 21.7 percent at around 30 quarters after the shock. It then converges towards its steady-state value from below.

Second, the cumulative increase in total public spending in response to the shock is substantial, amounting to 11.6 percent of steady-state output. This result contrasts sharply with that obtained under scenario B — and in the literature cited above — in which public spending plays no productive role. Third, most of the increase in public spending involves public investment and is almost entirely neoclassical in nature. In other words, the stimulus component of the fiscal expansion is negligible.
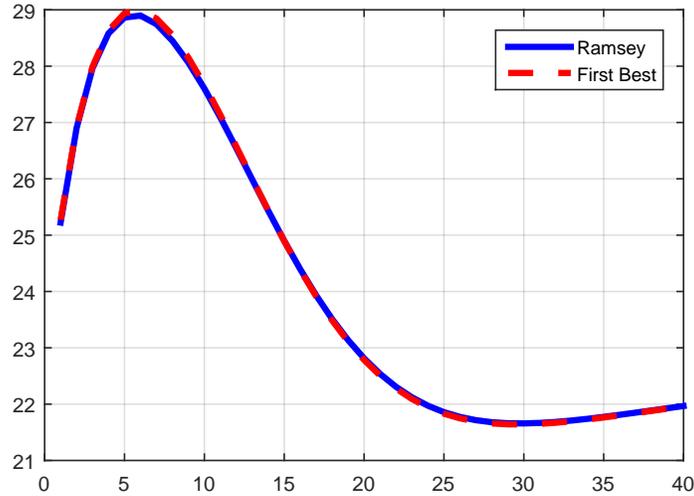
Figure 5: Optimal share of public investment in total public spending (in percent).

The latter observation is illustrated in Figure 6, which depicts the stimulus component of public consumption and investment in response to the shock. In both cases, stimulus spending — albeit positive at the time of the shock — is tiny, reaching a maximum of roughly 0.085 percent of steady-state output for public consumption and 0.015 percent of steady-state output for public investment. In cumulative terms, total stimulus spending amounts to -0.12 percent of steady-state output.

Intuitively, the stimulus component of public investment is positive because the latter helps close the output gap while preventing public consumption from deviating too much from the Samuelson condition. In other words, the burden of stabilizing output is now shared between the two fiscal instruments. Note that due to the convex adjustment costs, it is not optimal to use only public investment as a stabilization tool.

In terms of welfare, Table 1 shows that optimally adjusting public investment to the shock allows the economy to achieve a sizable welfare gain relative to the scenario in which public spending is kept constant, which amounts to 0.082 percent of steady-state consumption. This gain is two orders of magnitude larger than the gain achieved by only adjusting public consumption while keeping public investment constant.

These findings highlight the following important point: the conclusion one draws about the optimal size of a fiscal expansion and its welfare implications crucially depends on the set of instruments available to the policymaker. Whenever public investment is possible, the optimal fiscal expansion is rather sizable and the welfare gains it achieves are significantly larger than those associated with a plan that abstracts from public investment.
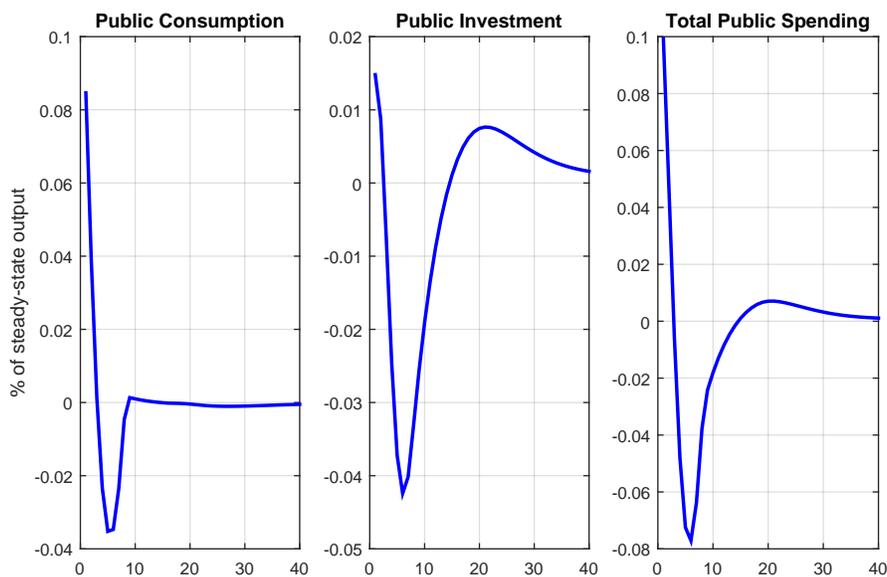
Figure 6: Optimal stimulus spending.

Notes: Stimulus spending is defined as the difference between the spending level obtained under the Ramsey allocation and that obtained under the flexible-price allocation. Stimulus spending is therefore only due to the presence of an output gap.

4.3   Discussion

Bachmann & Sims (2012) present evidence that, conditional on a positive government spending shock — that is, an exogenous and unanticipated increase in public expenditures — the ratio of U.S. public investment to public consumption rises more during recessions than during expansions. In our model, public spending is chosen optimally and there is no such thing as a public spending shock. Nonetheless, one can still regard the evidence reported by Bachmann & Sims (2012) as being broadly consistent with the implication of our model that a fiscal expansion that occurs during an economic downturn involves a change in the composition of government spending in a way that assigns a larger weight to public investment relative to public consumption.

Our results also give credence (at least partially) to the two largest fiscal plans that were implemented in 2008–2009 to cope with the global economic downturn, namely, the American Recovery and Reinvestment Act and the European Economic Recovery Plan. Roughly 40 percent of the authorized spending (excluding transfers) within the ARRA was allocated to public investment projects in infrastructure (see Drautzburg & Uhlig (2013)). This fraction is nearly twice as large as the historical average share of public investment in total public spending in the U.S. (23 percent). Likewise, Coenen et al. (2013) report that the European Economic Recovery Plan allocated

71 percent of the additional government spending to public investment,[20] whereas the historical average of this fraction is approximately 11.5 percent (see also Cwik & Wieland (2011)).

Finally, the path of U.S. federal public investment observed during the post-ARRA period is remarkably consistent with the prescriptions of our model. Figure 7 indeed shows that the share of federal investment to federal total spending in the U.S. rose for 10 quarters after 2009Q1, reaching a peak at 2011Q3, before declining persistently during the subsequent years and eventually falling below its pre-2009 level. This path closely resembles that illustrated in Figure 5.
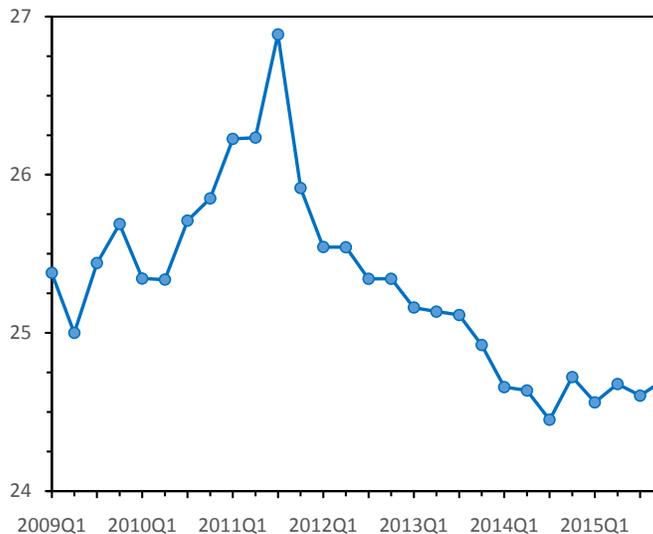
Figure 7: Share of federal non-defense gross investment in total federal non-defense spending on goods and services in the U.S. (in percent).

Source: U.S. Bureau of Economic Analysis (authors' calculations).

4.4 Robustness Analysis

This section studies the robustness of our results along three different dimensions: the use of a non-linear solution method, the size of the shock that makes the ZLB bind, and the inclusion of private capital.

4.4.1 Non-linear solution method

In our baseline analysis, we have relied on Guerrieri & Iacoviello (2014)'s piecewise linear algorithm to solve the model under an occasionally binding ZLB constraint. This required us to linearize the model around the zero-inflation deterministic steady state, which in turn implies that the Rotemberg (1982)-type quadratic adjustment costs vanish from the resource constraint. Guerrieri

---

[20]See Table 5 in Coenen et al. (2013).

& Iacoviello (2014) show that their algorithm approximates reasonably well the global solution in a number of models with occasionally binding constraints. Bilbiie et al. (2014), however, argue that one may obtain misleading results if a new-Keynesian model with a ZLB constraint is solved up to a first-order approximation that excludes price adjustment costs from the resource constraint.

To see whether and to what extent our results are affected by taking a linear approximation, we now work with the full, non-linear model. In solving the model, we assume that agents have perfect foresight over the simulation horizon.[21] We compute the Ramsey allocation following a negative preference shock using the same parameter values as in the baseline simulations. The results of this experiment are shown in Figure 8.
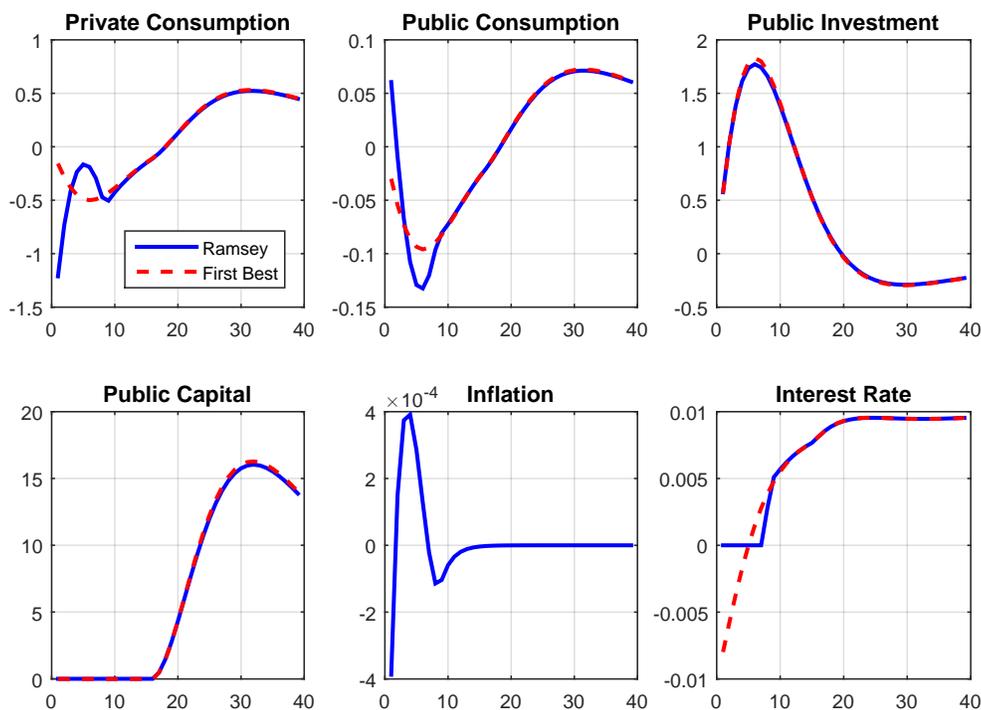
Figure 8: Ramsey optimal monetary and fiscal policy after a negative preference shock in the non-linear version of the model.

Notes: The figure shows the economy's response to a negative preference shock when the nominal interest rate, public consumption, and public investment are chosen optimally by a Ramsey planner. The response of private consumption, public consumption, public investment, and public capital are expressed as percentage deviations from steady-state output.

Comparing Figure 8 with Figure 4, one can see that the results are essentially unchanged. Quantitatively, the differences between the linear and non-linear allocations are negligible. For example, the cumulative increase in total public spending is 11.2 percent of steady-state output

---

[21] This approach has also been followed by Christiano et al. (2011), Werning (2011), and Bhattarai & Egorov (2016), among others.

Table 3: Cumulative Increase in Total and Stimulus Spending.

Case	Total spending	Stimulus spending
Baseline	11.60	-0.12
Non-linear solution	11.23	-0.36
Large preference shock	26.47	-1.93
Model with private capital	12.87	-0.18

Notes: The table shows the cumulative increase in total public spending and in total stimulus spending (in percent of steady-state output) after a negative preference shock that causes the ZLB to bind.

in the non-linear case, whereas the corresponding number is 11.6 percent in the linear solution (see Table 3). The same observation holds for the stimulus component of public spending, shown in Figure 9. At the time of the shock, stimulus spending is identical in the linear and non-linear versions of the model, and follows similar patterns in the subsequent periods. Given the similarity of the results based on the non-linear solution method and the piecewise linear solution algorithm, we only report the results based on the latter approach in the remainder.
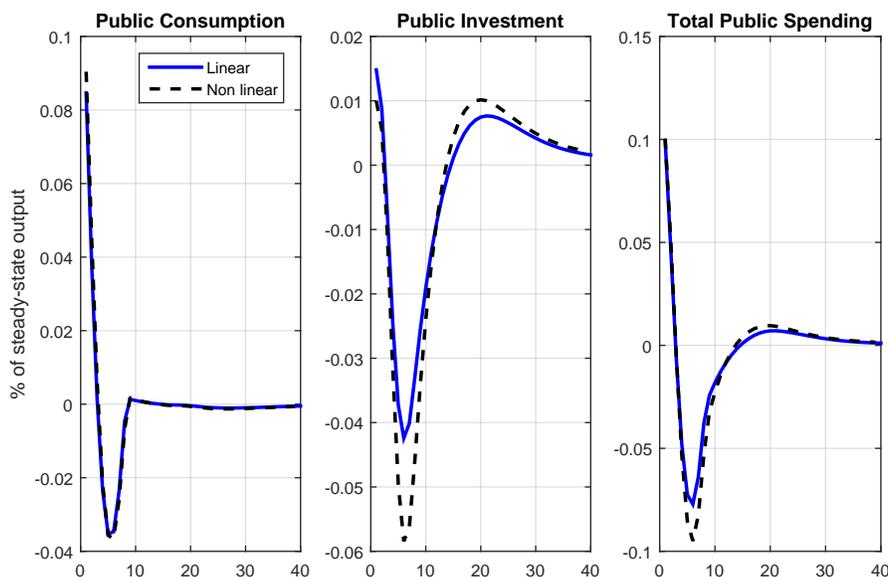
Figure 9: Optimal stimulus spending in the non-linear version of the model.

Notes: Stimulus spending is defined as the difference between the spending level obtained under the Ramsey allocation and that obtained under the flexible-price allocation. Stimulus spending is therefore only due to the presence of an output gap.

4.4.2 Size of the shock

The results discussed so far have shown that the optimal policy response to a preference shock that would otherwise generate a contraction of the same magnitude as the Great Recession entails raising public investment significantly and persistently above normal. Yet, the fraction of this fiscal expansion intended to fill the negative output gap — stimulus spending — is negligible. Would this still be the case if the economy faced an even larger shock? To investigate this question, we follow Woodford (2011) and Bilbiie et al. (2014) and consider a scenario akin to the Great Depression, that is, one in which the preference shock is large enough to generate an output decline of 28.8 percent in the decentralized economy. Figure 10 depicts the economy's response to the shock in the first-best and Ramsey equilibria.
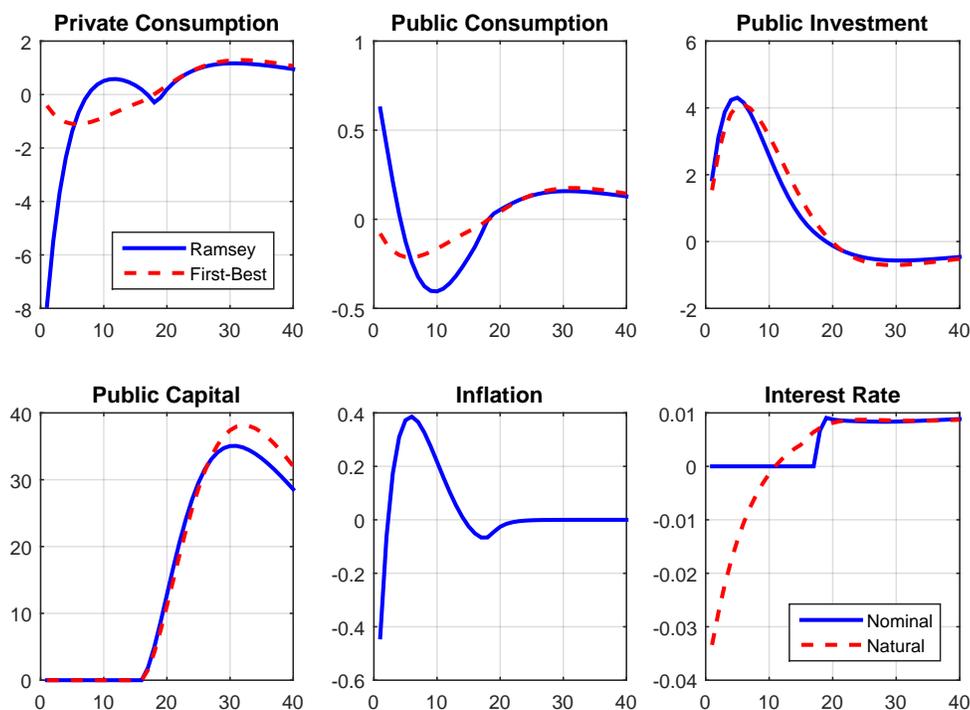
Figure 10: Ramsey optimal monetary and fiscal policy after a large negative preference shock (Great Depression calibration).

Notes: The figure shows the economy's response to a negative preference shock when the nominal interest rate, public consumption, and public investment are chosen optimally by a Ramsey planner. The response of private consumption, public consumption, public investment, and public capital are expressed as percentage deviations from steady-state output.

Qualitatively, the results are similar to those shown in Figure 4. There are, however, important quantitative differences. Under the Great-Depression scenario, the natural rate of interest remains negative for 10 quarters. In the Ramsey equilibrium, consumption falls by 8 percent and the

24

policymaker keeps the nominal interest rate at zero for 17 quarters. Public consumption rises initially by roughly 0.6 percent of steady-state output and remains above its steady-state level for 4 quarters; it then falls below that level during the subsequent 12 quarters. Public investment also rises in a hump-shaped manner during the first 18 quarters after the shock, with a peak response of 4.2 percent of steady-state output. These results have two implications. First, the optimal composition of public spending is even more heavily skewed towards public investment than in the baseline case: in the Ramsey allocation, the share of public investment in total public spending surges to roughly 29 percent on impact and reaches a peak of 37 percent 4 quarters after the shock (figure not reported). Second, the fiscal expansion is larger than that occurring in the baseline case: as a percentage of steady-state output, the cumulative increase in total public spending exceeds 26 percent in the Great-Depression scenario (see Table 3).

Figure 10 also shows that both public consumption and public investment are larger in the Ramsey allocation than in the first-best during the first 4 quarters after the shock, thus implying that stimulus spending is initially positive. Figure 11 quantifies this component. At the time of the shock, stimulus spending amount to, respectively, 0.7 and 0.3 percent of steady-state output for public consumption and public investment. This increase, however, is more than fully offset by a drop in stimulus spending during the subsequent quarters. Table 3 shows that cumulative stimulus spending is -1.93 percent of steady-state output.
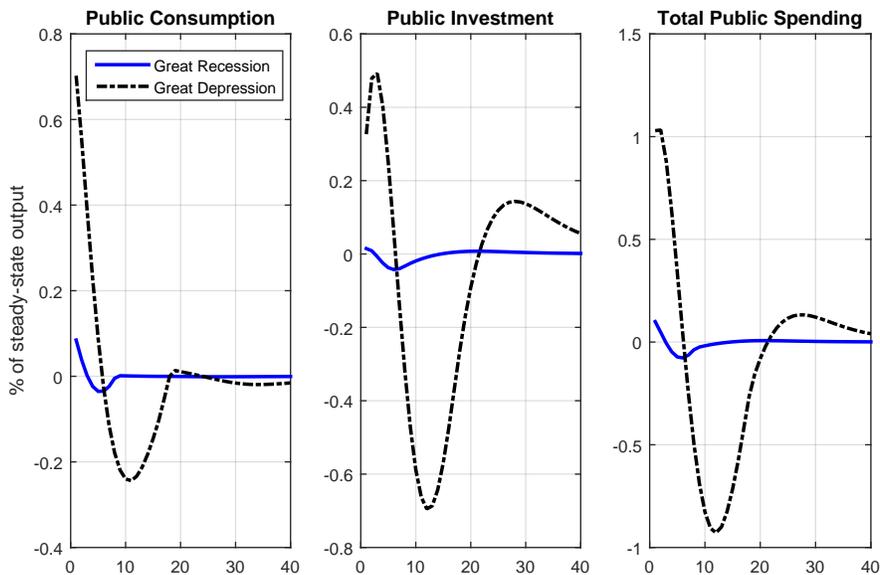
Figure 11: Optimal stimulus spending in response to a large shock (Great Depression Calibration).

Notes: Stimulus spending is defined as the difference between the spending level obtained under the Ramsey allocation and that obtained under the flexible-price allocation. Stimulus spending is therefore only due to the presence of an output gap.

4.4.3 Model with private capital

In the simple model studied so far, the optimal response to the adverse shock involved raising public spending and shifting its composition towards public investment because the latter enables the intertemporal allocation of resources as well as consumption smoothing. Would this policy response still be warranted if we also allowed the private sector to accumulate capital? To investigate this question, we augment the baseline model with private capital, which is accumulated by private households and rented to firms. As is the case with public capital, the accumulation of private capital is also subject to investment-adjustment costs. For simplicity, however, we abstract from time-to-build delays. In this extended version of the model, the production function takes the following form:

$$F(N_t, K_t, K_{G,t}) = N_t^a K_t^{1-a} K_{G,t}^b,$$

where $a, b \in [0,1]$. To conserve space, we leave out the full description of the model and the equilibrium conditions.[22] In our simulations, we set a to 2/3. We also assume that private and public capital have the same depreciation rate and adjustment-cost parameter. The remaining parameters are assigned identical values to those in the baseline simulations. The preference shock is again calibrated such that the resulting fall in output matches the observed decline in U.S. GDP from peak to trough during the Great Recession. Figure 12 shows the economy's response to the shock in the first-best and Ramsey equilibria.

The figure shows that both public and private investment rise in response to the shock, but the former rises less than in the baseline model, peaking at roughly 0.7% of steady-state output under the Ramsey allocation — compared to 1.7% in the baseline model. On the other hand, the response of public investment is now more persistent and converges to 0 from above. Public consumption also rises under the Ramsey plan but only by a negligible amount, as is the case in the model without private capital. This means that even in the presence of private capital, the optimal composition of public spending is still shifted towards public investment, the share of which increases to 24% on impact and reaches a maximum of 25.7% before converging back to its steady-state value (figure not reported). The cumulative increase in total public spending is slightly larger in this version of the model (12.9 percent of steady-state output) than in the baseline case (11.6 percent of steady-state output).

---

[22]The full description of the model with private capital is available upon request.
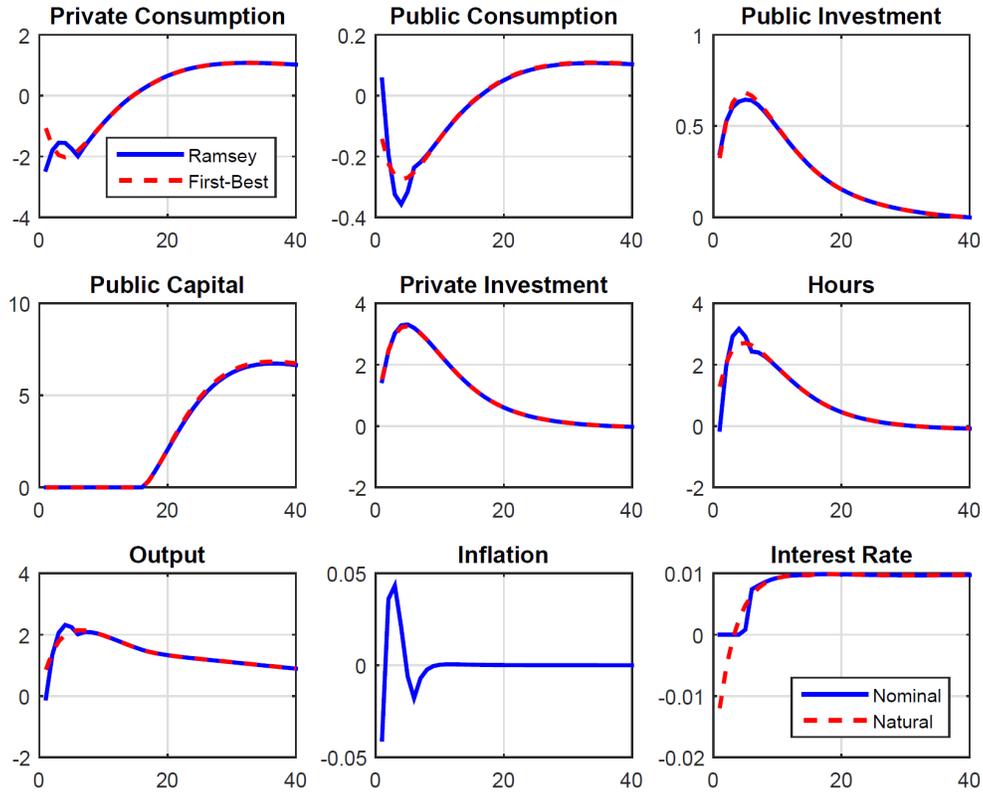
Figure 12: Ramsey monetary and fiscal policy after a negative preference shock in the model with private capital.

Notes: The figure shows the economy's response to a negative preference shock when the nominal interest rate, public consumption, and public investment are chosen optimally by a Ramsey planner. The response of private consumption, public consumption, public investment, and public capital are expressed as percentage deviations from steady-state output.

From Figure 12, one can also easily infer that the fraction of government spending that is carried out for stimulus purposes is also small in this version of the model. This is confirmed by Figure 13, which depicts the stimulus component of public consumption and investment. Observe that the stimulus component of both public consumption and investment are larger on impact in the model with private capital. As shown by Christiano et al. (2011), the presence of private capital amplifies the output loss associated with a binding ZLB. Therefore, there is a larger scope for increasing public spending to help close the larger output gap in this case. Total stimulus spending represents 0.22 percent of steady-state output in at the time of the shock, and cumulates to −0.18 percent of steady-state output (see Table 3).

In sum, these three experiments confirm the robustness of the main conclusions drawn from the baseline model. The optimal policy response to a shock that causes the ZLB to bind is to raise public spending persistently while changing its composition in way that assigns a larger weight to public investment relative to public consumption. The size of this fiscal expansion is substantially

larger than predicted by models that abstract from public investment, and tends to increase with the size of the shock and when we allow for private-capital accumulation. The stimulus component of this expansion remains, however, negligible in all cases.
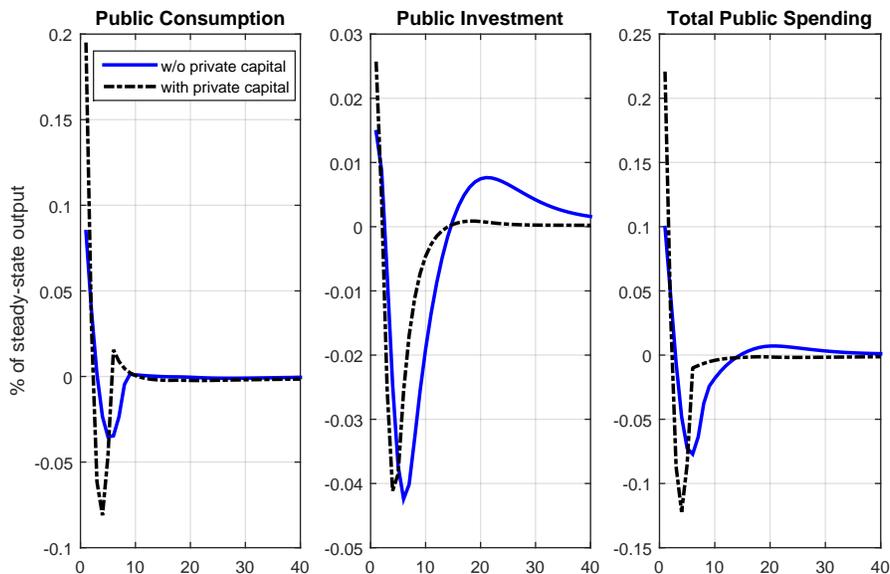
Figure 13: Optimal stimulus spending in the model with private capital.

Notes: Stimulus spending is defined as the difference between the spending level obtained under the Ramsey allocation and that obtained under the flexible-price allocation. Stimulus spending is therefore only due to the presence of an output gap.

5 Concluding Remarks

In the wake of the most severe economic crisis since the Great Depression and among fears of a prolonged "secular stagnation", there have been many calls to invest in public infrastructure. Surprisingly, while some earlier studies have examined the effectiveness of government investment from a positive standpoint, the questions of whether and to what extent public investment *should* be carried out in a deep recession have remained largely unexplored. The goal of this paper was to fill this gap in the literature.

In accordance with the popular wisdom, we have shown that it is optimal to increase public investment response to an adverse shock that may lead nominal interest rates to hit their ZLB. More specifically, the optimal policy response to such a shock is to shift the composition of government spending towards public investment in infrastructure. Importantly, however, we have shown that the increase in public investment hardly serves a stabilization purpose and is instead mostly warranted by the intertemporal allocation of resources that efficiency dictates even in the absence of an output gap.

An interesting extension of the analysis carried out in this paper would be to study the role of public investment and the optimal composition of public spending in a non-Ricardian economy in which firms and households are forced to deleverage. We leave this avenue for future work.

References

Adam, K. & Billi, R. M. (2006). Optimal Monetary Policy under Commitment with a Zero Bound on Nominal Interest Rates. *Journal of Money, Credit and Banking*, 38(7), 1877–1905.

Ambler, S. & Cardia, E. (1997). Optimal government spending in a business cycle model. In J.-O. Hairault, P.-Y. Hénin, & F. Portier (Eds.), *Business Cycles and Macroeconomic Stability: Should We Rebuild Built-in Stabilizers?* Kluwer Academic Press.

Ambler, S. & Paquet, A. (1996). Fiscal spending shocks, endogenous government spending, and real business cycles. *Journal of Economic Dynamics and Control*, 20(1-3), 237–256.

Aschauer, D. A. (1989). Is public expenditure productive? *Journal of Monetary Economics*, 23(2), 177–200.

Bachmann, R. & Sims, E. R. (2012). Confidence and the transmission of government spending shocks. *Journal of Monetary Economics*, 59(3), 235–249.

Baxter, M. & King, R. G. (1993). Fiscal policy in general equilibrium. *American Economic Review*, 83(3), 315–34.

Bhattarai, S. & Egorov, K. (2016). *Optimal monetary and fiscal policy at the zero lower bound in a small open economy*. Globalization and Monetary Policy Institute Working Paper 260, Federal Reserve Bank of Dallas.

Bilbiie, F. O., Monacelli, T., & Perotti, R. (2014). *Is Government Spending at the Zero Lower Bound Desirable?* NBER Working Papers 20687, National Bureau of Economic Research, Inc.

Blanchard, O. & Galí, J. (2007). Real Wage Rigidities and the New Keynesian Model. *Journal of Money, Credit and Banking*, 39(s1), 35–65.

Bom, P. R. & Ligthart, J. E. (2013). What have we learned from three decades of research on the productivity of public capital? *Journal of Economic Surveys*, (pp. n/a–n/a).

Bouakez, H., Guillard, M., & Roulleau-Pasdeloup, J. (2015). *Public Investment, Time to Build, and the Zero Lower Bound*. Cahiers de Recherches Economiques du Département d'Econométrie et d'Economie politique (DEEP) 14.06, Université de Lausanne, Faculté des HEC, DEEP.

Christiano, L., Eichenbaum, M., & Rebelo, S. (2011). When is the government spending multiplier large? *Journal of Political Economy*, 119(1), 78 – 121.

Christiano, L. J., Eichenbaum, M., & Evans, C. L. (2005). Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy. *Journal of Political Economy*, 113(1), 1–45.

Coenen, G., Straub, R., & Trabandt, M. (2013). Gauging the effects of fiscal stimulus packages in the euro area. *Journal of Economic Dynamics and Control*, 37(2), 367–386.

Cwik, T. & Wieland, V. (2011). Keynesian government spending multipliers and spillovers in the euro area. *Economic Policy*, 26(67), 493–549.

Drautzburg, T. & Uhlig, H. (2013). *Fiscal stimulus and distortionary taxation.* Working Papers 13-46, Federal Reserve Bank of Philadelphia.

Eberly, J., Rebelo, S., & Vincent, N. (2012). What explains the lagged-investment effect? *Journal of Monetary Economics*, 59(4), 370–380.

Eggertsson, G. B. (2011). What fiscal policy is effective at zero interest rates? In *NBER Macroeconomics Annual 2010, Volume 25*, NBER Chapters (pp. 59–112). National Bureau of Economic Research, Inc.

Eggertsson, G. B. & Woodford, M. (2003). *Optimal Monetary Policy in a Liquidity Trap.* NBER Working Papers 9968, National Bureau of Economic Research, Inc.

Fernald, J. G. (1999). Roads to Prosperity? Assessing the Link between Public Capital and Productivity. *American Economic Review*, 89(3), 619–638.

Guerrieri, L. & Iacoviello, M. (2014). *OccBin: A Toolkit for Solving Dynamic Models With Occasionally Binding Constraints Easily.* Technical report.

Jung, T., Teranishi, Y., & Watanabe, T. (2005). Optimal Monetary Policy at the Zero-Interest-Rate Bound. *Journal of Money, Credit and Banking*, 37(5), 813–35.

Kydland, F. E. & Prescott, E. C. (1982). Time to build and aggregate fluctuations. *Econometrica*, 50(6), 1345–70.

Lansing, K. J. (1998). Optimal Fiscal Policy in a Business Cycle Model with Public Capital. *Canadian Journal of Economics*, 31(2), 337–364.

Leduc, S. & Wilson, D. (2013). Roads to Prosperity or Bridges to Nowhere? Theory and Evidence on the Impact of Public Infrastructure Investment. *NBER Macroeconomics Annual*, 27(1), 89 – 142.

Leeper, E. M., Walker, T. B., & Yang, S.-C. S. (2010). Government investment and fiscal stimulus. *Journal of Monetary Economics*, 57(8), 1000–1012.

Nakata, T. (2013). *Optimal fiscal and monetary policy with occasionally binding zero bound constraints.* Finance and Economics Discussion Series 2013-40, Board of Governors of the Federal Reserve System (U.S.).

Nakata, T. (2015). *Optimal Government Spending at the Zero Lower Bound: A Non-Ricardian Analysis.* Finance and Economics Discussion Series 2015-38, Board of Governors of the Federal Reserve System (U.S.).

Rotemberg, J. J. (1982). Monopolistic price adjustment and aggregate output. *Review of Economic Studies*, 49(4), 517–31.

Schmidt, S. (2013). Optimal Monetary and Fiscal Policy with a Zero Bound on Nominal Interest Rates. *Journal of Money, Credit and Banking*, 45(7), 1335–1350.

Sims, E. & Wolff, J. (2013). *The Output and Welfare Effects of Government Spending Shocks over the Business Cycle.* NBER Working Papers 19749, National Bureau of Economic Research, Inc.

Werning, I. (2011). *Managing a Liquidity Trap: Monetary and Fiscal Policy.* NBER Working Papers 17344, National Bureau of Economic Research, Inc.

Woodford, M. (2011). Simple analytics of the government expenditure multiplier. *American Economic Journal: Macroeconomics*, 3(1), 1–35.

Appendix

A. First-Best

A.1 Equilibrium conditions

The Lagrangian is given by

$$
\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ \xi_t \left[U\left(C_t, N_t\right) + V\left(G_t^c\right) \right]
$$
$$
+ \lambda_{1,t} \left[F(N_t, K_{G,t}) - C_t - G_t^c - G_t^i \right]
$$
$$
+ \lambda_{2,t} \left[\left(1 - S\left(\frac{G_t^i}{G_{t-1}^i} \right) \right) G_t^i + (1-\delta) K_{G,t+T-1} - K_{G,t+T} \right] \Bigg\}.
$$

The efficient allocation is the solution to the following set of equations:

$$
0 = \xi_t U_{C,t} - \lambda_{1,t}, \tag{29}
$$
$$
0 = \xi_t U_{N,t} + \lambda_{1,t} F_N(N_t, K_{G,t}), \tag{30}
$$
$$
0 = \xi_t V_{G,t} - \lambda_{1,t}, \tag{31}
$$
$$
0 = \left[1 - S\left(\frac{G_t^i}{G_{t-1}^i} \right) - \left(\frac{G_t^i}{G_{t-1}^i} \right) S'\left(\frac{G_t^i}{G_{t-1}^i} \right) \right] \lambda_{2,t} + \beta \mathbb{E}_t \lambda_{2,t+1} \left(\frac{G_{t+1}^i}{G_t^i} \right)^2 S'\left(\frac{G_{t+1}^i}{G_t^i} \right) - \lambda_{1,t}, \tag{32}
$$
$$
0 = \lambda_{2,t} - \beta(1-\delta) \mathbb{E}_t \lambda_{2,t+1} - \beta^T \mathbb{E}_t \lambda_{1,t+T} F_{K_G}\left(N_{t+T}, K_{G,t+T}\right), \tag{33}
$$
$$
0 = F(N_t, K_{G,t}) - C_t + G_t^C + G_t^i, \tag{34}
$$
$$
0 = \left(1 - S\left(\frac{G_t^i}{G_{t-1}^i} \right) \right) G_t^i + (1-\delta) K_{G,t+T-1} - K_{G,t+T}. \tag{35}
$$

A.2 Functional forms

We consider the following functional forms:

Utility function

$$
U\left(C_t, N_t\right) + V\left(G_t^c\right) = \frac{\left(C_t^{\gamma}(1-N_t)^{1-\gamma}\right)^{1-\sigma}}{1-\sigma} + \chi \frac{\left(G_t^c\right)^{1-\sigma}}{1-\sigma} \qquad \text{if } \sigma \neq 1
$$

$$
= \gamma \ln C_t + (1-\gamma) \ln(1-N_t) + \chi \ln\left(G_t^c\right) \qquad \text{if } \sigma = 1,
$$

which implies the following first and second derivatives:

$$
U_C = \gamma \frac{\left(C^{\gamma}(1-N)^{1-\gamma}\right)^{1-\sigma}}{C}, \qquad U_{CC} = -\left(1 + \gamma(\sigma-1)\right) \frac{U_C}{C},
$$
$$
U_N = -(1-\gamma) \frac{\left(C^{\gamma}(1-N)^{1-\gamma}\right)^{1-\sigma}}{1-N}, \qquad U_{NN} = \left(1 + (1-\gamma)(\sigma-1)\right) \frac{U_N}{1-N}, \qquad U_{CN} = (1-\gamma)(\sigma-1) \frac{U_C}{1-N},
$$
$$
V_G = \chi \left(G^c\right)^{-\sigma}, \qquad V_{GG} = -\sigma \chi \left(G^c\right)^{-(1+\sigma)}.
$$

Production function

$$F(N_t, K_{G,t}) = N_t^a K_{G,t}^b, \qquad 0 \le a, b \le 1,$$

which implies the following first and second derivatives:

$$F_N = a\frac{N^a K_G^b}{N}, \qquad F_{NN} = -(1-a)\frac{F_N}{N},$$
$$F_{K_G} = b\frac{N^a K_G^b}{K_G^b}, \qquad F_{K_G K_G} = -(1-b)\frac{F_{K_G}}{K_G}, \qquad F_{N K_G} = b\frac{F_N}{K_G}.$$

Investment-adjustment-cost function

$$S\left(\frac{G_t^i}{G_{t-1}^i}\right) = \frac{\varpi}{2}\left(1 - \frac{G_t^i}{G_{t-1}^i}\right)^2, \qquad \varpi \ge 0,$$

which implies the following first and second derivatives:

$$S'\left(\frac{G_t^i}{G_{t-1}^i}\right) = -\frac{\varpi}{G_{t-1}^i}\left(1 - \frac{G_t^i}{G_{t-1}^i}\right),$$
$$S''\left(\frac{G_t^i}{G_{t-1}^i}\right) = \frac{\varpi}{\left(G_{t-1}^i\right)^2}.$$

A.3 Steady state

We focus on a deterministic steady state in which the preference shock takes a value of 1. In what follows, variables without time-subscript denote steady-state values. Evaluating system (29)–(35) at steady state and using $Y = F(N, K_G)$ and the specifications of preferences and technology, one obtains:

$$U_C = V_G \quad \Longleftrightarrow \quad \gamma\frac{\left(C^\gamma (1-N)^{1-\gamma}\right)^{1-\sigma}}{C} = \chi\left(G^c\right)^{-\sigma}, \tag{36}$$

$$-\frac{U_N}{U_C} = F_N \quad \Longleftrightarrow \quad \frac{1-\gamma}{\gamma}\frac{C}{1-N} = a\frac{Y,}{N}, \tag{37}$$

$$F_{K_G} = \frac{1-\beta(1-\delta)}{\beta T} \quad \Longleftrightarrow \quad b\frac{Y}{K_G} = \frac{1-(1-\delta)\beta}{\beta T}, \tag{38}$$

and

$$Y = N^a K_G^b, \tag{39}$$
$$Y = C + G^C + G^i, \tag{40}$$
$$G^i = \delta K_G. \tag{41}$$

Theoretically, this system of equations allows one to find C, N, Y, G^c, G^i, and K_G for a given parameter set: $\{\chi, \gamma, \sigma, a, b, \beta, \delta, T\}$. In practice, and in order to discipline our calibration, we choose χ and γ in order to match the fraction of time worked in steady state, N, as well as the steady-state ratio of total public spending to GDP, given by

$$g \equiv \frac{G^c + G^i}{Y}. \tag{42}$$

In other words, our calibration strategy consists in assigning values to $\{\sigma, a, b, \beta, \delta, T, g, N\}$ in order to find the equilibrium values of $\{C, Y, G^c, G^i, K_G, \chi, \gamma\}$ from equations (36) to (42). In particular, using (36), (40),

and (42), one finds

$$\gamma = \frac{(1-g)\,N}{(1-g)\,N + a\,(1-N)},$$

and from (38) and (41), we obtain

$$g^i \equiv \frac{G^i}{Y} = \frac{b\delta\beta^T}{1-(1-\delta)\beta}.$$

The resource constraint (40) can be re-written as

$$Y = N^a \left(\frac{K_G}{Y}\right)^b Y^b = N^{\frac{a}{1-b}} \left(\frac{g^i}{\delta}\right)^{\frac{b}{1-b}},$$

where we have used (41) again. The remaining endogenous variables K_G, G^i, G^c, C, and χ can be found recursively (and in this order) from equations (39), (41), (42), (40), and (36).

B. The Ramsey Problem

B.1 Equilibrium conditions

The Lagrangian can be expressed as

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \xi_t \left[U\left(C_t, N_t\right) + V\left(G_t^c\right) \right] \right.$$

$$+ \phi_{1,t} \left[\Lambda_{t+1} \left(\frac{1+R_t}{1+\pi_{t+1}}\right) - 1 \right]$$

$$+ \phi_{2,t} \left[\theta\left(mc_t - 1\right) - \psi\left(d\left(\pi_t\right) - \Lambda_{t+1} \frac{F\left(N_{t+1}, K_{G,t+1}\right)}{F\left(N_t, K_{G,t}\right)} d\left(\pi_{t+1}\right) \right) \right]$$

$$+ \phi_{3,t} \left[\left(1 - \frac{\psi}{2}\pi_t^2\right) F\left(N_t, K_{G,t}\right) - C_t - G_t^c - G_t^i \right]$$

$$+ \phi_{4,t} \left[(1-\delta)\,K_{G,t+T-1} + \left(1 - S\left(\frac{G_t^i}{G_{t-1}^i}\right)\right) G_t^i - K_{G,t+T} \right]$$

$$\left. + \phi_{5,t} \left[R_t - 0 \right] \right\},$$

where $mc_t = -\frac{U_{N,t}}{U_{C,t}F_{N,t}}$, and where we have defined $d(\pi_t) \equiv \pi_t(1+\pi_t)$ and $\Lambda_{t+1} \equiv \beta \frac{\xi_{t+1}}{\xi_t} \frac{U_{C,t+1}}{U_{C,t}}$.

The first-order conditions for this problem are

$$0 = \xi_t U_{C,t} - \left(\phi_{1,t} - \beta^{-1}\phi_{1,t-1}\right)\frac{U_{CC,t}}{U_{C,t}} + \theta\phi_{2,t}\left(\frac{U_{CN,t}}{U_{N,t}} - \frac{U_{CC,t}}{U_{C,t}}\right)mc_t - \psi\left(\phi_{2,t}\mathbb{E}_t\Omega_{t+1} - \beta^{-1}\phi_{2,t-1}\Omega_t\right)\frac{U_{CC,t}}{U_{C,t}} - \phi_{3,t},$$

$$0 = \xi_t U_{N,t} - \left(\phi_{1,t} - \beta^{-1}\phi_{1,t-1}\right)\frac{U_{CN,t}}{U_{C,t}} + \theta\phi_{2,t}\left(\frac{U_{NN,t}}{U_{N,t}} - \frac{U_{CN,t}}{U_{C,t}} - \frac{F_{NN,t}}{F_{N,t}}\right)mc_t$$

$$-\psi\left(\phi_{2,t}\mathbb{E}_t\Omega_{t+1} - \beta^{-1}\phi_{2,t-1}\Omega_t\right)\left(\frac{U_{CN,t}}{U_{N,t}} - \frac{F_{N,t}}{F_t}\right) + \phi_{3,t}F_{N,t}\left(1 - \psi\frac{\pi_t^2}{2}\right),$$

$$0 = \xi_t V_{G,t} - \phi_{3,t},$$

$$0 = \phi_{3,t} - \left[1 - S\left(\frac{G_t^i}{G_{t-1}^i}\right) - \frac{G_t^i}{G_{t-1}^i}S'\left(\frac{G_t^i}{G_{t-1}^i}\right)\right]\phi_{4,t} - \beta\mathbb{E}_t\phi_{4,t+1}\left(\frac{G_{t+1}^i}{G_t^i}\right)^2 S'\left(\frac{G_{t+1}^i}{G_t^i}\right),$$

$$0 = \phi_{4,t} - \beta(1-\delta)\mathbb{E}_t\phi_{4,t+1} + \theta\beta^T\mathbb{E}_t\phi_{2,t+T}\left(\frac{F_{NK_G,t+T}}{F_{N,t+T}}\right)mc_{t+T}$$

$$+\psi\beta^T\mathbb{E}_t\left(\phi_{2,t+T}\Omega_{t+T+1} - \beta^{-1}\phi_{2,t+T-1}\Omega_{t+T}\right)\left(\frac{F_{K_G,t+T}}{F\left(N_{t+T},K_{G,t+T}\right)}\right) - \beta^T\mathbb{E}_t\phi_{3,t+T}\left(1 - \psi\frac{\pi_{t+T}^2}{2}\right)F_{K_G,t+T},$$

$$0 = \beta^{-1}\phi_{1,t-1} + \psi\left[\phi_{2,t} - \beta^{-1}\phi_{2,t-1}\frac{\Omega_t}{d\left(\pi_t\right)}\right]d'\left(\pi_t\right) + \phi_{3,t}\psi F\left(N_t,K_{G,t}\right)\pi_t,$$

$$0 = \phi_{5,t} + \phi_{1,t}\mathbb{E}_t\left(\frac{\Lambda_{t+1}}{1+\pi_{t+1}}\right),$$

$$0 = \Lambda_{t+1}\left(\frac{1+R_t}{1+\pi_{t+1}}\right) - 1,$$

$$0 = \psi\Lambda_{t+1}\frac{F\left(N_{t+1},K_{G,t+1}\right)}{F\left(N_t,K_{G,t}\right)}d\left(\pi_{t+1}\right) - \psi d\left(\pi_t\right) - \theta\left(mc_t - 1\right),$$

$$0 = \left(1 - \frac{\psi}{2}\pi_t^2\right)F(N_t,K_{G,t}) - C_t - G_t^c - G_t^i,$$

$$0 = (1-\delta)K_{G,t+T-1} + \left[1 - S\left(\frac{G_t^i}{G_{t-1}^i}\right)\right]G_t^i - K_{G,t+T},$$

$$0 = \min\left(\phi_{5,t},R_t\right),$$

where we have defined $\Omega_t = \Lambda_t\frac{F(N_t,K_{G,t})}{F(N_{t-1},K_{G,t-1})}d\left(\pi_t\right)$.

B.2 Steady state

In zero-inflation steady state, the Ramsey allocation is identical to the first best. This implies that

$$\phi_1 = \phi_2 = \phi_5 = 0,$$

$$\phi_3 = \phi_4 = U_C,$$

and

$$\pi = 0,$$
$$mc = 1.$$