EUROPEAN CENTRAL BANK

WORKING PAPER SERIES

NO. 376 / JULY 2004

ECB-CFS RESEARCH NETWORK ON CAPITAL MARKETS AND FINANCIAL INTEGRATION IN EUROPE

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by Cornelia Holthausen
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In 2004 all publications will carry a motif taken from the €100 banknote.

1 We thank Marc Bayle, Giampiero Carla, Charlie Kahn, Elias Kazarian, Cyril Monnet, Rodrigo Oliveira Soares, Daniela Russo and Andreas Schoenenberger as well as seminar participants at the Annual Meeting of the Verein für Socialpolitik and the ECB-CFS Athens Workshop for helpful discussions. The views expressed in this paper are our own and do not necessarily reflect the view of the European Central Bank.
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ECB-CFS Research Network on
"Capital Markets and Financial Integration in Europe"

This paper is part of the research conducted under the ECB-CFS Research Network on "Capital Markets and Financial Integration in Europe". The Network aims at stimulating top-level and policy-relevant research, significantly contributing to the understanding of the current and future structure and integration of the financial system in Europe and its international linkages with the United States and Japan. After two years of work, the ECB Working Paper Series is issuing a selection of papers from the Network. This selection is covering the priority areas "European bond markets", "European securities settlement systems", "Bank competition and the geographical scope of banking activities", "international portfolio choices and asset market linkages" and "start-up financing". It also covers papers addressing the impact of the euro on financing structures and the cost of capital.

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ISSN 1561-0810 (print)
ISSN 1725-2806 (online)

CONTENTS

Abstract

The competition between a central securities depository (CSD) and a custodian bank is analysed in a Stackelberg model. The CSD sets its prices first, the custodian bank follows. There are many investor banks each of which has to decide whether to use the service of the CSD or of the custodian bank. This decision depends on the prices and the investor bank's preferences for the inhomogeneous services of the two service providers. Since the custodian bank uses services provided by the CSD as input, the CSD can raise its rival's costs. However, due to network externalities, the CSD's equilibrium market share is not necessarily higher than socially optimal. This result has important policy implications that are related to a discussion currently taking place in the securities settlement industry.

Keywords: Securities settlement, network competition, raising rival's cost

JEL Codes: G10, G20, L14

Non-technical summary

Every securities trade involves settlement of the securities, that is, the transfer of securities from the seller to the buyer. This transfer is normally done electronically on the books of the institution where the securities are held. Nowadays, securities are usually stored electronically at so-called Central Securities Depositories (CSDs). In most countries, there is one national CSD. Additionally, several international CSDs are active in the storekeeping of securities.

Investors who do not have a direct account with a CSD can trade securities using the services of a custodian bank. In this case, the custodian bank holds an account with the CSD and executes buy- or sell-orders on the books of the CSD on behalf of the investor. The investor then settles on the books of this custodian bank.

Increasingly, CSDs and custodian banks compete for customers. The nature of competition is tricky because of two complications: first, because the custodian bank has to settle all net orders on the books of the CSD, it is at the same time a competitor and a customer of the CSD. Second, there are network effects involved: the more customers use a given service provider (i.e. either the CSD or the custodian bank) for their services, the more profitable it is to use that provider.

This paper analyses whether competition between CSD and custodian banks leads to a desirable outcome, or whether the equilibrium market share of either competitor is too high. We analyse a model in which one CSD competes with one custodian bank, and an infinite number of investor banks choose where to hold a securities account. Investors have some initial preferences which service provider to use.

We show that generally, there are two possible welfare-maximising allocations: one in which the CSD, and one in which the custodian bank serves the majority of customers. The driving force behind this result are network effects. Whenever the majority of investors are customers of one service provider, it is beneficial that the next investor chooses the same provider, because in this way, more trades can be netted out with the other customers. The reason why it is not efficient that all investors choose the same provider is that investors have individual preferences regarding the service provider.

It is shown that the equilibrium solution is quite different from the optimal one. First, in equilibrium, the CSD is always able to obtain a higher market share than the custodian bank. The reason is that the CSD is in a unique position as the custodian bank's competitor and supplier. It can use its pricing policy to make settlement on its books relative unattractive for custodian banks. In particular, it can choose its marginal fees relatively high compared to its fixed fees. Because the custodian bank settles more trades than an investor bank, this strategy makes settlement on the books of the CSD relatively expensive for the custodian bank. Consequently, the custodian bank is not able to offer attractive conditions to its customers, and only those investors with a strong preference to settle with the custodian, chooses it.

Second, it may be the case that the equilibrium market share of the CSD is either smaller or larger than the optimal one. In particular, we find that the market share chosen by the CSD is larger than the socially optimal one if and only if network externalities are

relatively small (compared to investor preferences). This result reflects the fact that the CSD and a social planner care differently about the costs arising from consumer preferences and network externalities. In particular, the CSD is not concerned with the costs from settling across institutions, which are borne by the custodian bank. Still, these costs matter for welfare.

In sum, we show that the CSD is able to exploit its monopoly position as central depository by raising the custodian bank's costs to attract more investor banks as customers. However, it is also shown that the market share of the CSD in equilibrium is not necessarily higher than its socially optimal market share.

Our analysis appears to be relevant in two ways. Firstly, it contributes to the current discussion between market participants, especially CSDs and custodian banks, on whether and how competition in the securities settlement industry is distorted. Since we have shown that the CSD's equilibrium market share is not necessarily higher than socially optimal, the model provides no case for regulatory interventions aiming at reducing the CSD's market share. Secondly, it is theoretically relevant in that it considers a case of asymmetric network competition that has not been analysed yet.

1 Introduction

Every financial market transaction in which securities are traded involves settlement, i.e. the transfer of the securities from the seller to the buyer, and the related payment from the buyer to the seller. Before settlement can take place, each trading party needs to choose a settlement service provider that carries out asset transfers on behalf of the respective party. The different settlement service providers compete with each other for customers. In this paper, we model the competition between two very different settlement service providers, a central securities depository (CSD) and a custodian bank.

Today, a physical transfer of securities certificates or cash between the seller and the buyer hardly takes place anymore. Instead, assets are transferred electronically by account entries. To facilitate the electronic transfer of securities, most industrialized countries have established a CSD. CSDs are central store houses for securities. If an issuer wants to issue securities in a given country, he usually deposits the entire issue with the national CSD.[1] An investor who owns shares of a security deposited in the CSD must hold them either on a securities account directly with the CSD; or on a securities account with an intermediary that has a securities account with the CSD[2]. By construction, the number of shares deposited in the CSD must equal the number of shares on accounts with the CSD. Furthermore, the number of shares some entity A holds on accounts with other entities (intermediaries or the CSD) must equal the number of shares owned by A plus the number of shares on account of other entities with A.

This "double booking principle" determines the booking procedure required for the transfer of securities from the seller to the buyer, i.e. the securities settlement process. If for example the seller and the buyer have securities accounts with the same entity (the CSD or an intermediary), then the transfer is settled simply by debiting the seller's and crediting the buyer's account. If the seller has an account with the CSD and the buyer has an account with an intermediary that has an account with the CSD, the seller's account with the CSD has to be debited and the intermediary's account with the CSD and the buyer's account with the intermediary have to be credited. I.e. in this case, three securities accounts are involved.

An entity that holds securities on accounts with other entities mainly on behalf of (small or institutional) investors or on its own behalf is called an investor bank throughout this paper. Institutions that hold securities on accounts with other institutions mainly on behalf of investor banks are called custodian banks. I.e. custodian banks act as intermediaries between investor banks and CSDs.

In the past ten to fifteen years, custodian banking has become increasingly important.[3] Technological progress in information technology and an increasing globalization contributed to a rapidly growing complexity of financial markets. Many especially smaller investor banks responded to these developments by outsourcing activities to other banks. Some big banks became custodian banks specialized in the securities custody and settlement business. Investor banks

[1] Today, the securities are usually deposited electronically in the CSD, not in form of physical paper certificates.

[2] Or on a securities account with an intermediary that has a securities account with another intermediary that has a securities account with the CSD etc

[3] See for example ECSDA (2002), page 15ff.

now have the choice either to have securities accounts directly with the CSD or with a custodian bank. I.e. the CSD and the custodian bank compete with each other for investor banks.

When elaborating on the competiton between CSDs and custodian banks, a special feature of this competition has to be noted: in order to settle securities stored in a CSD, custodian banks use services provided by the CSD as input, but not vice versa. If both the seller and the buyer in a securities transaction have an account with the custodian bank, the custodian bank can settle the transaction internally as described above without routing it to the CSD. However, if an investor bank that has a securities account with a custodian bank trades with another investor bank that has an account with the CSD, then internalizations of settlement within the custodian bank is not possible. The transaction is settled through the custodian bank's account with the CSD as described above and the custodian bank has to pay a price to the CSD for having its securities account with the CSD credited or debited. I.e. the CSD can raise the costs of the custodian bank by increasing the price the custodian bank has to pay to the CSD.

In this paper, we present a simple model describing the competition between a CSD and a custodian bank. We assume that there are only one CSD, one custodian bank and a continuum of investor banks. The CSD moves first. It sets a price q_C for opening a securities account with the CSD and another price p_C for debiting or crediting this account, i.e. for the settlement of a securities transaction on this account. Note that we assume that the CSD is not able to price discriminate directly by setting one settlement prices $p_{C,1}$ to be paid by investor banks and another settlement price $p_{C,2}$ to be paid by the custodian bank. This assumption can be justified because in reality competition authorities would probably not allow this kind of direct price discrimination. The custodian bank moves second. It also sets a price q_A for opening and another price p_A for debiting or crediting a securities account with the custodian bank taking into account the prices set by the CSD. Next, each investor bank opens an account with either the CSD or the custodian bank. Finally, banks are randomly matched to trade with each other and the transactions are settled through account with the CSD and the custodian bank.

We show that the CSD can raise the costs of the custodian bank in a very subtle way. Compare the custodian bank with an investor bank that has opened an account with the CSD. Since both have only one securities account with the CSD, both have to pay to the CSD the price q_C. But the number of transactions settled on an investor bank's account with the CSD is low compared to the number of transactions settled on a custodian bank's account with the CSD unless the custodian bank can by chance internalize the settlement of almost all transactions of its customers. Thus, the relative relevance of the price p_C compared to the price q_C is higher for the custodian bank than for the investor bank. If the CSD raises p_C and simultaneously reduces q_C, the overall costs for investor banks for using the service of the CSD may remain unchanged. However, the overall costs for the custodian bank for using the services of the CSD rise. Thus, with this strategy, the CSD can raise its rivals costs without losing investor banks as customers. To the contrary, the custodian bank has to raise its own prices to cover the additional costs it has to pay to the CSD and thus looses investor banks as customers to the CSD.

As can be expected from the reasoning above, we show that in equilibrium the market share of the CSD is higher than the market share of the custodian bank though we assume that the CSD and the custodian bank face the same exogeneous cost and demand parameters (symmetry). Most importantly, we compare the equilibrium market shares with the socially optimal market shares. We show that the CSD's market share is not always higher than socially optimal. Depending on the parameter constellation, it can be higher than, equal to or lower than the social optimum. I.e. it is often socially desirable to have relatively many investor bank that go to the CSD. The reason is the presence of network externalities. The CSD and the custodian bank are two different settlement networks. Settling transactions across the two networks is socially expensive. It is better to pool to a certain extend many investor banks in one of the two networks, for example in the CSD.

This result has important policy implications that are related to a discussion currently taking place in the securities settlement industry.[4] CSDs argue that the competition between CSDs and custodian banks is distorted in favour of custodian banks since CSDs are regulated by public authorities that aim to reduce the risks of financial instability while the settlement business of custodian banks is not regulated in a similar way.[5] Custodian banks argue that there is also a distortion in favour of CSDs since CSDs have a monopoly as central depository that enables them to raise the costs of custodian banks as described above.[6] However, this reasoning is obviously not supported by our model. In our model, it is true that the CSD can exploit its monopoly position as a central depository by raising the custodian bank's costs to gain a higher market share. But the equilibrium market share of the CSD without regulatory intervention is not necessarily higher than socially optimal. Thus, our model provides no reason for regulatory intervention favouring custodian banks as long as CSDs are not allowed to price discriminate between custodian banks and investor banks.

Our model is obviously closely related to the theoretical literature on network industries. This literature can be separated into two branches. The first branch analyses industries like the electricity or gas industry with only one network provider (the owner of the cable or pipeline network). Firms specialized in supplying consumers with the commodity conveyed through the network (electricity or gas) need to buy access to the network from the network provider.[7] The network provider itself may also offer the commodity to consumers. In this case, it is both an input supplier for and a direct competitor of the other firms at the market. In this respect, the competition between the network provider and

[4] The discussion is closely related to the more prominent discussion between stock exchanges and other financial market participants on the internalizations of securities buy and sell orders by custodian banks. See Euronext (2002) and APCIMS-EASD et. al. (2002).

[5] See ECSDA (2002). Custodian banks are regulated as banks, but not as settlement service providers.

[6] See for example BNP Paribas Securities Services (2002), Citigroup (2003) and Fair & Clear Group (2003).

[7] This literature focuses mainly on access price regulation. See for example Armstrong, Doyle and Vickers (1996), Laffont and Tirole (1994) and Vickers (1995).

other suppliers of the commodity resembles the competition between a CSD and custodian banks. However, in the gas industry for example, a supplier of gas has to use the more services of the provider of the network the more consumers the supplier can attract. But the more investor banks a custodian bank can attract, the more likely is the case that the buyer and the seller of a securities transaction both have an account with this custodian bank so that the custodian bank can internalize the settlement without routing it through its account with the CSD. In the extreme case that all buyers and sellers of securities have an account with the same custodian bank, the custodian bank can internalize the settlement of all transactions and no settlement is routed through its account with the CSD.

The second branch of literature on network industries looks at competition between networks.[8] The most prominent example is the competition between two mobile phone networks. Consumers have the choice between both networks. If a customer of one network wants to call a customer of another network, the call has to be routed through a link between the two networks. This situation obviously resembles the competition between a CSD and a custodian bank. However, while the competition between mobile phone networks is symmetric, the competition between the CSD and the custodian bank is asymmetric insofar as only the custodian bank has an account with the CSD, not vice versa.

The competition between a CSD and a custodian bank in the securities settlement industry therefore deserves a special analysis. However, there are only a few research papers on the securities settlement industry. Kauko (2002, 2003) concentrates on the competition between two CSDs. These paper appears to be more in line with the above described second branch of literature on network industries. Koeppl and Monnet (2004) and Tapking and Yang (2004) analyse mergers of CSDs and thus discuss issues related to financial market integration. Other academic papers on securities settlement are empirical and deal mainly with the costs of domestic versus cross-border settlement, for example Lannoo and Levin (2001) and Schmiedel, Malkamaeki and Tarkka (2002). Custodian banking and the competition between CSDs and custodian banks has to our knowledge not yet been analysed from an academic perspective.

The paper is organized as follows: The assumptions of the model are described in section 2. In section 3, we derive the social welfare optimum. The payoff functions of the players are described in section 4.1. In the sections 4.2, 4.3 and 4.4, we analyse the equilibrium behaviour of the players. Section 5 is devoted to an alternative version of our model in which the CSD and the custodian bank set their prices simultaneously. We show that the simultaneous move game has no equilibrium in pure strategies. This result can be seen as a good justification of our assumption that the CSD moves first.

[8] See Laffont, Rey and Tirole (1996a and 1996b).

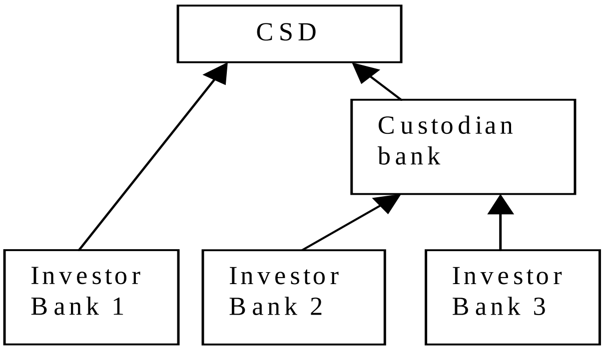

Figure 1: The structure of settlement relationships

2 The model

In this section, we describe the assumptions of our model. We assume that there is one CSD, one custodian bank A and a continuum $[0,1]$ of investor banks. An investor bank can maintain a securities account either with the CSD, or with the custodian bank, which in turn must have an account with the CSD. Decisions are taken in three stages.

(1) The CSD moves first. It sets a price q_C a bank has to pay if it wants to open a securities account with the CSD and a price p_C a bank has to pay when a securities transaction is settled on its account with the CSD. Note that we do not allow for direct price discrimination, i.e. the CSD is not allowed to charge investor banks a settlement price $p_{C,1}$ and to charge the custodian bank another settlement price $p_{C,2} \neq p_{C,1}$. In reality, competition authorities would most likely not allow price discrimination of this kind. The CSD's marginal costs of maintaining an account for a bank are c_a. When securities are transferred from one account with the CSD to another account with the CSD, the CSD has marginal costs of $2c_s$ (one account has to be debited, the other has to be credited).

(2) Custodian bank A moves second. It sets a price q_A an investor bank has to pay if it wants to open a securities account with the custodian bank and a price p_A an investor bank has to pay when a securities transaction is settled on its account with the custodian bank. The custodian bank's marginal costs of maintaining an account for a bank are $\widetilde{c}_a$. When securities are transferred from one account with the custodian bank to another account with the custodian bank, the custodian bank has costs of $2\widetilde{c}_s$ (again, one account has to be debited, the other has to be credited). In most parts of this paper, we will assume $\widetilde{c}_a = c_a$ and $\widetilde{c}_s = c_s$. We assume that custodians charge the same price for its services, irrespective of whether internalisation of settlement is possible or not. This seems to correspond best to the real world, where large custodians tend to charge the same fee in both cases.

(3) The investor banks move next. Each of these banks has to open a securities account with either the CSD or the custodian bank. Besides the differences in prices, the quality of the services of the CSD on the one hand and the custodian bank on the other hand matters. This is reflected in our model by the assumption that bank $i \in [0,1]$ has additional costs $c(\frac{1}{2} - i)$, $c > 0$, when choosing to open an account with the custodian bank. Thus, if the prices of the CSD and the custodian bank were equal ($q_C = q_A$ and $p_C = p_A$), then all banks in $[0, \frac{1}{2}[$ would go to the custodian bank and all banks in $]\frac{1}{2}, 1]$ would go to the CSD. This assumption basically implies that on average the quality of the services provided by the CSD and by the custodian bank is equally high.

After the investor banks have made their decision, a security is issued into the CSD and a number α is drawn randomly. This α is the proportion of investor banks with an account with the CSD that receive one unit of the security at the primary market. On top of this, $\frac{1}{2} - \alpha$ banks with an account with the custodian bank also receive one unit of the security at the primary market. Thus, one half of all investor banks is able to buy a unit of the security at the primary market.

Denote by k the proportion of investor banks with an account with the CSD, i.e. $1 - k$ is the proportion of investor banks with an account with the custodian bank. Notice that our model setup implies several limitations on possible realizations on the parameter α: α can neither exceed k nor $\frac{1}{2}$; similarly, $\frac{1}{2} - \alpha$ cannot exceed $1 - k$. Therefore, we need to make restrictions on the distribution of α and assume that α is uniformly distributed over the interval

$$[\max\{0, k - \frac{1}{2}\}; \min\{\frac{1}{2}, k\}].$$

Finally, we assume that the settlement costs of transactions at the primary market can be neglected.

Each investor bank that bought the security at the primary market has to sell its unit and each other investor bank has to buy one unit of the security at the secondary market. We assume that those banks with an account with the custodian bank simply give their sell or buy order to the custodian bank. The custodian bank acts as a broker and executes the orders of its customer banks. It of course internalizes as many trades as possible. Whether there is a net transfer of securites from the CSD to the custodian bank or vice versa, depends on the realization of α: If $\alpha < \frac{1}{2}k$, that is, less than half of the banks with an account with the CSD wish to sell the security, then α banks with an account with the CSD sell to α banks with an account with the CSD, the other $k - 2\alpha$ banks with an account with the CSD buy from $k - 2\alpha$ banks with an account with the custodian bank. Finally $\frac{1}{2}[1 - k - (k - 2\alpha)] = \frac{1}{2} - k + \alpha$ banks with an account with the custodian bank sell to $\frac{1}{2} - k + \alpha$ banks with an account with the custodian bank. If instead $\alpha > \frac{1}{2}k$, then $k - \alpha$ banks with an account in the CSD buy from $k - \alpha$ banks with an account with the CSD, the other $k - 2(k - \alpha) = 2\alpha - k$ banks with an account with the CSD sell to $2\alpha - k$ banks with an account with the custodian bank. Finally, $\frac{1}{2}[1 - k - (2\alpha - k)] = \frac{1}{2} - \alpha$

banks with an account with the custodian bank buy from $\frac{1}{2} - \alpha$ banks with an account with the custodian bank.

It is essential to clearly understand how trades are settled. Take for example the case that an investor bank with an account with the CSD sells to another investor bank that also has an account with the CSD. In this case, the account of the former is debited and the account of the latter is credited. The CSD incurs marginal costs of $2c_s$ and both banks pay p_C as price for the settlement to the CSD. If instead a bank with an account with the CSD sells to another bank with an account with the custodian bank, the transaction has to be settled across the two settlement service providers. The account of the seller with the CSD is debited and the (so called omnibus) account of the custodian bank with the CSD and the account of the buyer with the custodian bank are both credited. Here, the CSD faces marginal costs $2c_s$ and the custodian bank $\widetilde{c}_s$. The seller and the custodian bank both pay p_C to the CSD and the buyer pays p_A to the custodian bank.

3 Social welfare

In this section, we derive the social welfare optimum, i.e. the socially optimal allocation of investor banks to the two settlement service providers. The social costs of settlement for given α and k are

$$S = \begin{cases} k(c_a + c_s) + (1-k)(\widetilde{c}_a + \widetilde{c}_s) + (k - 2\alpha)c_s + \int_k^1 c(\frac{1}{2} - i)di, & \text{if } \alpha \leq \frac{1}{2}k \\ k(c_a + c_s) + (1-k)(\widetilde{c}_a + \widetilde{c}_s) + (2\alpha - k)c_s + \int_k^1 c(\frac{1}{2} - i)di, & \text{if } \alpha \geq \frac{1}{2}k \end{cases}$$

Each investor bank has to open one account, either with the CSD or with the custodian. This implies social costs incurred at the CSD of kc_a and social costs incurred at the custodian bank of $(1-k)\widetilde{c}_a$. Also, because investor banks either send or receive exactly once the security, their accounts have to be either debited once or credited once. This implies social costs incurred at the CSD of kc_s and social costs incurred at the custodian bank of $(1-k)\widetilde{c}_s$. Furthermore, if $\alpha \leq \frac{1}{2}k$, a proportion of $(k - 2\alpha)$ transactions has to be settled through the custodian bank's account with the CSD, i.e. this account has to be credited $(k-2\alpha)$ times. The social costs for this settlement across the two settlement service providers are $(k - 2\alpha)c_s$ (incurred at the CSD). Analoguously, if $\alpha \geq \frac{1}{2}k$, a proportion of $(2\alpha - k)$ transactions has to be settled on the custodian bank's account with a CSD, i.e. this account has to be debited $(2\alpha - k)$ times. The social costs are $(2\alpha - k)c_s$ (again incurred at the CSD). Finally, the social costs incurred at the investor banks (those that go to the custodian bank) are $\int_k^1 c(\frac{1}{2} - i)di$.

Taking expectations with respect to α easily leads to

$$E(S) = \begin{cases} k(c_a + c_s) + (1-k)(\widetilde{c}_a + \widetilde{c}_s) + \frac{1}{2}kc_s - \frac{1}{2}ck(1-k) & \text{if } k \leq \frac{1}{2} \\ k(c_a + c_s) + (1-k)(\widetilde{c}_a + \widetilde{c}_s) + \frac{1}{2}(1-k)c_s - \frac{1}{2}ck(1-k) & \text{if } k \geq \frac{1}{2} \end{cases} \tag{1}$$

Here, the third term ($\frac{1}{2}kc_s$ resp. $\frac{1}{2}(1-k)c_s$) represents the expected social costs of settling transactions across the two service providers, i.e. on the account of

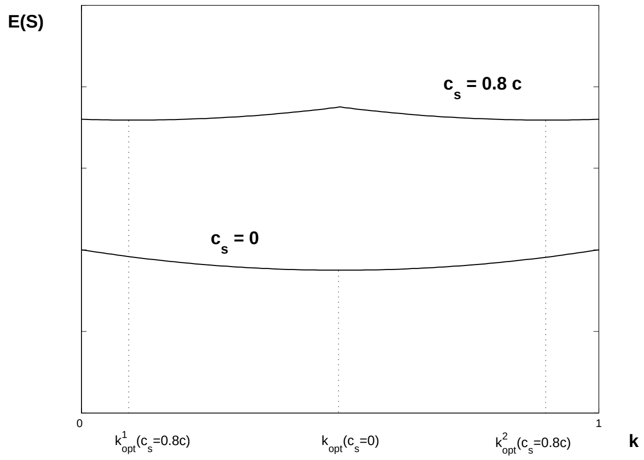

Figure 2: Assume $\widetilde{c}_a + \widetilde{c}_s = c_a + c_s$. When c_s is low relative to c, investor bank's preferences are crucial for the allocation of banks across institutions, and k_{opt} is close to $\frac{1}{2}$. For high c_s, the investor banks' preferences become less important relative to the costs of settling across institutions and k_{opt} moves away from $\frac{1}{2}$.

the custodian bank with the CSD. This term is decreasing as k moves away from its midpoint $\frac{1}{2}$. The reason is that settlement across institutions is costly. If many investor banks are concentrated in either the CSD or in the custodian bank (in other words, k is close to either 1 or 0), then most trades can be settled internally and these costs are avoided. In a sense, the term can be interpreted as network externalities that arise from concentrating accounts in one institution.

To derive the socially optimal k, $E(S)$ has to be minimised with respect to k and subject to $0 \leq k \leq 1$. In the appendix, we prove

Proposition 1 The socially optimal market share of the CSD k is given by $k = k_{opt}$ with

$$k_{opt}\begin{cases} = \max\left\{\frac{c-c_s}{2c} + \dfrac{\widetilde{c}_a + \widetilde{c}_s - c_a - c_s}{c}, 0\right\} & \text{if } \widetilde{c}_a + \widetilde{c}_s < c_a + c_s \\ \in \left\{\frac{c-c_s}{2c}, \frac{c+c_s}{2c}\right\} & \text{if } \widetilde{c}_a + \widetilde{c}_s = c_a + c_s \\ = \min\left\{\dfrac{c+c_s}{2c} + \dfrac{\widetilde{c}_a + \widetilde{c}_s - c_a - c_s}{c}, 1\right\} & \text{if } \widetilde{c}_a + \widetilde{c}_s > c_a + c_s \end{cases}$$

Looking at the middle case first, if the CSD and the custodian bank have the same cost structure ($\widetilde{c}_a + \widetilde{c}_s = c_a + c_s$), then there are two socially optimal allocations, namely $k^1_{opt} = \frac{1}{2}\frac{c-c_s}{c} < 1/2$ and $k^2_{opt} = \frac{1}{2}\frac{c+c_s}{c} > 1/2$. Both have the same distance from $k = 1/2$ and are in in this sense symmetric ($k^1_{opt} = 1 - k^2_{opt}$). To understand the meaning of these solutions, let us focus on the extreme case where $c_s = 0$. In this case, settlement across institutions is not socially costly, so the only relevant cost parameter that a social planner would care about are the investor banks' preferences, which are maximized for $k = \frac{1}{2}$. In this case, it is optimal that any investor bank $i \in [0, \frac{1}{2}[$ goes to the custodian bank and any investor bank $i \in]\frac{1}{2}, 1]$ goes to the CSD.

If c_s increases, it is more and more important to avoid settlement across the two service providers, i.e. to pool many investor banks in one service provider. By doing so, network externalities can be exploited. If c_s is sufficiently high and c sufficiently low, it is optimal that all investor banks have accounts with the same institution ($k = 0$ or $k = 1$) so that settlement across service providers does not take place at all. In other words, the social planner has to balance the costs of forcing investor banks to settle with the institution that is not their preferred one (as measured by c) against the cost of settling across institutions c_s. Note that as long as the cost structure is equal for both institutions, it is irrelevant for the social optimum at which institution more settlement takes place, i.e. whether $k < \frac{1}{2}$ or $k > \frac{1}{2}$. Figure 2 displays expected social costs for different realizations of c_s.

Proposition 1 also tells us that when the cost structures differ for the CSD and the custodian bank so that $\widetilde{c}_a + \widetilde{c}_s \neq c_a + c_s$, there is a correcting term in the definition of k_{opt} that implies that the number of customers settling at the more costly institution should be further reduced. Notice that in this case, there is no multiplicity of equilibria: if the custodian bank has a more efficient cost structure, then the optimal allocation predicts that $k_{opt} < \frac{1}{2}$ while the opposite is true if the CSD is more efficient.

From now on, we focus on the case $\widetilde{c}_a + \widetilde{c}_s = c_a + c_s$ to reduce the mathematical complexity of the analysis. Also, to avoid corner solutions with $k^1_{opt} = 0$ and $k^2_{opt} = 1$, we assume for the rest of the paper $c \geq c_s$.

4 The equilibrium

4.1 The payoff functions

We now present the payoff functions of the players for given k and prices p_A, q_A, p_C and q_C. The payoff function of some investor bank $i \in [0, 1]$ is simply

$$\pi_i = \begin{cases} -[q_A + p_A + c(\frac{1}{2} - i)] & \text{if } i \text{ is customer of the custodian bank} \\ -[q_C + p_C] & \text{if } i \text{ is customer of the CSD} \end{cases}$$

Now consider the custodian bank. For each investor bank that maintains an account with it and uses it to make one transaction, the custodian bank receives $q_A + p_A$, but incurs settlement costs of $c_a + c_s$. Moreover, for those $k - 2\alpha$ (resp. $2\alpha - k$) transactions where the receiver (the sender) is customer of the CSD, the custodian bank needs to pay a fee p_C to the CSD for settlement. All other trades are settled internally. The profit of the custodian bank is thus given by

$$\pi_A = \begin{cases} (1 - k)(q_A - c_a + p_A - c_s) - (k - 2\alpha)p_C\ , & \text{if } \alpha < \frac{1}{2}k \\ (1 - k)(q_A - c_a + p_A - c_s) - (2\alpha - k)p_C\ , & \text{if } \alpha > \frac{1}{2}k \end{cases}$$

Taking expectations with respect to α, we obtain the custodian bank's payoff function

$$E[\pi_A] = \begin{cases} (1-k)(q_A - c_a + p_A - c_s) - \frac{1}{2}kp_C \ , \ \text{if } k \leq \frac{1}{2} \\ (1-k)(q_A - c_a + p_A - c_s) - \frac{1}{2}(1-k)p_C \ , \ \text{if } k \geq \frac{1}{2} \end{cases} \tag{2}$$

Again, $\frac{1}{2}k$ is the expected number of trades to be settled through the custodian bank's account with the CSD if $k \leq \frac{1}{2}$ (and $\frac{1}{2}(1-k)$ if $k \geq \frac{1}{2}$). Obviously, it peaks at $k = 1/2$, that is, when just one half of all investor banks have an account with either institution. In the other extreme, for $k = 0$, all trades can be settled internally on accounts with the custodian bank. Similarly, if $k = 1$, all banks maintain an account with the CSD, so the custodian bank settles no trades.

Finally, consider the CSD. Similar considerations as above lead to the CSD's profit

$$\pi_C = \begin{cases} k(q_C - c_a + p_C - c_s) + (k-2\alpha)(p_C - c_s) \ , \ \text{if } \alpha < \frac{1}{2}k \\ k(q_C - c_a + p_C - c_s) + (2\alpha - k)(p_C - c_s) \ , \ \text{if } \alpha > \frac{1}{2}k \end{cases}$$

Taking expectations with respect to α gives the CSD's payoff function

$$E[\pi_C] = \begin{cases} k(q_C - c_a + p_C - c_s) + \frac{1}{2}k(p_C - c_s) \ , \ \text{if } k \leq \frac{1}{2} \\ k(q_C - c_a + p_C - c_s) + \frac{1}{2}(1-k)(p_C - c_s) \ , \ \text{if } k \geq \frac{1}{2} \end{cases} \tag{3}$$

4.2 The decision of the investor banks

We solve the model backwards and start with stage (3). Here, each bank $i \in [0,1]$ has to decide with which service provider it wants to have an account given the prices q_c, p_C, q_A, p_A. Bank i is indifferent if and only if

$$\begin{aligned} q_C + p_C &= c(\frac{1}{2} - i) + q_A + p_A \\ &\Longleftrightarrow \\ i &= \frac{1}{2} + \frac{q_A + p_A - q_C - p_C}{c} \end{aligned}$$

The proportion of banks with an account with the CSD is thus

$$k = \frac{1}{2} + \frac{q_A + p_A - q_C - p_C}{c} \tag{4}$$

Here and throughout the paper, we assume that corner solutions with $k = 0$ or $k = 1$ will not occur in equilibrium.

If $q_A + p_A = q_C + p_C$, we of course have $k = \frac{1}{2}$. If the CSD and the custodian bank charge different prices, i.e. $q_A + p_A >$ (or $<)q_C + p_C$, we have $k > (<)\frac{1}{2}$ so that more investor banks settle at the institution with lower prices. Finally, notice the impact of the investors' cost c of having to settle at a different institution than the preferred one: If c increases, then k moves closer to $\frac{1}{2}$ since it gets more expensive for banks in $[0, \frac{1}{2}[$ and less expensive for banks in $]\frac{1}{2}, 1]$ to use the custodian bank. Thus, k is decreasing (increasing) in c if $q_A + p_A > (<)q_C + p_C$.

4.3 The decision of the custodian bank

The payoff function of the custodian bank is obviously a function of $q_A + p_A$. Thus, to derive the best response correspondence of the custodian bank on given prices q_C and p_C, $E[\pi_A]$ is to be maximized with respect to $q_A + p_A$ only (not with respect to q_A and p_A) and subject to equation (4), $q_A + p_A \geq 0$ and $0 \leq k \leq 1$. In the appendix, we prove

Proposition 2 The best response correspondence of the custodian bank is given by the following:

(i) If $2q_C + p_C \geq 2(c_a + c_s) - c$ and $2q_C + 3p_C \leq 3c + 2(c_a + c_s)$, then

$$q_A + p_A \begin{cases} = \frac{1}{4}c + \frac{1}{2}(c_a + c_s) + \frac{1}{2}q_C + \frac{1}{4}p_C \equiv X & \text{if } q_C + p_C > \widehat{c} \\ \in \{X, Y\} & \text{if } q_C + p_C = \widehat{c} \\ = \frac{1}{4}c + \frac{1}{2}(c_a + c_s) + \frac{1}{2}q_C + \frac{3}{4}p_C \equiv Y & \text{if } q_C + p_C < \widehat{c} \end{cases}$$

(ii) If $2q_C + p_C < 2(c_a + c_s) - c$, then

$$q_A + p_A \begin{cases} = V & \text{if } 2q_C + 3p_C > 3c + 2(c_a + c_s) \text{ and } q_C + p_C > \widehat{c} \\ \in \{V, W\} & \text{if } 2q_C + 3p_C > 3c + 2(c_a + c_s) \text{ and } q_C + p_C = \widehat{c} \\ = W & \text{otherwise} \end{cases}$$

(iii) If $2q_C + 3p_C > 3c + 2(c_a + c_s)$, then

$$q_A + p_A \begin{cases} = W & \text{if } 2q_C + p_C < 2(c_a + c_s) - c \text{ and } q_C + p_C < \widehat{c} \\ \in \{V, W\} & \text{if } 2q_C + p_C < 2(c_a + c_s) - c \text{ and } q_C + p_C = \widehat{c} \\ = V & \text{otherwise} \end{cases}$$

with $\widehat{c} \equiv \frac{1}{2}c + c_a + c_s$, $V \equiv q_C + p_C - \frac{1}{2}c$ and $W \equiv q_C + p_C + \frac{1}{2}c$.

Case (i) represents the normal case with an interior solution where $0 < k < 1$. Here, there exist two possible optimal responses by the custodian bank: if the prices set by the CSD are rather high, the custodian bank is able to attract customers by choosing rather low prices $q_A + p_A = X$. If the CSD sets low prices, on the other hand, the custodian bank knows that is will not be able to attract a large mass of depositors and chooses high prices $q_A + p_A = Y$.

Cases (ii) and (iii) represent corner solutions in which either $k = 0$ or $k = 1$. However, as we will see later on, only case (i) is relevant in equilibrium.

Furthermore, it is important to note that in $q_C + p_C = \frac{1}{2}c + (c_a + c_s)$ the best response correspondence is discontinuous (at least if $p_C > 0$). The custodian bank is indifferent between $q_A + p_A = X$ ($k = \frac{1}{2} - \frac{p_C}{4c} < \frac{1}{2}$) and $q_A + p_A = Y$ ($k = \frac{1}{2} + \frac{p_C}{4c} > \frac{1}{2}$) or between $q_A + p_A = V$ ($k = 0$) and $q_A + p_A = W$ ($k = 1$). This is due to the fact that the custodian bank's payoff function $E[\pi_A]$ is not quasi-concave in $q_A + p_A$. For this reason, there is no equilibrium in the simultaneous move game, i.e. if the CSD and the custodian bank choose their prices simultaneously. (See section 5 below.)

4.4 The decision of the CSD

The CSD moves first so that it maximizes $E[\pi_C]$ with respect to q_C and p_C taking into account both $q_A + p_A$ as given in proposition 2 and k as derived in section 4.2. We show in the appendix that the maximization leads to the following proposition:

Proposition 3 (1) If $c_a + \frac{1}{2}c_s \leq c$, then the CSD chooses $q_C = 0$ and $p_C = \frac{1}{2}c + c_a + c_s$. In this case, the prices of the custodian bank are given by any q_A-p_A combination satisfying $q_A + p_A = \frac{5}{8}c + \frac{5}{4}(c_a + c_s)$, $q_A \geq 0$ and $p_A \geq 0$. Finally, we have

$$k = \frac{\frac{5}{8}c + \frac{1}{4}(c_a + c_s)}{c} \equiv k_1$$

(2) If $c \leq c_a + \frac{1}{2}c_s$, then the CSD chooses $p_C = \frac{3}{2}c + \frac{1}{2}c_s$ and $q_C = c_a + \frac{1}{2}c_s - c$. In this case, the prices of the custodian bank are given by any q_A-p_A combination satisfying $q_A + p_A = \frac{7}{8}c + c_a + \frac{9}{8}c_s$, $q_A \geq 0$ and $p_A \geq 0$. Finally, we have

$$k = \frac{\frac{7}{8}c + \frac{1}{8}c_s}{c} \equiv k_2$$

From proposition 3, we can draw some remarkable conclusions. In the first place, we see that $q_C + p_C < q_A + p_A$ so that investor banks always face lower prices when settling at the CSD than if they settled with the custodian bank. As a result, $k > 1/2$, i.e. the CSD is able to attract more customers than the custodian bank although we have assumed symmetry in marginal costs ($\widetilde{c}_a = c_a$ and $\widetilde{c}_s = c_s$) and in the quality of the settlement services of the two service providers.

The main reason is that the CSD can raise the costs of the custodian bank without raising the costs of its other customers in the following way:[9] The CSD can increase p_C and simultaneously decrease q_C by the same amount. The costs of investor banks with the CSD remain unchanged since these banks have one account with the CSD and want to have only one trade settled. However, the costs of the custodian bank increase since it has also only one account, but many trades settled through its account with the CSD. The resulting higher costs of the custodian bank normally force the custodian bank to increase its prices so that it loses customers to the CSD. If c is high (case (1) in the proposition), then in equilibrium we have $q_C = 0$. In other words, the CSD applies the above described strategy of raising-rival's-costs as far as possible. Note that $k = \frac{5}{8} > 1/2$ even if $c \to \infty$. This is because if c approaches infinity, the prices of both service providers do the same.

Generally, the prices set by the CSD is increasing in its costs parameters (c_a and c_s). Moreover, prices are increasing in c: the more costly it is for investor banks to switch between service providers (i.e. the higher c), the higher the CSD can set prices be set without having to fear that many customers move to the custodian bank.

[9] Another reason may be the first mover advantage of the CSD.

If c is low compared to c_s and c_a (case (2) in the proposition), we find $q_C > 0$, i. e. the CSD has less incentives to raise the cost of the custodian bank. The reason appears to be the following: If c is low and $q_C + p_C$ is only slightly lower than $q_A + p_A$, it is attractive to choose the CSD even for those investor banks that are located close to the bottom of the interval $[0, 1]$. In this case, it is sufficient for the CSD to raise the costs of the custodian bank only little.

Notice that k is decreasing in c.[10] This makes sense given that we have $k > 1/2$ in equilibrium (recall the results of section 4.2). If c increases, banks in $[0, \frac{1}{2}[$ increasingly prefer to go to the custodian bank and banks in $]\frac{1}{2}, 1]$ increasingly prefer to go to the CSD, provided prices remain unchanged. Consequently, if $k > 1/2$ and c increases, then the CSD will lose customers. Finally, k is increasing in c_a and c_s, i.e. technological progress leads to a lower k, thus to more custodian banking.

4.5 Welfare assessment

We now compare the equilibrium with the results of our welfare analysis (proposition 1). As discussed in section 3, there are two welfare maximizers, namely $k^1_{opt} = \frac{1}{2}\frac{c-c_s}{c} < 1/2$ and $k^2_{opt} = \frac{1}{2}\frac{c+c_s}{c} > 1/2$. Since both equilibrium solutions k_1 and k_2 are above $1/2$, it is obvious that in equilibrium we have

$$k > k^1_{opt}.$$

Comparison with k^2_{opt} is less straightforward. Since $c \geq c_s$, we also have $k_2 \geq k^2_{opt}$. However, we can not say that the equilibrium value of k is always higher than the socially optimal value, because it is not always true that $k_1 \geq k^2_{opt}$: Indeed, it is easy to check that

$$k_1 \geq k^2_{opt} \Leftrightarrow c \geq 2(c_s - c_a). \tag{5}$$

In other words, the market share chosen by the CSD is larger than the socially optimal one if and only if network externalities (c_s) are relatively small. The result reflects the fact that the CSD and a social planner care differently about the costs arising from consumer preferences and network externalities. In particular, comparing the functional form for social welfare (1) and the CSD's profit function (3), we see that the CSD is less concerned than the planner about the cost arising from settlement across institutions. Still, these costs (which are borne by the custodian bank) are relevant for social welfare.

The implications of this are illustrated in figure 3, which displays all relevant conditions in the $(c, c_s - c_a)$-graph. As we only consider parameter values for which $c \geq c_s$, the relevant area is the one on the left of the dotted line. The dashed line separates the regions for which the different equilibria are obtained in equilibrium: k_1 results for high values of c, while k_2 is obtained for parameter combinations below the line. We have already shown that $k_2 \geq k^2_{opt}$, so let us focus on the area above the line, for which k_1 is obtained. Here, the solid

[10] k is a continuous function of c, but not differentiable in $c = c_a + \frac{1}{2}c_s$.

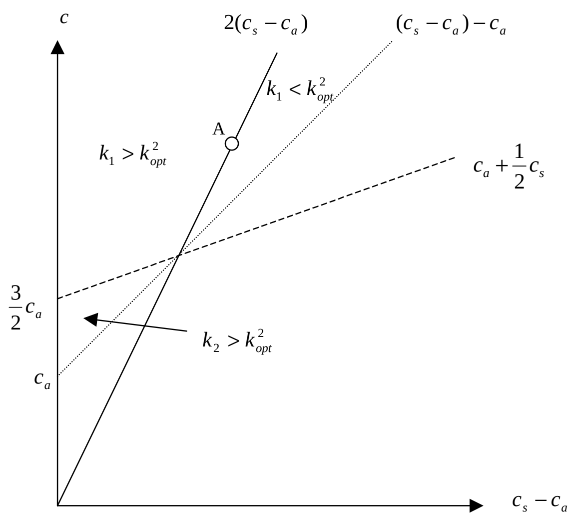

Figure 3: The equilibrium value of k is higher (lower) than socially optimal on the left (right) of the solid line.

line indicates those parameter combinations for which exactly $k_1 = k_{opt}^2$. To understand condition 5, start from a point on this line (point A), and suppose that c_s increases. Given that we are considering $k > \frac{1}{2}$, these higher externalities imply that it is socially optimal that the CSD obtains a higher market share, so $k < k_{opt}^2$. On the other hand, if c_s decreases then the investor bank's preferences play a higher role for the socially optimal allocation, so k_{opt}^2 moves closer to $\frac{1}{2}$. Therefore, on the left of point A in the figure we have $k > k_{opt}^2$.

To summarize, we have $k < k_{opt}^2$ in equilibrium if and only if c is sufficiently low. Since k can be higher than, equal to or lower than k_{opt}^2, there is no case for regulatory interventions aiming at reducing the CSD's market share.

5 The simultaneous move game

In this final section, we show that our model would have no equilibrium if we were assuming that the CSD and the custodian bank set their prices simultaneously. Firstly note as mentioned earlier that the payoff function $E[\pi_A]$ of the custodian bank is not quasi concave in $q_A + p_A$. For given prices q_C and p_C of the CSD, $E[\pi_A]$ has a local maximum with $q_A + p_A$ somewhere between $q_C + p_C - \frac{1}{2}c$ and $q_C + p_C$ and another local maximum with $q_A + p_A$ somewhere between $q_C + p_C$ and $q_C + p_C + \frac{1}{2}c$. Thus, the sufficient conditions for the existance of a Nash equilibrium are not satisfied. However, that does not neccesarily imply that there is no Nash equilibrium.

To show that there is indeed no Nash equilibrium, we have to derive the best response function of the CSD which is given by

Proposition 4 The best response function of the CSD is

$$p_C \begin{cases} \in [0, q_A + p_A - \frac{1}{2}c] & \text{if } \frac{5}{2}c + 2c_a + c_s \leq q_A + p_A \\ = \frac{3}{4}c + \frac{1}{2}(q_A + p_A) + c_a + \frac{1}{2}c_s & \text{if } \frac{3}{2}c + 2c_a + c_s \leq q_A + p_A \leq \frac{5}{2}c + 2c_a + c_s \\ = q_A + p_A & \text{if } \frac{1}{2}c + \frac{2}{3}c_a + c_s \leq q_A + p_A \leq \frac{3}{2}c + 2c_a + c_s \\ = \frac{1}{4}c + \frac{1}{2}(q_A + p_A) + \frac{1}{3}c_a + \frac{1}{2}c_s & \text{if } \frac{2}{3}c_a + c_s - \frac{1}{2}c \leq q_A + p_A \leq \frac{1}{2}c + \frac{2}{3}c_a + c_s \\ \in [0, \frac{1}{2}c + q_A + p_A] & \text{if } q_A + p_A \leq \frac{2}{3}c_a + c_s - \frac{1}{2}c \end{cases}$$

$$q_C \begin{cases} = q_A + p_A - \frac{1}{2}c - p_C & \text{if } \frac{5}{2}c + 2c_a + c_s \leq q_A + p_A \\ = 0 & \text{if } \frac{2}{3}c_a + c_s - \frac{1}{2}c \leq q_A + p_A \leq \frac{5}{2}c + 2c_a + c_s \\ = q_A + p_A + \frac{1}{2}c - p_C & \text{if } q_A + p_A \leq \frac{2}{3}c_a + c_s - \frac{1}{2}c \end{cases}$$

We see from this proposition that in the simultaneous move game the CSD still has clear incentives to set $q_C = 0$ and p_C relatively high to raise the custodian bank's costs without raising the overall costs of the investor banks. Furthermore, note that p_C is increasing in $q_A + p_A$ and continuous in $q_A + p_A$, while the custodian banks best response correspondence $q_A + p_A$ is discontinuous according to proposition 2. As mentioned before, this is the reason why the simultaneous move game has no equilibrium as stated in

Proposition 5 If the CSD and the custodian bank set their prices simultaneously, then there is no equilibrium in pure strategies.

With numeric examples, it is easy to show that in a diagramm with $q_A + p_A$ on the horizontal and p_C on the vertical axis, the best response of the custodian bank runs right of the best response of the CSD as long as $p_C < \frac{1}{2}c + c_a + c_s$(and $q_C = 0$). For $p_C > \frac{1}{2}c + c_a + c_s$(and $q_C = 0$) however, it runs left of the CSD's best response. I.e. it jumps from the right to the left of the CSD's best response in $p_C = \frac{1}{2}c + c_a + c_s$. For that reason, there is no interception of the two best responses and no equilibrium in pure staregies.

6 Concluding remarks

We have discussed a simple model of the competition between a CSD and a custodian bank. It has been shown that the CSD is able to exploit its monopoly position as central depository by raising the custodian bank's costs to attract more investor banks as customers. However, we have also shown that the market share of the CSD in equilibrium is not necessarily higher than its socially optimal market share.

Our analysis appears to be relevant in two ways. Firstly, it contributes to the current discussion between market participants, especially CSDs and custodian banks, on whether and how competition in the securities settlement industry is distorted. Since we have shown that the CSD's equilibrium market share is not necessariliy higher than socially optimal, the model provides no case for regulatory interventions aiming at reducing the CSD's market share. Secondly, it is theoretically relevant in that it considers a case of asymmetric network competition which has not been analyzed yet.

Extensions of the model could go into different directions. Firstly, it might be interesting to allow the custodian bank to price discriminate by charging investor banks with an account with the custodian bank a higher price if they trade with an investor bank with an account with the CSD. This idea is in line with Laffont, Rey and Tirol (1996b). Secondly, one may want to analyze our model with the assumption that the CSD can charge a progressive settlement price, i.e. the more trades a customer of the CSD wants to have settled through accounts with the CSD, the higher the price per settlement this customer has to pay. Since the custodian bank is the customer of the CSD with the highest number of trades to be settled through accounts with the CSD, this would clearly adversely effect the custodian bank and give the CSD power to raise its rival's cost even higher. Finally, it might be interesting to assume that there are more than one custodian banks so that there is competition between several custodian banks. If the services of all custodian banks are perfect substitutes for all investor banks, the price competition between custodian banks would reduce settlement prices significantly.

7 Appendix

Proof of proposition 1:

Let $E[S^u] = k(c_a + c_s) + (1 - k)(\tilde{c}_a + \tilde{c}_s) + \frac{1}{2}kc_s - \frac{1}{2}ck(1 - k)$ be the upper branch of $E[S]$. This is a convex function in k with a minimum in

$$k = \frac{c - c_s}{2c} + \frac{\tilde{c}_a + \tilde{c}_s - c_a - c_s}{c} \equiv k_1$$

with

$$E[S^u]^* = \tilde{c}_a + \tilde{c}_s - \frac{1}{2}\frac{[\frac{1}{2}c + \tilde{c}_a + \tilde{c}_s - c_a - \frac{3}{2}c_s]^2}{c}$$

Let $E[S^L] = k(c_a + c_s) + (1 - k)(\tilde{c}_a + \tilde{c}_s) + \frac{1}{2}(1 - k)c_s - \frac{1}{2}ck(1 - k)$. This is also a convex function in k with a minimum in

$$k = \frac{c + c_s}{2c} + \frac{\tilde{c}_a + \tilde{c}_s - c_a - c_s}{c} \equiv k_2$$

with

$$E[S^L]^* = \tilde{c}_a + \tilde{c}_s + \frac{1}{2}c_s - \frac{1}{2}\frac{[\frac{1}{2}c + \tilde{c}_a + \tilde{c}_s - c_a - \frac{1}{2}c_s]^2}{c}$$

It is easy to show that

$$\begin{array}{rcl} E[S^u]^* & \geq & E[S^L]^* \\ & \Leftrightarrow & \\ \tilde{c}_a + \tilde{c}_s & \geq & c_a + c_s \end{array}$$

Assume $c_a + c_s \leq \widetilde{c}_a + \widetilde{c}_s$. In this case, we have $E[S^u]^* \geq E[S^L]^*$ and $k_1 \leq \frac{1}{2}$, i.e. $k_{opt} = k_1$. Assume instead $\widetilde{c}_a + \widetilde{c}_s \leq c_a + c_s$. In this case, we have $E[S^u]^* \leq E[S^L]^*$ and $\frac{1}{2} \leq k_2$, i.e. $k_{opt} = k_2$. This completes the proof of the proposition.

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Proof of proposition 2:

For given prices q_C and p_C, we have to maximise $E[\pi_A]$ as given in equation 2 with respect to $q_A + p_A$ subject to $q_A + p_A \geq 0$ and $0 \leq k \leq 1$, where k is given by equation 4.

Let $E[\pi_A^U] = (1-k)(q_A - c_a + p_A - c_s) - \frac{1}{2}kp_C$ be the upper branch of $E[\pi_A]$. In a first step, we maximise $E[\pi_A^U]$ subject to $q_A + p_A \geq 0$ and $0 \leq k \leq \frac{1}{2}$. We easily get the following maximiser and maximum:

$$q_A + p_A = \begin{cases} q_C + p_C & \text{if } q_C + \frac{3}{2}p_C < c_a + c_s + \frac{1}{2}c \\ \frac{1}{4}c + \frac{1}{2}(c_a + c_s) + \frac{1}{2}q_C + \frac{1}{4}p_C & \text{if } c_a + c_s + \frac{1}{2}c \leq q_C + \frac{3}{2}p_C \leq c_a + c_s + \frac{3}{2}c \\ q_C + p_C - \frac{1}{2}c & \text{if } q_C + \frac{3}{2}p_C > c_a + c_s + \frac{3}{2}c \end{cases}$$

and

$$E[\pi_A^U]^* = \begin{cases} \frac{1}{2}q_C + \frac{1}{4}p_C - \frac{1}{2}(c_a + c_s) \equiv E_1 & \text{if } q_C + \frac{3}{2}p_C < c_a + c_s + \frac{1}{2}c \\ \frac{[\frac{1}{4}c - \frac{1}{2}(c_a + c_s) + \frac{1}{2}q_C + \frac{3}{4}p_C]^2}{c} - \frac{1}{2}p_C \equiv E_2 & \text{if } c_a + c_s + \frac{1}{2}c \leq q_C + \frac{3}{2}p_C \leq c_a + c_s + \frac{3}{2}c \\ q_C + p_C - \frac{1}{2}c - (c_a + c_s) \equiv E_3 & \text{if } q_C + \frac{3}{2}p_C > c_a + c_s + \frac{3}{2}c \end{cases}$$

Let $E[\pi_A^L] = (1-k)(q_A - c_a + p_A - c_s) - \frac{1}{2}(1-k)p_C$ be the lower branch of $E[\pi_A]$. We maximise $E[\pi_A^L]$ subject to $q_A + p_A \geq 0$ and $\frac{1}{2} \leq k \leq 1$. Again we easily get the maximiser and maximum:

$$q_A + p_A = \begin{cases} q_C + p_C + \frac{1}{2}c & \text{if } q_C + \frac{1}{2}p_C < c_a + c_s - \frac{1}{2}c \\ \frac{1}{4}c + \frac{1}{2}(c_a + c_s) + \frac{1}{2}q_C + \frac{3}{4}p_C & \text{if } c_a + c_s - \frac{1}{2}c \leq q_C + \frac{1}{2}p_C \leq c_a + c_s + \frac{1}{2}c \\ q_C + p_C & \text{if } q_C + \frac{1}{2}p_C > c_a + c_s + \frac{1}{2}c \end{cases}$$

and

$$E[\pi_A^U]^* = \begin{cases} 0 \equiv E_4 & \text{if } q_C + \frac{1}{2}p_C < c_a + c_s - \frac{1}{2}c \\ \frac{[\frac{1}{4}c - \frac{1}{2}(c_a + c_s) + \frac{1}{2}q_C + \frac{1}{4}p_C]^2}{c} \equiv E_5 & \text{if } c_a + c_s - \frac{1}{2}c \leq q_C + \frac{1}{2}p_C \leq c_a + c_s + \frac{1}{2}c \\ \frac{1}{2}q_C + \frac{1}{4}p_C - \frac{1}{2}(c_a + c_s) \equiv E_6 & \text{if } q_C + \frac{1}{2}p_C > c_a + c_s + \frac{1}{2}c \end{cases}$$

We now have to consider the following nine cases:

A) $q_C + \frac{3}{2}p_C < c_a + c_s + \frac{1}{2}c$ and $q_C + \frac{1}{2}p_C < c_a + c_s - \frac{1}{2}c$. It follows immediately that $q_C + p_C < c_a + c_s$, i.e. $E_1 < 0 = E_4$. The best response is thus given by $q_A + p_A = q_C + p_C + \frac{1}{2}c$.

B) $q_C + \frac{3}{2}p_C < c_a + c_s + \frac{1}{2}c$ and $c_a + c_s - \frac{1}{2}c \leq q_C + \frac{1}{2}p_C \leq c_a + c_s + \frac{1}{2}c$. Because $E_5 = \frac{1}{c}[\frac{1}{4}c + E_1]^2$, we know that $E_5 \leq E_1 \Leftrightarrow [E_1 - \frac{1}{4}c]^2 \leq 0$. This is only possible if $E_1 = \frac{1}{4}c$, i.e. $q_C + \frac{1}{2}p_C = c_a + c_s + \frac{1}{2}c$ which is in contradiction to $q_C + \frac{3}{2}p_C < c_a + c_s + \frac{1}{2}c$. The best response is thus given by $q_A + p_A = \frac{1}{4}c + \frac{1}{2}(c_a + c_s) + \frac{1}{2}q_C + \frac{3}{4}p_C$.

C) $q_C + \frac{3}{2}p_C < c_a + c_s + \frac{1}{2}c$ and $q_C + \frac{1}{2}p_C > c_a + c_s + \frac{1}{2}c$. This case is not possible.

D) $c_a + c_s + \frac{1}{2}c \leq q_C + \frac{3}{2}p_C \leq c_a + c_s + \frac{3}{2}c$ and $q_C + \frac{1}{2}p_C < c_a + c_s - \frac{1}{2}c$. First note that $\frac{1}{4}c - \frac{1}{2}(c_a + c_s) + \frac{1}{2}q_C + \frac{3}{4}p_C \geq 0$ so that E_2 is maximised by minimising $(c_a + c_s)$. We now have to consider two sub-cases: (i) $2c \geq p_C$. Since $c_a + c_s > q_C + \frac{1}{2}p_C + \frac{1}{2}c$, we get $E_2 < \frac{1}{2}p_C[\frac{1}{2c}p_C - 1] \leq 0$. (ii) $2c < p_C$. Since $c_a + c_s \geq q_C + \frac{3}{2}p_C - \frac{3}{2}c$, we get $E_2 \leq c - \frac{1}{2}p_C < 0$. I.e. we have $E_2 < E_4$ and the best response is $q_A + p_A = q_C + p_C + \frac{1}{2}c$.

E) $c_a + c_s + \frac{1}{2}c \leq q_C + \frac{3}{2}p_C \leq c_a + c_s + \frac{3}{2}c$ and $c_a + c_s - \frac{1}{2}c \leq q_C + \frac{1}{2}p_C \leq c_a + c_s + \frac{1}{2}c$. It is easy to see that $E_2 \geq E_5 \Leftrightarrow q_C + p_C \geq \frac{1}{2}c + c_a + c_s$. The best response is therefore $q_A + p_A = \frac{1}{4}c + \frac{1}{2}(c_a + c_s) + \frac{1}{2}q_C + \frac{1}{4}p_C$ if $q_C + p_C \geq \frac{1}{2}c + c_a + c_s$ and $q_A + p_A = \frac{1}{4}c + \frac{1}{2}(c_a + c_s) + \frac{1}{2}q_C + \frac{3}{4}p_C$ if $q_C + p_C \leq \frac{1}{2}c + c_a + c_s$.

F) $c_a + c_s + \frac{1}{2}c \leq q_C + \frac{3}{2}p_C \leq c_a + c_s + \frac{3}{2}c$ and $q_C + \frac{1}{2}p_C > c_a + c_s + \frac{1}{2}c$. Because $E_2 = \frac{1}{c}[\frac{1}{4}c + \frac{1}{2}p_C + E_6]^2 - \frac{1}{2}p_C$, we know that $E_2 \leq E_6 \Leftrightarrow [\frac{1}{4}c - E_6 - \frac{1}{2}p_C]^2 \leq 0$. This is only possible if $E_6 = \frac{1}{4}c - \frac{1}{2}p_C$, i.e. $q_C + \frac{3}{2}p_C = c_a + c_s + \frac{1}{2}c$ which is in contradiction to $q_C + \frac{1}{2}p_C > c_a + c_s + \frac{1}{2}c$. Thus we get $E_2 > E_6$ and $q_A + p_A = \frac{1}{4}c + \frac{1}{2}(c_a + c_s) + \frac{1}{2}q_C + \frac{1}{4}p_C$ as best response.

G) $q_C + \frac{3}{2}p_C > c_a + c_s + \frac{3}{2}c$ and $q_C + \frac{1}{2}p_C < c_a + c_s - \frac{1}{2}c$. Here we have $E_3 \geq E_4 \Leftrightarrow q_C + p_C \geq \frac{1}{2}c + c_a + c_s$. The best response is therefore $q_A + p_A = q_C + p_C - \frac{1}{2}c$ if $q_C + p_C \geq \frac{1}{2}c + c_a + c_s$ and $q_A + p_A = q_C + p_C + \frac{1}{2}c$ if $q_C + p_C \leq \frac{1}{2}c + c_a + c_s$.

H) $q_C + \frac{3}{2}p_C > c_a + c_s + \frac{3}{2}c$ and $c_a + c_s - \frac{1}{2}c \leq q_C + \frac{1}{2}p_C \leq c_a + c_s + \frac{1}{2}c$. First note that $\frac{1}{4}c - \frac{1}{2}(c_a + c_s) + \frac{1}{2}q_C + \frac{1}{4}p_C \geq 0$. Moreover, we have $\frac{1}{4}c - \frac{1}{2}(c_a + c_s) + \frac{1}{2}q_C + \frac{1}{4}p_C \geq c \Leftrightarrow q_C + \frac{1}{2}p_C \geq c_a + c_s + \frac{3}{2}c$ which is in contradiction to $q_C + \frac{1}{2}p_C \leq c_a + c_s + \frac{1}{2}c$. Thus, we have $E_5 < \frac{1}{4}c - \frac{1}{2}(c_a + c_s) + \frac{1}{2}q_C + \frac{1}{4}p_C$. Since $\frac{1}{4}c - \frac{1}{2}(c_a + c_s) + \frac{1}{2}q_C + \frac{1}{4}p_C \geq E_3 \Leftrightarrow q_C + \frac{3}{2}p_C \leq c_a + c_s + \frac{3}{2}c$ which is in contradiction to above, we know that $E_3 > E_5$ and get as best response $q_A + p_A = q_C + p_C - \frac{1}{2}c$.

I) $q_C + \frac{3}{2}p_C > c_a + c_s + \frac{3}{2}c$ and $q_C + \frac{1}{2}p_C > c_a + c_s + \frac{1}{2}c$. Since $E_3 \leq E_6 \Leftrightarrow q_C + \frac{3}{2}p_C \leq c_a + c_s + c$ which is in contradiction to $q_C + \frac{3}{2}p_C > c_a + c_s + \frac{3}{2}c$, we get $E_3 > E_6$ and $q_A + p_A = q_C + p_C - \frac{1}{2}c$ as best response.

It is easy to verify that these results finalize the proof of proposition 2.

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Proof of proposition 3:

(I) To begin with, we determine the best the CSD can do under the restrictions $q_C \geq 0$, $p_C \geq 0$, $q_C + p_C \leq c_a + c_s + \frac{1}{2}c$, $2q_C + p_C \geq 2(c_a + c_s) - c$, $2q_C + 3p_C \leq 2(c_a + c_s) + 3c$. Here, the custodian bank chooses according to proposition 2 $q_A + p_A = \frac{1}{4}c + \frac{1}{2}(c_a + c_s) + \frac{1}{2}q_C + \frac{3}{4}p_C$ so that $k = \frac{1}{c}[\frac{3}{4}c + \frac{1}{2}(c_a + c_s) - \frac{1}{2}q_C - \frac{1}{4}p_C] \geq \frac{1}{2}$. Thus, we have to solve

$$\begin{aligned} MaxE[\pi_C] &= k(q_C - c_a + p_C - c_s) + \frac{1}{2}(1-k)(p_C - c_s) \\ s.t.\ q_C &\geq 0,\ p_C \geq 0,\ q_C + p_C \leq c_a + c_s + \frac{1}{2}c \\ 2q_C + p_C &\geq 2(c_a + c_s) - c,\ 2q_C + 3p_C \leq 2(c_a + c_s) + 3c \end{aligned}$$

We proceed as follows: We ignore the last two constraints and then show that the solution of the reduced problem satisfies the last two constraints, i.e. these constraints are not binding. It is easy to show that the solution of the reduced problem is the following:

(ii) If $c \geq c_a + \frac{1}{2}c_s$, then $q_C = 0$ and $p_C = \frac{1}{2}c + c_a + c_s$. In this case, we have $2q_C + p_C < 2(c_a + c_s) - c \Leftrightarrow \frac{1}{2}c + c < c_a + \frac{1}{2}c_s + \frac{1}{2}c_s$ and $2q_C + 3p_C > 2(c_a + c_s) + 3c \Leftrightarrow \frac{1}{2}c + c < c_a + \frac{1}{2}c_s + \frac{1}{2}c_s$ which is in contradiction to $c \geq c_a + \frac{1}{2}c_s$ and $c \geq c_s$ so that the solution satisfies the constraints $2q_C + p_C \geq 2(c_a + c_s) - c$ and $2q_C + 3p_C \leq 2(c_a + c_s) + 3c$. Furthermore, we get $q_A + p_A = \frac{5}{8}c + \frac{5}{4}(c_a + c_s), k = \frac{1}{c}[\frac{5}{8}c + \frac{1}{4}(c_a + c_s)]$ and $E[\pi_C] = \frac{1}{c}[\frac{5}{8}c + \frac{1}{4}(c_a + c_s)][\frac{1}{4}c - \frac{1}{2}c_a] + \frac{1}{4}c + \frac{1}{2}c_a \equiv E_{I(i)}$.

(ii) If $c \leq c_a + \frac{1}{2}c_s$, then $q_C = c_a + \frac{1}{2}c_s - c$ and $p_C = \frac{3}{2}c + \frac{1}{2}c_s$. In this case, we have $2q_C + p_C < 2(c_a + c_s) - c \Leftrightarrow c < c_s$ and $2q_C + 3p_C > 2(c_a + c_s) + 3c \Leftrightarrow c < c_s$ which is in contradiction to $c \geq c_s$ so that the solution satisfies the constraints $2q_C + p_C \geq 2(c_a + c_s) - c$ and $2q_C + 3p_C \leq 2(c_a + c_s) + 3c$. Furthermore, we get $q_A + p_A = \frac{7}{8}c + c_a + \frac{9}{8}c_s, k = \frac{1}{c}[\frac{7}{8}c + \frac{1}{8}c_s]$ and $E[\pi_C] = \frac{1}{2c}[\frac{1}{4}c - \frac{1}{4}c_s]^2 + \frac{1}{2}c \equiv E_{I(ii)}$.

(II) Now we show that the best the CSD can do under the restrictions $q_C \geq 0$, $p_C \geq 0$, $q_C + p_C \geq c_a + c_s + \frac{1}{2}c$, $2q_C + p_C \geq 2(c_a + c_s) - c$, $2q_C + 3p_C \leq 2(c_a + c_s) + 3c$ would make the CSD worse off. Here, the custodian bank chooses according to proposition 2 $q_A + p_A = \frac{1}{4}c + \frac{1}{2}(c_a + c_s) + \frac{1}{2}q_C + \frac{1}{4}p_C$ so that $k = \frac{1}{c}[\frac{3}{4}c + \frac{1}{2}(c_a + c_s) - \frac{1}{2}q_C - \frac{3}{4}p_C] \leq \frac{1}{2}$ and

$$E[\pi_C] = \frac{1}{c}[\frac{3}{4}c + \frac{1}{2}(c_a + c_s) - \frac{1}{2}(q_C + \frac{3}{2}p_C)][q_C + \frac{3}{2}p_C - c_a - \frac{3}{2}c_s]$$

We ignore the constraints and maximise $E[\pi_C]$ with respect to $q_C + \frac{3}{2}p_C$. It is very easy to check that the maximum is given by $E[\pi_C] = \frac{1}{2c}[\frac{3}{4}c - \frac{1}{4}c_s]^2 \equiv E_{II}$. It is clear that the maximum under the constraints of (II) cannot be higher than E_{II}. Thus, we only have to compare

(α) E_{II} with $E_{I(i)}$ under $c \geq c_a + \frac{1}{2}c_s$. It is easy to see that $E_{II} \geq E_{I(i)} \Leftrightarrow c[c + 2c_a + 2c_s] \leq [c_a + \frac{1}{2}c_s]^2$ which is in contradiction to $c \geq c_a + \frac{1}{2}c_s$ so that $E_{II} < E_{I(i)}$.

(β) E_{II} with $E_{I(ii)}$ under $c \leq c_a + \frac{1}{2}c_s$. It is easy to see that $E_{II} \geq E_{I(ii)} \Leftrightarrow c + \frac{1}{2}c_s \leq 0$ so that $E_{II} < E_{I(ii)}$.

(III) Now we show that the best the CSD can do under the restrictions that $q_A + p_A = V$ would make the CSD worse off compared to the maximum derived under (I). If $q_A + p_A = V$, then $E[\pi_C] = 0 \equiv E_{III}$. Thus we have to compare

(α) $E_{III} = 0$ with $E_{I(i)}$ under $c \geq c_a + \frac{1}{2}c_s$. As long as $\frac{1}{4}c - \frac{1}{2}c_a \geq 0$, we have $E_{I(i)} > 0$, i.e. $E_{III} < E_{I(i)}$. If $\frac{1}{4}c - \frac{1}{2}c_a < 0$, then $E_{I(i)} \geq \frac{1}{4}c - \frac{1}{2}c_a + \frac{1}{4}c + \frac{1}{2}c_a > 0$. Thus, we again have$E_{III} < E_{I(i)}$.

(β) E_{III} with $E_{I(ii)}$ under $c \leq c_a + \frac{1}{2}c_s$. Since we obviously have $E_{I(ii)} > 0$, we immediately get $E_{III} < E_{I(ii)}$.

(IV) Finally, we show that the best the CSD can do under the restrictions that $q_A + p_A = W$ would make the CSD worse off compared to the maximum derived under (I). If $q_A + p_A = W$, then $E[\pi_C] = q_C + p_C - c_a - c_s$.

(α) $c \geq c_a + \frac{1}{2}c_s$. According to proposition 2, we have $q_A + p_A = W$ only if $2q_C + p_C < 2(c_a + c_s) - c$. Thus, the best the CSD can do is to maximise $q_C + p_C - c_a - c_s$ subject to $q_C \geq 0$, $p_C \geq 0$, $2q_C + p_C < 2(c_a + c_s) - c$, i.e. to choose $q_C = 0$, $p_C = 2(c_a + c_s) - c$ with $E[\pi_C] = c_a + c_s - c \equiv E_{IV(\alpha)}$. (a) If

$\frac{1}{4}c - \frac{1}{2}c_a > 0$, we have $E_{I(i)} > \frac{1}{2}[\frac{1}{4}c - \frac{1}{2}c_a] + \frac{1}{4}c + \frac{1}{2}c_a$. With $c \geq c_a + \frac{1}{2}c_s$ we easily get $\frac{1}{2}[\frac{1}{4}c - \frac{1}{2}c_a] + \frac{1}{4}c + \frac{1}{2}c_a < E_{IV(\alpha)} \Rightarrow c < c_s$ which is in contradiction to our assumption $c \geq c_s$. (b) If $\frac{1}{4}c - \frac{1}{2}c_a = 0$, we have $E_{I(i)} = \frac{1}{2}c$ and $E_{IV(\alpha)} = c_s - \frac{1}{2}c$ so that $E_{I(i)} > E_{IV(\alpha)}$ as long as $c > c_s$. For $c = c_s$, the cases (I) and (IV)(α) are equivalent and do not need to be distinguished. (c) If $\frac{1}{4}c - \frac{1}{2}c_a < 0$, we have $E_{I(i)} > \frac{1}{2}c$ and $E_{IV(\alpha)} \leq \frac{1}{2}c$ so that again $E_{I(i)} > E_{IV(\alpha)}$.

(β) $c \leq c_a + \frac{1}{2}c_s$. According to proposition 2, we have $q_A + p_A = V$ only in two cases: (a) $2q_C + p_C < 2(c_a + c_s) - c$ and $2q_C + 3p_C \leq 2(c_a + c_s) + 3c$. This implies $q_C + p_C < c_a + c_s + \frac{1}{2}c$, i.e. $E[\pi_C] < \frac{1}{2}c \leq E_{I(ii)}$. (b) $2q_C + p_C < 2(c_a + c_s) - c$, $2q_C + 3p_C > 2(c_a + c_s) + 3c$ and $q_C + p_C \leq c_a + c_s + \frac{1}{2}c$. Again, this implies $E[\pi_C] < \frac{1}{2}c \leq E_{I(ii)}$.

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Proof of proposition 4:

For given prices $q_A + p_A$, we have to maximise $E[\pi_C]$ as given in equation 3 with respect to q_C and p_C subject to $q_C \geq 0$, $p_C \geq 0$ and $0 \leq k \leq 1$, where k is given by equation 4.

Let $E[\pi_C^L] = k(q_C - c_a + p_C - c_s) + \frac{1}{2}(1-k)(p_C - c_s)$ be the lower branch of $E[\pi_C]$. In a first step, we maximise $E[\pi_C^L]$ subject to $q_C \geq 0$, $p_C \geq 0$ and $\frac{1}{2} \leq k \leq 1$. We get the following maximiser and maximum:

$$p_C \begin{cases} = q_A + p_A & \text{if } q_A + p_A < 2c_a + c_s + \frac{3}{2}c \\ = \frac{3}{4}c + c_a + \frac{1}{2}c_s + \frac{1}{2}(q_A + p_A) & \text{if } 2c_a + c_s + \frac{3}{2}c \leq q_A + p_A \leq 2c_a + c_s + \frac{5}{2}c \\ \in [0, q_A + p_A - \frac{1}{2}c] & \text{if } q_A + p_A > 2c_a + c_s + \frac{5}{2}c \end{cases}$$

$$q_C \begin{cases} = 0 & \text{if } q_A + p_A < 2c_a + c_s + \frac{5}{2}c \\ = q_A + p_A - \frac{1}{2}c - p_C & \text{if } q_A + p_A \geq 2c_a + c_s + \frac{5}{2}c \end{cases}$$

and

$$E[\pi_C^L]^* = \begin{cases} \frac{3}{4}(q_A + p_A) - \frac{1}{2}c_a - \frac{3}{4}c_s \equiv \widetilde{E}_1 & \text{if } q_A + p_A < 2c_a + c_s + \frac{3}{2}c \\ \frac{1}{2}\frac{[\frac{3}{4}c + \frac{1}{2}(q_A + p_A) - c_a - \frac{1}{2}c_s)]^2}{c} + c_a \equiv \widetilde{E}_2 & \text{if } 2c_a + c_s + \frac{3}{2}c \leq q_A + p_A \leq 2c_a + c_s + \frac{5}{2}c \\ q_A + p_A - \frac{1}{2}c - c_a - c_s \equiv \widetilde{E}_3 & \text{if } q_A + p_A > 2c_a + c_s + \frac{5}{2}c \end{cases}$$

Let $E[\pi_C^U] = k(q_C - c_a + p_C - c_s) + \frac{1}{2}k(p_C - c_s)$ be the upper branch of $E[\pi_C]$. We maximise $E[\pi_C^U]$ subject to $q_C \geq 0$, $p_C \geq 0$ and $0 \leq k \leq \frac{1}{2}$. We get the following maximiser and maximum:

$$p_C \begin{cases} = q_A + p_A & \text{if } q_A + p_A > \frac{2}{3}c_a + c_s + \frac{1}{2}c \\ = \frac{1}{4}c + \frac{1}{3}c_a + \frac{1}{2}c_s + \frac{1}{2}(q_A + p_A) & \text{if } \frac{2}{3}c_a + c_s - \frac{1}{2}c \leq q_A + p_A \leq \frac{2}{3}c_a + c_s + \frac{1}{2}c \\ \in [0, q_A + p_A + \frac{1}{2}c] & \text{if } q_A + p_A < \frac{2}{3}c_a + c_s - \frac{1}{2}c \end{cases}$$

$$q_C \begin{cases} = 0 & \text{if } q_A + p_A \geq \frac{2}{3}c_a + c_s - \frac{1}{2}c \\ = q_A + p_A + \frac{1}{2}c - p_C & \text{if } q_A + p_A \leq \frac{2}{3}c_a + c_s - \frac{1}{2}c \end{cases}$$

and

$$E[\pi_C^U]^* = \begin{cases} \frac{3}{4}(q_A + p_A) - \frac{1}{2}c_a - \frac{3}{4}c_s \equiv \widetilde{E}_4 & \text{if } q_A + p_A > \frac{2}{3}c_a + c_s + \frac{1}{2}c \\ \frac{3}{2}\frac{[\frac{1}{4}c + \frac{1}{2}(q_A + p_A) - \frac{1}{3}c_a - \frac{1}{2}c_s)]^2}{c} \equiv \widetilde{E}_5 & \text{if } \frac{2}{3}c_a + c_s - \frac{1}{2}c \leq q_A + p_A \leq \frac{2}{3}c_a + c_s + \frac{1}{2}c \\ 0 \equiv \widetilde{E}_6 & \text{if } q_A + p_A < \frac{2}{3}c_a + c_s - \frac{1}{2}c \end{cases}$$

We now have to consider the following five cases:

A) $q_A + p_A > 2c_a + c_s + \frac{5}{2}c$. We easily find that $\widetilde{E}_3 > \widetilde{E}_4$, i.e. the best response is $p_C \in [0, q_A + p_A - \frac{1}{2}c]$, $q_C = q_A + p_A - \frac{1}{2}c - p_C$.

B) $2c_a + c_s + \frac{3}{2}c \leq q_A + p_A \leq 2c_a + c_s + \frac{5}{2}c$. Here we have $\widetilde{E}_2 > \widetilde{E}_4$, i.e. $p_C = \frac{3}{4}c + c_a + \frac{1}{2}c_s + \frac{1}{2}(q_A + p_A)$, $q_C = 0$.

C) $\frac{2}{3}c_a + c_s + \frac{1}{2}c \leq q_A + p_A \leq 2c_a + c_s + \frac{3}{2}c$. Here we have $\widetilde{E}_1 = \widetilde{E}_4$, i.e. $p_C = q_A + p_A$, $q_C = 0$.

D) $\frac{2}{3}c_a + c_s - \frac{1}{2}c \leq q_A + p_A \leq \frac{2}{3}c_a + c_s + \frac{1}{2}c$. Here we have $\widetilde{E}_1 < \widetilde{E}_5$, i.e. $p_C = \frac{1}{4}c + \frac{1}{3}c_a + \frac{1}{2}c_s + \frac{1}{2}(q_A + p_A)$, $q_C = 0$.

E) $q_A + p_A \leq \frac{2}{3}c_a + c_s - \frac{1}{2}c$. Here we have $\widetilde{E}_1 < \widetilde{E}_6$, i.e. $p_C \in [0, q_A + p_A + \frac{1}{2}c]$, $q_C = q_A + p_A + \frac{1}{2}c - p_C$.

This finalises the proof.

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Proof of proposition 5:

To prove this proposition, one would need to consider 20 cases. Since the proof is simple, but tedious, we discuss only the first two cases:

(1) $q_A + p_A = \frac{1}{4}c + \frac{1}{2}(c_a + c_s) + \frac{1}{2}q_C + \frac{1}{4}p_C$ and $q_C + p_C = q_A + p_A - \frac{1}{2}c$. This implies $q_C + p_C + \frac{1}{2}c = \frac{1}{4}c + \frac{1}{2}(c_a + c_s) + \frac{1}{2}q_C + \frac{1}{4}p_C$, i.e. $2q_C + 3p_C = 2(c_a + c_s) - c$. Since $q_A + p_A = \frac{1}{4}c + \frac{1}{2}(c_a + c_s) + \frac{1}{2}q_C + \frac{1}{4}p_C$ requires $2q_C + p_C \geq 2(c_a + c_s) - c$, this is only possible if $p_C = 0$, i.e. $q_C = c_a + c_s - \frac{1}{2}c$. Since $q_A + p_A = \frac{1}{4}c + \frac{1}{2}(c_a + c_s) + \frac{1}{2}q_C + \frac{1}{4}p_C$ also requires $q_C + p_C \geq \frac{1}{2}c + c_a + c_s$, this is not possible.

(2) $q_A + p_A = \frac{1}{4}c + \frac{1}{2}(c_a + c_s) + \frac{1}{2}q_C + \frac{1}{4}p_C$, $q_C = 0$ and $p_C = \frac{3}{4}c + \frac{1}{2}(q_A + p_A) + c_a + \frac{1}{2}c_s$. This implies $q_A + p_A = \frac{1}{2}c + \frac{6}{7}c_a + \frac{5}{7}c_s$. This is not possible since $q_C = 0$ and $p_C = \frac{3}{4}c + \frac{1}{2}(q_A + p_A) + c_a + \frac{1}{2}c_s$ requires $q_A + p_A \geq 2c_a + c_s + \frac{3}{2}c$.

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