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BANK LEVERAGE CYCLES

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**MACROPRUDENTIAL
RESEARCH NETWORK**

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This paper presents research conducted within the Macroprudential Research Network (MaRs). The network is composed of economists from the European System of Central Banks (ESCB), i.e. the 27 national central banks of the European Union (EU) and the European Central Bank. The objective of MaRs is to develop core conceptual frameworks, models and/or tools supporting macro-prudential supervision in the EU.

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Abstract

We document the cyclical dynamics in the balance sheets of US leveraged financial intermediaries in the post-war period. Leverage has contributed more than equity to fluctuations in total assets. All three variables are several times more volatile than GDP. Leverage has been positively correlated with assets and (to a lesser extent) GDP, and negatively correlated with equity. These findings are robust across financial subsectors. We then build a general equilibrium model with banks subject to endogenous leverage constraints, and assess its ability to replicate the facts. In the model, banks borrow in the form of collateralized risky debt. The presence of moral hazard creates a link between the volatility in bank asset returns and bank leverage. We find that, while standard TFP shocks fail to replicate the volatility and cyclical of leverage, volatility shocks are relatively successful in doing so.

Keywords: financial intermediaries, short-term collateralized debt, limited liability, call option, put option, moral hazard, leverage, cross-sectional volatility.

JEL codes: E20, G10, G21

Non-technical Summary

The 2007-09 financial crisis witnessed a severe disruption of financial intermediation in many industrialized economies. This has led to a surge in both empirical and theoretical research aimed at understanding the causes and consequences of the financial crisis.

A particularly influential strand of the literature has focused on the role played by the 'shadow banking' sector, which comprises all those financial intermediaries (investment banks, off-balance investment vehicles, etc.) that are not subject to regulatory capital requirements. Many of these financial intermediaries funded their assets primarily by means of collateralized debt with very short maturity. Thus, the initial losses suffered by some of the assets that served as collateral, together with the uncertainty surrounding individual exposures to such assets, led the holders of that short-term debt (mostly institutional investors) to largely stop rolling over their lending. This funding freeze forced the shadow financial intermediaries to deleverage, with the resulting contraction in financing flows to the real economy.

In fact, the observed deleveraging of shadow intermediaries during the 2007-09 financial crisis is not an isolated episode. In the postwar period, the leverage ratio of some financial intermediaries in the US has exhibited a markedly procyclical pattern, in the sense that expansions (contractions) in balance sheet size have gone hand in hand with increases (decreases) in leverage. This points to the importance of endogenous leverage fluctuations for the cyclical behavior of intermediated credit.

The aim of our paper is both empirical and theoretical. On the empirical front, we perform a systematic analysis of the cyclical fluctuations in US financial intermediaries' balance sheets. Our empirical findings can be summarized as follows. First, leverage fluctuations contribute at least as much as equity fluctuations to the cyclical movements in total assets. Second, leverage, equity capital and total assets have roughly the same volatility, and are themselves several times more volatile than GDP. Third, leverage is positively correlated with total assets and (to a lesser extent) GDP, and negatively correlated with equity capital.

On the theoretical front, we construct a general equilibrium model of financial intermediation and endogenous leverage, and assess its ability to match the evidence. The model incorporates a financial intermediation sector consisting of banks that borrow from institutional investors in the form of short-term collateralized risky debt. The source of risk in banks' debt is the following. Banks invest in the firm sector. Banks and firms are segmented across islands, and firms are hit by island-specific shocks. Therefore, banks are exposed to island-specific risk, such that a fraction of them declare bankruptcy and default on their debt in each period.

Banks' leverage is endogenously determined by market forces. In particular, we assume the existence of a moral hazard problem on the part of banks. Due to limited liability, the payoff structure of a bank resembles that of a call option on island-specific risk. That is, banks enjoy the upside risk in their assets over and above the face value of their debt, leaving institutional investors to bear the downside risk. This provides banks with an incentive to engage in inefficiently risky lending practices. Such an incentive increases with the assumed debt commitment relative to the size of the bank's balance sheet. In order to induce each bank to invest efficiently, institutional investors restrict their lending to a certain ratio of the bank's net worth, i.e. they impose a leverage constraint.

We then calibrate our model to the US economy and analyze its dynamic properties. We consider two exogenous driving forces: total factor productivity (TFP), and time-varying volatility of island-specific shocks. Our results show that TFP shocks by themselves are unable to replicate the volatility of leverage in the data. They also fail to replicate the procyclicality of leverage with respect to GDP. On the contrary, shocks to cross-sectional volatility are able to produce fluctuations in leverage of a realistic size, and a positive comovement with GDP. The mechanism is as follows. Consider e.g. an increase in island-specific volatility. Higher uncertainty regarding asset returns makes it more attractive for banks to engage in inefficiently risky lending practices. In order to prevent them from doing so, institutional investors impose a tighter constraint on banks' leverage. This forces banks to contract their balance sheets, thus reducing intermediated credit. This leads

to a fall in capital investment by firms, and in aggregate output.

Finally, we study how the average level of cross-sectional volatility affects the volatility of aggregate economic activity in our model. We find that lower cross-sectional uncertainty raises the volatility of GDP. A reduction in cross-sectional volatility allows banks to increase their leverage on average. This generates larger fluctuations in total intermediated assets and hence in aggregate output. This result is reminiscent of Minsky's 'financial instability hypothesis,' according to which a lower perception of uncertainty leads to riskier investment practices, thus creating the conditions for the emergence of a financial crisis. In our model, lower perceived risk leads financial intermediaries to raise their leverage ratios, thus making the economy more vulnerable to the effects of negative aggregate shocks.

1 Introduction

The 2007-2009 financial crisis witnessed a severe disruption of financial intermediation in many industrialized economies. This has led to a surge in both empirical and theoretical research aimed at understanding the causes and consequences of the financial crisis, evaluating the policy measures put in place to tackle its effects, and proposing further policy actions and new regulatory frameworks.

A particularly influential strand of the literature has focused on the role played by the 'shadow banking' sector in the origin and propagation of the financial turmoil. The latter sector comprises all those financial intermediaries (investment banks, hedge funds, finance companies, off-balance-sheet investment vehicles, etc.) that have no access to central bank liquidity or public sector credit guarantees, and that are not subject to regulatory capital requirements.¹ Many of these financial intermediaries funded their asset purchases primarily by means of collateralized debt with very short maturity, such as sale and repurchase (*repo*) agreements or asset backed commercial paper (ABCP). As argued by Brunnermeier (2009), Gorton and Metrick (2010, 2011), Krishnamurthy, Nagel and Orlov (2012) and others, the initial losses suffered by some of the assets that served as collateral in repo or ABCP transactions, together with the uncertainty surrounding individual exposures to such assets, led the holders of that short-term debt (mostly institutional investors, such as money market funds) to largely stop rolling over their lending. This funding freeze forced the shadow financial intermediaries to deleverage, with the resulting contraction in their balance sheets and ultimately in the credit flow to the real economy.

In fact, the observed deleveraging of shadow intermediaries during the 2007-2009 financial crisis is not an isolated episode. As documented by Adrian and Shin (2010, 2011b), since the 1960s the leverage ratio of important segments of the financial intermediation sector has exhibited a markedly procyclical pattern, in the sense that expansions (contractions) in balance sheet size have gone hand in hand with increases (decreases) in leverage. This procyclicality has been particularly strong in the case of security brokers and dealers, a category that used to include investment banks. Overall, these findings point to the importance of endogenous leverage fluctuations for the cyclical behavior of financial intermediation.

The aim of our paper is both empirical and theoretical. On the empirical front, we perform a systematic analysis of the cyclical fluctuations in the balance sheets of US financial intermediaries, with a special attention to the leverage ratio (i.e. the ratio between total assets and equity capital). Our analysis comprises the main subsectors in what Greenlaw et al. (2008) have termed the 'leveraged sector', including depository intermediaries such as US-chartered commercial banks and savings institutions, as well as non-depository intermediaries such as security broker/dealers and finance companies. Our empirical findings can be summarized as follows. First, leverage contributes at least as much as equity to the cyclical movements in total assets. Second, leverage, equity capital and total assets are several times more volatile than GDP. Third, leverage is negatively correlated with equity. Finally, leverage is positively correlated with total assets and (to a lesser extent) GDP.² Importantly, these facts are robust across subsectors, regardless of whether they represent depository or non-depository institutions. In addition, these findings are fairly robust to the filtering process, the type of assets considered (total or financial) and the sample period.

On the theoretical front, we construct a general equilibrium model of financial intermediation and endogenous leverage, and assess its ability to match the evidence discussed above. The model incorporates a financial intermediation sector consisting of banks that borrow from institutional investors in the form of short-term collateralized risky debt. The source of risk in banks' debt

¹See Pozsar et al. (2012) for an in-depth analysis of 'shadow banking' in the United States.

²In the case of security broker/dealers, the procyclicality of leverage with respect to assets confirms the original findings by Adrian and Shin (2010). Like them, we use Flow of Funds data. Our analysis of the data is somewhat different though. Whereas Adrian and Shin focus on the growth rates of leverage and assets, we focus on their cyclical components as implied by a standard bandpass filter. We also consider real rather than nominal assets, given our interest in their comovements with real GDP and for consistency with our theoretical model. Using data from the FDIC Call Reports, Adrian and Shin (2011b) obtain a positive comovement between assets and leverage also for commercial banks.

is the following. Banks invest in the nonfinancial corporate (firm) sector. Banks and firms are segmented across islands, and firms are hit by island-specific shocks. Therefore, banks are exposed to island-specific risk, such that a fraction of them declare bankruptcy and default on their debt in each period.

Banks' leverage is endogenously determined by market forces. In particular, we assume the existence of a moral hazard problem based on the one developed by Adrian and Shin (2011a) in a static, partial equilibrium context.³ Due to limited liability, the payoff structure of a bank resembles that of a call option on island-specific risk.⁴ That is, banks enjoy the upside risk in their assets over and above the face value of their debt, leaving institutional investors to bear the downside risk. This provides banks with an incentive to engage in inefficiently risky lending practices. Such an incentive increases with the assumed debt commitment relative to the size of the bank's balance sheet. In order to induce each bank to invest efficiently, institutional investors restrict their lending to a certain ratio of the bank's net worth, i.e. they impose a leverage constraint.

We then calibrate our model to the US economy and analyze its dynamic properties. In particular, we study the model economy's response to two exogenous driving forces: total factor productivity (TFP), and time-varying volatility of island-specific shocks. While TFP shocks are fairly standard in the real business cycle literature, changes in cross-sectional volatility have received considerable attention recently as a source of aggregate fluctuations.⁵

Our results show that TFP shocks by themselves are unable to replicate the volatility of leverage in the data, as well as its procyclicality with respect to GDP. Intuitively, TFP shocks barely affect banks' risk-taking incentives. On the contrary, shocks to cross-sectional volatility are able to produce fluctuations in leverage of a realistic size, as well as a positive comovement between leverage and GDP. The mechanism is as follows. Consider e.g. an increase in island-specific volatility. Higher uncertainty regarding asset returns, coupled with limited liability, makes it more attractive for banks to engage in inefficiently risky lending practices. In order to prevent them from doing so, institutional investors impose a tighter constraint on banks' leverage. For given net worth, this deleveraging forces banks to contract their balance sheets, thus producing a fall in intermediated credit. This leads to a fall in capital investment by firms, and in aggregate output. The consequence is a positive comovement between leverage, assets and GDP. In fact, volatility shocks alone generate a procyclicality of leverage above the empirical one. Combining the latter shocks with TFP shocks improves the model's performance, by reducing the correlation with GDP to a level that is comparable with that in the data.

Finally, we use our model as a laboratory for studying how the steady-state level of cross-sectional volatility affects both the mean level and the volatility of economic activity. We find that lower cross-sectional volatility raises the mean level of banks' leverage, through a channel very similar to the one described above. This produces an increase in the mean levels of intermediated assets, and hence in the mean levels of capital investment and GDP. Perhaps more surprisingly, lower cross-sectional uncertainty *raises* the volatility of GDP. A reduction in cross-sectional volatility allows banks to increase their leverage, which generates larger fluctuations in total intermediated assets and hence in aggregate output. This result, which may be thought of as a 'risk diversification paradox', is reminiscent of Minsky's (1992) 'financial instability hypothesis,' according to which a lower perception of uncertainty leads to riskier investment practices, thus creating the conditions for the emergence of a financial crisis. In our model, lower perceived risk leads financial intermediaries to raise their leverage ratios, thus making the economy more vulnerable to the effects of negative aggregate shocks.

Our paper contributes to the literature on the macroeconomic effects of financial frictions. On the one hand, a recent literature has provided theoretical explanations for the 'leverage cycles'

³Adrian and Shin's (2011a) moral hazard problem is in turn inspired by Holmström and Tirole (1997).

⁴For a pioneering analysis of the payoff structure of defaultable debt claims, equity stakes, and their relationship to option derivatives, see Merton (1974).

⁵See e.g. Curdia (2007), Christiano, Motto and Rostagno (2010), Gilchrist, Sim and Zakrajsek (2010), Bloom (2009), Bloom et al. (2011) and Arellano, Bai and Kehoe (2012). Christiano et al. (2010) refer to such disturbances as 'risk shocks', whereas Bloom (2009) labels them 'uncertainty shocks'.

discussed above, with contributions by Adrian and Shin (2011a), Ashcraft, Garleanu and Pedersen (2011), Brunnermeier and Pedersen (2009), Brunnermeier and Sannikov (2011), Dang, Gorton and Holmström (2011), Geanakoplos (2010), and Gorton and Ordoñez (2011), among others.⁶ Most of these models consider some type of link between changes in ‘uncertainty’, typically defined as changes in the volatility of shocks, and the emergence of these leverage cycles. While these models provide important insights on the equilibrium behavior of leverage, they are primarily aimed at illustrating theoretical mechanisms and are thus mainly qualitative. In particular, most of these papers consider two- or three-period economies, or two-period-lived agents (i.e. an OLG structure); they also assume a partial equilibrium structure.⁷ We build on this literature by analyzing endogenous leverage cycles in a fully dynamic, general equilibrium model that can be compared to aggregate data and, more generally, be useful for quantitative analysis.

On the other hand, our paper is related to a growing literature on the role of financial intermediaries in DSGE models. Early contributions, such as Carlstrom and Fuerst (1997), Bernanke, Gertler and Gilchrist (1999) and Kiyotaki and Moore (1997), emphasized the importance of financial frictions for the macroeconomy, but largely obviated the role played by financial intermediaries. In recent years, and largely motivated by the recent financial crisis, a number of papers study how frictions arising in the financial intermediation sector affect credit flows to the real economy. In the model of Christiano, Motto and Rostagno (2010), banks incur a cost when creating liabilities that provide liquidity services, such as deposits. In Gertler and Karadi (2011) and Gertler and Kiyotaki (2010), banks are leverage constrained due to a moral hazard problem different from the one used here.⁸ The authors focus their discussion on how changes in bank equity capital affect bank credit supply for given leverage, and how unconventional policy interventions can mitigate the effects of adverse shocks on financial intermediation. He and Krishnamurthy (2012) propose a framework with financial intermediaries subject to occasionally binding capital constraints, and use it to explain the nonlinear behavior of risk premia during crises and to evaluate different policy interventions. Boissay, Collard and Smets (2012) consider a model of heterogeneous financial intermediaries where moral hazard and asymmetric information may generate sudden interbank market freezes.

The paper proceeds as follows. Section 2 presents empirical evidence on the cyclical behavior of GDP, assets and leverage of financial intermediaries in the US. Section 3 lays out the model. Section 4 calibrates and simulates the model, assessing its ability of replicate the data. Section 5 concludes.

2 Bank leverage cycles in the US economy

By definition, the size of a financial intermediary’s balance sheet is the product of two components: its equity capital, and its leverage ratio. We thus have the following identity: $A = \phi N$, where A denotes total assets, ϕ represents the leverage ratio, and N is equity capital. In logs, we have

$$\log(A) = \log(\phi) + \log(N). \tag{1}$$

Applying a linear trend-cycle filter on both sides of (1) yields the same identity for the cyclical components of the three variables. Such an identity raises a number of empirical questions. First, it is interesting to ask what is the relative contribution of leverage and equity capital to the cyclical fluctuations in total assets. Second, one may wonder how leverage and equity correlate with

⁶Some of these authors focus on the behavior of ‘margins’ or ‘haircuts’ in short-term collateralized debt contracts, which are closely related to the concept of ‘leverage’.

⁷An exception is Brunnermeier and Sannikov (2011), who present an infinite-horizon, general equilibrium model in continuous time.

⁸In particular, moral hazard arises due to bankers’ ability to divert a fraction of deposits for personal use. Gertler and Kiyotaki (2010) also consider idiosyncratic bank liquidity shocks, which give rise to an interbank market for funding.

each other, and how each component correlates with total assets.⁹ A related question is how the leverage ratio and the assets of financial intermediaries comove with aggregate economic activity, as represented by real GDP. Last but not least, the size of fluctuations in leverage, assets and equity relative to those in real economic activity is itself a matter of empirical interest.

Table 1 displays a number of statistics regarding the cyclical fluctuations in leverage, equity capital, total assets and GDP in the United States, for the period 1963:Q1-2011:Q3.¹⁰ Our leverage, equity and assets series are constructed using data from the US Flow of Funds.¹¹ We consider four leveraged financial subsectors: US-chartered commercial banks, savings institutions, security brokers and dealers and finance companies. US-chartered commercial banks and savings institutions are both depository institutions, whereas security broker/dealers and finance companies are non-depository and belong to what Pozsar et al. (2012) have defined as the 'shadow banking' sector.

Given the information in Table 1, four stylized facts should be underlined:

1. **Leverage is volatile.** A first conclusion to extract is that the leverage ratios of the different subsectors are fairly volatile. In all cases, leverage fluctuates *more* than equity capital, as shown by the standard deviations. Leverage of security broker/dealers and finance companies (both non-depository subsectors) is about 7 and 3 times larger than that of GDP, respectively. The leverage of savings institutions (a depository subsector) displays a high volatility too, due mainly to the savings and loans crisis episode in the 1980s. For commercial banks, the leverage ratio fluctuates comparatively less, although its standard deviation is still about twice that of GDP.
2. **Leverage and equity comove negatively.** A second stylized fact is that leverage and assets tend to comove *negatively* over the business cycle. This can be easily understood by taking into account that $var(\log A) = var(\log \phi) + var(\log N) + 2cov(\log \phi, \log N)$. If the variance of leverage is roughly of the same order of magnitude than that of assets and equity, then necessarily it must be negatively correlated with equity. Thus, equity typically increases in deleveraging periods, and viceversa. This negative correlation is large for the four subsectors, ranging from -0.88 for savings institutions to -0.54 for security broker/dealers.
3. **Leverage and assets comove positively.** A third lesson to draw is that assets and leverage tend to comove *positively* over the business cycle. This pattern is particular strong for security broker/dealers and finance companies. As shown in Table 1, for the latter subsectors both variables have a contemporaneous correlation of 0.65 at business cycle frequencies. This observation confirms the original finding of Adrian and Shin (2010), albeit with a different treatment of the data.¹² As explained by these authors, such a strong comovement reveals an active management of leverage as a means of expanding and contracting the size of balance sheets. For the other subsectors, the correlation coefficients are smaller, but statistically significant in all cases.
4. **Leverage (and assets) are procyclical.** The final lesson to draw is that both leverage and assets tend to comove *positively* with GDP. In particular, the leverage of financial intermediaries displays a mildly procyclical behavior. The correlation of the different leverage ratios with GDP ranges from 0.12 to 0.36, and while they are relatively small, they are all

⁹For a seminal study of cyclical comovements between leverage and total assets of US financial intermediaries, see Adrian and Shin (2010).

¹⁰The cyclical component is obtained by detrending each series with a bandpass filter that preserves cycles of 6 to 32 quarters and with lag length of $K = 12$ quarters (Baxter and King, 1999). Notice that the linear identity in (1) is preserved by the bandpass filter.

¹¹See the Data Appendix for details on the sources and the treatment of the data.

¹²Adrian and Shin (2010) focus on the comovement between the *growth rates* of leverage and *nominal* total assets. Here, we focus on the behavior of *real* total assets, due both to our interest in the comovement of financial variables with real GDP and for consistency with our subsequent theoretical model. Also, we use a standard band-pass filter so as to extract the cyclical component of assets and leverage. Our results show that Adrian and Shin's (2010) findings are robust to this different transformation of the data.

statistically significant (with the exception of savings institutions). The correlation between assets and GDP is higher in all cases, ranging from 0.42 in the case of security traders to 0.71 for savings institutions.

As a graphical illustration, Figure 1 shows the cyclical components of total assets and leverage for the two largest leveraged financial subsectors in the United States: US-chartered commercial banks, and security brokers and dealers.¹³ The recession starting in 2007 witnessed a sharp decline in the leverage ratio of security broker/dealers, and an incipient decline in that of commercial banks. A similar deleveraging process was observed during the mid-70s recession. However, other recessions such as the 1981-82 one have not had any noticeable effect on the leverage of these two subsectors. This explains their relatively low cyclicalities with respect to GDP. Notice also that the strong correlation of commercial banks' assets and leverage at the beginning of the sample has weakened somewhat over time, while such comovement seems to have been more stable for security broker/dealers. Figure 2 displays the cyclical comovement between leverage and equity capital, again for commercial banks and security broker/dealers. The negative correlation between both variables is evident in the case of commercial banks, and less clear for broker/dealers. For instance, the recent recession featured an increase in equity of commercial banks, unlike that of broker/dealers. For the latter subsector, the negative correlation in Table 2 is the product of earlier historical episodes, such as the two recessions in the 1970s.

Table 1 reveals a fair degree of heterogeneity among financial subsectors. To illustrate this, the last column of Table 1 reports the range of values for each moment. In light of this heterogeneity, it would be interesting to consolidate the balance sheets of the different subsectors so as to study the cyclical properties of the leveraged financial system as a whole. Unfortunately, the Flow of Funds data does not allow this possibility, because asset and liability positions *between* the different subsectors are not netted out. As a result, simply adding assets and equity would lead to a double-counting of such cross positions. Nevertheless, it is important to emphasize that the stylized facts discussed above are robust across financial subsectors.

The above empirical findings are also robust in other dimensions. First, we have repeated the analysis using a Hodrick–Prescott filter instead of a bandpass one. Second, we have replaced ‘total assets’ by ‘total financial assets’, which are also available in the Flow of Funds. In both cases quantitative results change very little.¹⁴ Finally, we have restricted the sample period by starting it in 1984, instead of in 1963. Our motivation for doing so is the fact the US financial system has experienced substantial structural transformations during the postwar period, which raises the question as to how robust the business cycle statistics in Table 1 are to considering different subsamples. Results are shown in Table 2. In this case, the stylized facts hold qualitatively. The only exception is that the correlations of commercial banks' assets and leverage with GDP are no longer statistically significant.

Notice finally that the equity capital series from the Flow of Funds are of book equity, i.e. the difference between the value of intermediaries' portfolio of claims and their liabilities. An alternative measure of equity is the market capitalization, i.e. the market value of intermediaries' traded shares. As argued by Adrian, Colla and Shin (2012), book equity is the appropriate notion of equity if one is interested in the supply of bank credit, as we are here, whereas market capitalization would have been more appropriate if one were interested in new share issuance or mergers and acquisitions decisions.

To summarize, our empirical analysis reveals four main findings regarding the US leveraged financial sector. First, the leverage ratio of the different subsectors display large fluctuations, contributing more than equity capital to cyclical movements in total assets. Second, leverage comoves negatively with equity. Third, leverage comoves positively with total assets. Finally, leverage is mildly procyclical with respect to GDP. In what follows, we present a general equilibrium model aimed at explaining the fluctuations in the leverage ratio of US financial intermediaries.

¹³The sample period used for the figures runs through 2011:Q4. However, the lag length of the bandpass filter ($K = 12$ quarters) implies that the last filtered observation corresponds to 2008:Q4.

¹⁴Results are available upon request.

3 Model

The model economy is composed of five types of agents: households, final good producers (‘firms’ for short), capital producers, institutional investors, and banks.¹⁵ On the financial side, the model structure is as follows. Households lend to institutional investors in the form of deposits and equity. Institutional investors use the latter funds to lend to banks in the form of short-term, collateralized debt. Banks combine their external funding (short-term debt) and their own accumulated net worth to invest in firms. We assume no frictions in the relationship between banks and firms, such that the Modigliani-Miller theorem applies to firm financing. For simplicity, following Gertler and Karadi (2011) and Gertler and Kiyotaki (2010) we assume that firms issue perfectly state-contingent debt only, which can be interpreted as equity. Banks and firms are segmented across islands, where the latter are subject to idiosyncratic shocks. Banks are thus exposed to island-specific risk, such that a fraction of them declare bankruptcy and default on their debt each period. Banks’ debt is not guaranteed, and is therefore risky. Institutional investors operate economy-wide and diversify perfectly across islands; in fact, their only role in our model is to insulate households from island-specific risk, which allows us to make use of the representative household construct.

The real side of the model is fairly standard. At the end of each period, after production has taken place, firms use borrowed funds to purchase physical capital from capital producers. At the beginning of the following period, firms combine their stock of capital and households’ supply of labor to produce a final good. The latter is purchased by households for consumption purposes, and by capital producers. After production, firms sell their depreciated capital stock to capital producers, who use the latter and the final goods to produce new capital. The markets for labor, physical capital and the final good are all nation-wide.

We now analyze the behavior of each type of agent. All variables are expressed in real terms, with the final good acting as the numeraire.

3.1 Households

The representative household’s utility is

$$E_0 \sum_{t=0}^{\infty} \beta^t [u(C_t) - v(L_t)],$$

where C_t is consumption and L_t is labor supply. The budget constraint is

$$C_t + N_t^{II} + D_t = W_t L_t + R_t^N N_{t-1}^{II} + R_{t-1}^D D_{t-1} + \Pi_t^b,$$

where D_t and N_t^{II} are deposits and equity holdings at institutional investors, R_{t-1}^D is the riskless gross deposit rate, R_t^N is the gross return on institutional investor equity, W_t is the wage, and Π_t^b are lump-sum net dividend payments from the household’s ownership of banks. As we will see later on, Π_t^b incorporates any equity injections by households into banks. The first order conditions are

$$\begin{aligned} 1 &= E_t [\Lambda_{t,t+1} R_t^D], \\ 1 &= E_t [\Lambda_{t,t+1} R_{t+1}^N], \\ W_t &= \frac{v'(L_t)}{u'(C_t)}, \end{aligned}$$

where

$$\Lambda_{t,t+1} = \beta \frac{u'(C_{t+1})}{u'(C_t)}$$

¹⁵In an earlier version of this paper (Nuño and Thomas, 2012), we consider a version of our model with two banking sectors that operate in parallel: an ‘unregulated’ sector that finances itself with equity and collateralized debt (which corresponds to the banking sector here), and a ‘regulated’ sector that finances itself with equity and deposits, whereby the latter are guaranteed by the government and banks are subject to minimum capital requirements. In that model version, endogenous fluctuations in leverage take place in the unregulated sector but not in the regulated one.

is the stochastic discount factor.

3.2 Firms

The final good is produced by perfectly competitive firms. As in Kiyotaki and Moore (2008) and Gertler and Kiyotaki (2010), we assume that firms are segmented across a continuum of 'islands', indexed by $j \in [0, 1]$. These islands may be interpreted as regions, or alternatively as sectors. The representative firm in island j starts period t with a stock K_t^j of physical capital, purchased at the end of period $t - 1$. The firm then receives an island-specific shock ω_t^j that changes the amount of effective capital to $\omega_t^j K_t^j$. The shock ω_t^j is iid over time and across islands. Let $F(\omega; \sigma_{t-1}) \equiv F_{t-1}(\omega)$ denote the cumulative distribution function of island-specific shocks at time t , where σ_{t-1} denotes the standard deviation of $\log \omega_t^j$. The latter standard deviation follows an exogenous process. Notice that the standard deviation of island-specific shocks in a given period is known one period in advance. We also assume that ω^j has a unit mean, $E[\omega^j] = 1$.

Effective capital is combined with labor to produce units of final good, Y_t^j , according to a Cobb-Douglas technology,

$$Y_t^j = Z_t (\omega^j K_t^j)^\alpha (L_t^j)^{1-\alpha}, \quad (2)$$

where Z_t is an exogenous aggregate total factor productivity (TFP) process. The firm maximizes operating profits, $Y_t^j - W_t L_t^j$, subject to (2). The first order condition is

$$W_t = (1 - \alpha) Z_t \left(\frac{\omega^j K_t^j}{L_t^j} \right)^\alpha. \quad (3)$$

Therefore, the effective capital-labor ratio is equalized across islands: $\omega^j K_t^j / L_t^j = [W_t / (1 - \alpha) Z_t]^{1/\alpha}$ for all j . The firm's profits are given by

$$Y_t^j - W_t L_t^j = \alpha Z_t (\omega^j K_t^j)^\alpha (L_t^j)^{1-\alpha} = R_t^k \omega^j K_t^j,$$

where

$$R_t^k \equiv \alpha Z_t \left[\frac{(1 - \alpha) Z_t}{W_t} \right]^{(1-\alpha)/\alpha}$$

is the return on effective capital, which is equalized too across islands. After production, the firm sells the depreciated effective capital $(1 - \delta) \omega^j K_t^j$ to capital producers at price one. The total cash flow from the firm's investment project, equal to the sum of operating profits and proceeds from the sale of depreciated capital, is given by

$$R_t^k \omega^j K_t^j + (1 - \delta) \omega^j K_t^j = [R_t^k + (1 - \delta)] \omega^j K_t^j. \quad (4)$$

The capital purchase in the previous period was financed entirely by state-contingent debt. In particular, the cash flow in (4) is paid off entirely to the lending banks.

At the end of period t , the firm buys K_{t+1}^j units of new capital at price one for production in $t + 1$. In order to finance this purchase, the firm issues a number of claims on next period's cash flow equal to the number of capital units acquired, K_{t+1}^j . Following Gertler and Kiyotaki (2010), we assume that the firm can only borrow from banks located on the same island. In particular, the firm sells A_t^j claims to banks on island j . The firm's balance sheet constraint is thus simply

$$K_{t+1}^j = A_t^j.$$

3.3 Capital producers

There is a representative, perfectly competitive capital producer. At the beginning of each period, after production of final goods has taken place, the capital producer purchases the stock of depreciated capital $(1 - \delta) K_t$ from firms at price one. Used capital can be transformed into new capital

on a one-to-one basis at no cost. Capital producers also purchase final goods in the amount I_t , which are used to produce new capital goods on a one-to-one basis. At the end of the period, the new capital is sold to the firms at price one. In equilibrium, capital producers make zero profits.

3.4 Banks

In each island j there exists a representative bank. After production in period t , island j 's firm pays the bank its share of the cash flow from the investment project, $[R_t^k + (1 - \delta)] \omega^j A_{t-1}^j$. Therefore, the gross rate of return on the bank's assets is

$$\frac{[R_t^k + (1 - \delta)] \omega^j A_{t-1}^j}{A_{t-1}^j} = [R_t^k + (1 - \delta)] \omega^j \equiv R_t^A \omega^j.$$

Regarding the liabilities side of its balance sheet, the bank borrows from institutional investors by means of one-period collateralized risky debt contracts. The collateralized risky debt contracts may be thought of as sale and repurchase (*repo*) agreements. Under the risky debt contract, at the end of period $t - 1$ the bank sells its financial claims A_{t-1}^j (which serve as collateral) to the institutional investor at price B_{t-1}^j , and agrees to repurchase them at the beginning of time t at a non-state-contingent price \bar{B}_{t-1}^j . At the beginning of period t , the proceeds from the bank's assets, $R_t^A \omega^j A_{t-1}^j$, exceed the face value of its debt, \bar{B}_{t-1}^j , if and only if ω^j exceeds a threshold level $\bar{\omega}_t^j$ given by

$$\bar{\omega}_t^j \equiv \frac{\bar{B}_{t-1}^j}{R_t^A A_{t-1}^j}, \quad (5)$$

that is, the face value of debt normalized by the bank's assets times their aggregate return. If $\omega^j \geq \bar{\omega}_t^j$ the bank honors its debt, that is, it repurchases its assets at the pre-agreed price \bar{B}_{t-1}^j . If $\omega^j < \bar{\omega}_t^j$, the bank defaults and closes down, whereas the institutional investor simply keeps the collateral and cashes the resulting proceeds, $R_t^A \omega^j A_{t-1}^j$. Notice that the threshold $\bar{\omega}_t^j$ depends on R_t^A and is thus contingent on the aggregate state.

For non-defaulting banks, following Gertler and Karadi (2011) and Gertler and Kiyotaki (2010) we assume that a random fraction $1 - \theta$ of them close down for exogenous reasons each period, at which point the net worth accumulated in each bank is reverted to the household.¹⁶ The remaining fraction θ of banks continue operating. For the latter, the flow of dividends distributed to the household is given by

$$\Pi_t^j = R_t^A \omega^j A_{t-1}^j - \bar{B}_{t-1}^j - N_t^j, \quad (6)$$

where N_t^j is net worth after dividends have been paid. As in Gertler and Karadi (2011) and Gertler and Kiyotaki (2010), we assume that households inject equity in new banks, but cannot inject equity in continuing banks. Therefore, continuing banks are subject to a non-negativity constraint on dividends, $\Pi_t^j \geq 0$, or equivalently,

$$N_t^j \leq R_t^A \omega^j A_{t-1}^j - \bar{B}_{t-1}^j. \quad (7)$$

Once the bank has decided how much net worth to hold, it purchases claims on firm profits, A_t^j , subject to its balance sheet constraint,

$$A_t^j = N_t^j + B_t^j.$$

When borrowing from the institutional investor, the bank faces two constraints. First, a *participation constraint* requires that the institutional investor is willing to fund the bank. Indeed,

¹⁶ As we show below, in equilibrium banks have no incentive to pay dividends. The assumption of an exogenous exit probability for non-defaulting banks should thus be viewed as a short-cut for motivating dividend payments by such banks, which would otherwise accumulate net worth indefinitely.

the institutional investor may alternatively lend at the riskless deposit rate R_t^D . The latter investment has a present discounted value of $E_t \Lambda_{t,t+1} R_t^D B_t^j = B_t^j = A_t^j - N_t^j$, where we have used the household's Euler equation and the bank's balance sheet constraint. Therefore, the participation constraint takes the form

$$E_t \Lambda_{t,t+1} \left\{ R_{t+1}^A A_t^j \int^{\bar{\omega}_{t+1}^j} \omega dF_t(\omega) + \bar{B}_t^j \left[1 - F_t(\bar{\omega}_{t+1}^j) \right] \right\} \geq A_t^j - N_t^j. \quad (8)$$

Second, in the spirit of Adrian and Shin (2011a) we assume that once the bank has received the funding it may choose to invest in either of two firm segments within its island: a 'standard' segment, and a 'substandard' segment. Both segments differ only in the distribution of island-specific returns, given by $F_t(\omega)$ and $\tilde{F}_t(\omega) \equiv \tilde{F}(\omega; \sigma_t)$ respectively. The substandard technology has lower average payoff, $\int \omega dF_t(\omega) < \int \omega d\tilde{F}_t(\omega) = 1$, and is thus inefficient. Furthermore, $F_t(\omega)$ is assumed to first-order stochastically dominate $\tilde{F}_t(\omega)$: $\tilde{F}_t(\omega) > F_t(\omega)$ for all $\omega > 0$. Therefore, the substandard technology has higher *downside* risk. In order to induce the bank to invest in the standard segment, the institutional investor imposes an *incentive compatibility* (IC) constraint. Let $V_{t+1}(\omega, A_t^j, \bar{B}_t^j)$ denote the value function at time $t+1$ of a continuing bank, to be defined below. Then the IC constraint takes the following form,

$$\begin{aligned} & E_t \Lambda_{t,t+1} \int_{\bar{\omega}_{t+1}^j} \left\{ \theta V_{t+1}(\omega, A_t^j, \bar{B}_t^j) + (1-\theta) \left[R_{t+1}^k A_t^j \omega - \bar{B}_t^j \right] \right\} dF_t(\omega) \\ & \geq E_t \Lambda_{t,t+1} \int_{\bar{\omega}_{t+1}^j} \left\{ \theta V_{t+1}(\omega, A_t^j, \bar{B}_t^j) + (1-\theta) \left[R_{t+1}^k A_t^j \omega - \bar{B}_t^j \right] \right\} d\tilde{F}_t(\omega). \end{aligned} \quad (9)$$

To understand the bank's incentives to finance one firm segment or another, notice that its expected net payoff, conditional on a particular aggregate state at time $t+1$, can be expressed as

$$\int_{\bar{\omega}_{t+1}^j} \left(R_{t+1}^A A_t^j \omega - \bar{B}_t^j \right) dF_t(\omega) = R_{t+1}^A A_t^j \int_{\bar{\omega}_{t+1}^j} \left(\omega - \bar{\omega}_{t+1}^j \right) dF_t(\omega).$$

The integral represents the value of a *call option* on island-specific returns with strike price equal to the default threshold, $\bar{\omega}_{t+1}^j$, or equivalently to the (normalized) face value of debt, $\bar{B}_t^j / R_{t+1}^A A_t^j$. Intuitively, limited liability implies that the bank enjoys the upside risk in asset returns over and above the face value of its debt, but does not bear the downside risk, which is transferred to the institutional investor. Furthermore, the value of the call option on island-specific risk may be expressed as

$$\int_{\bar{\omega}_{t+1}^j} \left(\omega - \bar{\omega}_{t+1}^j \right) dF_t(\omega) = \int \omega dF_t(\omega) + \int^{\bar{\omega}_{t+1}^j} \left(\bar{\omega}_{t+1}^j - \omega \right) dF_t(\omega) - \bar{\omega}_{t+1}^j.$$

Therefore, *given* the (normalized) face value of its debt, the bank's expected net payoff increases with the mean island-specific return, $\int \omega dF_t(\omega)$, but also with the value of the *put option* on island-specific returns with strike price $\bar{\omega}_{t+1}^j$,¹⁷

$$\int^{\bar{\omega}_{t+1}^j} \left(\bar{\omega}_{t+1}^j - \omega \right) dF_t(\omega) \equiv \pi_t(\bar{\omega}_{t+1}^j) \equiv \pi(\bar{\omega}_{t+1}^j; \sigma_t). \quad (10)$$

The put option value under the substandard technology, which we denote by $\tilde{\pi}_t(\bar{\omega}_{t+1}^j)$, is defined analogously, with \tilde{F}_t replacing F_t . Given our assumptions on both distributions, it can be shown

¹⁷The relationship between the values of a European call option and a European put option is usually referred to as the 'put-call parity'.

that $\tilde{\pi}_t(\bar{\omega}_{t+1}^j) > \pi_t(\bar{\omega}_{t+1}^j)$.¹⁸ Therefore, when choosing between investment strategies, the bank trades off the *higher mean return* of investing in the standard firm segment against the *lower put option value*. Furthermore, letting $\Delta\pi_t(\bar{\omega}_{t+1}^j) \equiv \tilde{\pi}_t(\bar{\omega}_{t+1}^j) - \pi_t(\bar{\omega}_{t+1}^j)$ denote the difference in put option values, we have that $\Delta\pi'_t(\bar{\omega}_{t+1}^j) = \tilde{F}_t(\bar{\omega}_{t+1}^j) - F_t(\bar{\omega}_{t+1}^j) > 0$: the incentive to invest in the riskier firm segment increases with the (normalized) debt commitment.

We are ready to spell out the bank's maximization problem. Let $V_t(\omega, A_{t-1}^j, \bar{B}_{t-1}^j)$ denote the value function of a non-defaulting bank at time t before paying out dividends, and let $\bar{V}_t(N_t^j)$ denote the bank's value function after paying out dividends and at the time of borrowing from the institutional investor. We then have the following Bellman equations:

$$V_t(\omega, A_{t-1}^j, \bar{B}_{t-1}^j) = \max_{N_t^j} \left\{ \Pi_t^j + \bar{V}_t(N_t^j) \right\},$$

subject to (6) and (7); and

$$\bar{V}_t(N_t^j) = \max_{A_t^j, \bar{B}_t^j} E_t \Lambda_{t,t+1} \int_{\bar{\omega}_{t+1}^j} \left[\theta V_{t+1}(\omega, A_t^j, \bar{B}_t^j) + (1-\theta) \left(R_{t+1}^A A_t^j \omega - \bar{B}_t^j \right) \right] dF_t(\omega),$$

subject to (5), (8) and (9). Let $\bar{b}_t^j \equiv \bar{B}_t^j / A_t^j$ denote the face value of debt normalized by the bank's assets. This allows us to express the default threshold as $\bar{\omega}_t^j = \bar{b}_{t-1}^j / R_t^A$. Appendix B proves the following result.

Proposition 1 (Solution to the bank's problem) *Assume the model parameters satisfy*

$$0 < \beta R^A - 1 < (1-\theta) \beta R^A \int_{\bar{\omega}^j} (\omega - \bar{\omega}^j) dF(\omega),$$

where R^A and $\bar{\omega}^j$ are the steady-state values of R_t^A and $\bar{\omega}_t^j$, respectively. Then the equilibrium dynamics of bank j in a neighborhood of the deterministic steady state are characterized by the following features:

1. The bank optimally retains all earnings,

$$N_t^j = \left(\omega^j - \frac{\bar{b}_{t-1}^j}{R_t^A} \right) R_t^A A_{t-1}^j, \quad (11)$$

where \bar{b}_{t-1}^j is equalized across islands.

2. The IC constraint holds with equality. In equilibrium, the latter can be expressed as

$$1 - \int \omega d\tilde{F}_t(\omega) = E_t \left\{ \frac{\Lambda_{t,t+1} R_{t+1}^A (\theta \lambda_{t+1} + 1 - \theta)}{E_t \Lambda_{t,t+1} R_{t+1}^A (\theta \lambda_{t+1} + 1 - \theta)} [\tilde{\pi}(\bar{\omega}_{t+1}; \sigma_t) - \pi(\bar{\omega}_{t+1}; \sigma_t)] \right\}, \quad (12)$$

where $\bar{\omega}_{t+1} = \bar{b}_t / R_{t+1}^A$ and λ_{t+1} is the Lagrange multiplier associated to the participation constraint. Both $\bar{\omega}_{t+1}$ and λ_{t+1} are equalized across islands.

3. The participation constraint holds with equality,

$$A_t^j = \frac{1}{1 - E_t \Lambda_{t,t+1} R_{t+1}^A [\bar{\omega}_{t+1} - \pi(\bar{\omega}_{t+1}; \sigma_t)]} N_t^j \equiv \phi_t N_t^j. \quad (13)$$

¹⁸Using integration by parts, it is possible to show that $\tilde{\pi}_t(\bar{\omega}_{t+1}^j) = \int^{\bar{\omega}_{t+1}^j} F_t(\omega) d\omega$. First-order stochastic dominance of $F_t(\omega)$ over $\tilde{F}_t(\omega)$ implies second-order dominance: $\int^x \tilde{F}_t(\omega) d\omega > \int^x F_t(\omega) d\omega$ for all $x > 0$. It thus follows that $\tilde{\pi}_t(\bar{\omega}_{t+1}^j) > \pi_t(\bar{\omega}_{t+1}^j)$ for all $\bar{\omega}_{t+1}^j > 0$.

According to (12), the (normalized) repurchase price \bar{b}_t is set such that the gain in mean return from investing in the standard firm segment exactly compensates the bank for the loss in the put option value. According to (13), the bank's demand for assets equals its net worth times a *leverage ratio* ϕ_t which is equalized across islands. Notice that leverage decreases with the left tail risk of the bank's portfolio, as captured by the put option value $\pi(\bar{\omega}_{t+1}; \sigma_t)$. Intuitively, since all the downside risk in the bank's assets is born by the institutional investor, a higher perception of such risk leads the latter to impose a tighter leverage constraint.

Once \bar{b}_t and ϕ_t have been determined, it is straightforward to obtain the actual loan size, $B_t^j = (\phi_t - 1) N_t^j$; its face value, $\bar{B}_t^j = \bar{b}_t A_t^j = \bar{b}_t \phi_t N_t^j$; and the implicit gross 'repo' rate, $\bar{B}_t^j / B_t^j = \bar{b}_t \phi_t / (\phi_t - 1)$. The loan-to-value ratio is then $B_t^j / A_t^j = (\phi_t - 1) / \phi_t$, and the 'repo' haircut or margin is $1 - B_t^j / A_t^j = 1 / \phi_t$. Therefore, changes in leverage are mirrored by opposite changes in haircuts.

3.5 Institutional investors

A representative, perfectly competitive institutional investor collects funds from households in the form of deposits and equity, and lends these funds to banks through short-term collateralized debt. Its balance sheet is thus $N_t^{II} + D_t = B_t$, where $B_t = \int_0^1 B_t^j dj$. There is no friction in the relationship between households and institutional investors. We assume that equity is sufficiently high to absorb aggregate risk and thus make deposits effectively safe. The institutional investor operates economy-wide and hence perfectly diversifies its portfolio across islands. The institutional investor's return from financing the island- j bank is

$$\min \left\{ R_t^A \omega^j A_{t-1}^j, \bar{B}_{t-1}^j \right\} = R_t^A A_{t-1}^j \min \left\{ \omega^j, \frac{\bar{b}_{t-1}}{R_t^A} \right\} = R_t^A \phi_{t-1} N_{t-1}^j \min \left\{ \omega^j, \bar{\omega}_t \right\}.$$

Aggregating across islands and subtracting gross interest payments on deposits, we obtain the return on the institutional investor's equity,

$$\begin{aligned} R_t^N N_{t-1}^{II} &= R_t^A \phi_{t-1} \int_0^1 N_{t-1}^j \min \left\{ \omega^j, \bar{\omega}_t \right\} dj - R_{t-1}^D D_{t-1} \\ &= R_t^A \phi_{t-1} N_{t-1} \left\{ [1 - F_{t-1}(\bar{\omega}_t)] \bar{\omega}_t + \int^{\bar{\omega}_t} \omega dF_{t-1}(\omega) \right\} - R_{t-1}^D D_{t-1}, \end{aligned}$$

where in the second equality we have used the fact ω^j is distributed independently from N_{t-1}^j , and where $N_{t-1} \equiv \int_0^1 N_{t-1}^j dj$ is aggregate net worth of banks. The institutional investor distributes all earnings to the household in every period.

3.6 Aggregation and market clearing

Aggregate net worth of banks at the *end* of period t , N_t , is the sum of the net worth of continuing banks, N_t^{cont} , and that of new banks, N_t^{new} ,

$$N_t = N_t^{cont} + N_t^{new}.$$

From (11), $\bar{b}_{t-1} / R_t^A = \bar{\omega}_t$ and $A_{t-1}^j = \phi_{t-1} N_{t-1}^j$, we have that $N_t^j = R_t^A (\omega^j - \bar{\omega}_t) \phi_{t-1} N_{t-1}^j$. Aggregating across islands, we obtain the total net worth of continuing banks,

$$N_t^{cont} = \theta R_t^A \int_{\bar{\omega}_t} (\omega - \bar{\omega}_t) dF_{t-1}(\omega) \phi_{t-1} N_{t-1},$$

where we have used the fact that ω^j is distributed independently from N_{t-1}^j . Banks that default or exit the market exogenously are replaced by an equal number of new banks, $F_{t-1}(\bar{\omega}_t) +$

$[1 - F_{t-1}(\bar{\omega}_t)](1 - \theta) = 1 - \theta [1 - F_{t-1}(\bar{\omega}_t)]$. We assume that new banks are endowed by households with a fraction τ of total assets at the beginning of the period, $A_{t-1} \equiv \int_0^1 A_{t-1}^j dj$. Therefore,

$$N_t^{new} = \{1 - \theta [1 - F_{t-1}(\bar{\omega}_t)]\} \tau A_{t-1}.$$

We thus have

$$N_t = \theta R_t^A \int_{\bar{\omega}_t} (\omega - \bar{\omega}_t) dF_{t-1}(\omega) \phi_{t-1} N_{t-1} + \{1 - \theta [1 - F_{t-1}(\bar{\omega}_t)]\} \tau A_{t-1}. \quad (14)$$

New banks leverage their starting net worth with the same ratio as continuing banks. We thus have

$$A_t = \phi_t (N_t^{cont} + N_t^{new}) = \phi_t N_t.$$

Aggregate net dividends to households from banks are given, by

$$\Pi_t = (1 - \theta) R_t^A \int_{\bar{\omega}_t} (\omega - \bar{\omega}_t) dF_{t-1}(\omega) \phi_{t-1} N_{t-1} - N_t^{new}.$$

Market clearing for capital requires that total demand by firms equals total supply by capital producers, $\int_0^1 K_t^j dj = K_t$. The aggregate capital stock evolves as follows,

$$K_{t+1} = I_t + (1 - \delta) K_t.$$

The total issuance of state-contingent claims by firms must equal total demand by banks,

$$K_{t+1} = A_t.$$

From (3), firm j 's labor demand is $L_t^j = [(1 - \alpha) Z_t / W_t]^{1/\alpha} \omega^j K_t^j$. Aggregating across islands and imposing labor market clearing, we have

$$\int_0^1 L_t^j dj = \left(\frac{(1 - \alpha) Z_t}{W_t} \right)^{1/\alpha} \int_0^1 \omega^j K_t^j dj = \left(\frac{(1 - \alpha) Z_t}{W_t} \right)^{1/\alpha} K_t = L_t, \quad (15)$$

where we have used the fact that ω^j and K_t^j are distributed independently and the fact that ω^j has unit mean. Equations (3) and (15) then imply that $\omega^j K_t^j / L_t^j = K_t / L_t$. Using the latter and (2), aggregate supply of the final good by firms equals

$$Y_t = \int_0^1 Y_t^j dj = Z_t \left(\frac{L_t}{K_t} \right)^{1-\alpha} \int_0^1 \omega^j K_t^j dj = Z_t K_t^\alpha L_t^{1-\alpha}.$$

Finally, total supply of the final good must equal consumption demand by households and investment demand by capital producers,

$$Y_t = C_t + I_t.$$

3.7 Comparison to RBC model

Appendix C summarizes the equilibrium conditions in our model, and compares it with a standard RBC model. As we show there, the RBC model shares all its equilibrium conditions with our model, except for the investment Euler equation, given by

$$1 = E_t \{ \Lambda_{t,t+1} R_{t+1}^A \}. \quad (16)$$

The equation that determines leverage in our framework (13) can be rewritten in an analogous form,

$$1 = E_t \left\{ \Lambda_{t,t+1} R_{t+1}^A [\bar{\omega}_{t+1} - \pi(\bar{\omega}_{t+1}; \sigma_t)] \frac{\phi_t}{\phi_t - 1} \right\}. \quad (17)$$

A comparison of equations (17) and (16) reveals that the term

$$[\bar{\omega}_{t+1} - \pi(\bar{\omega}_{t+1}; \sigma_t)] \frac{\phi_t}{\phi_t - 1} \equiv \Theta(\bar{\omega}_{t+1}, \phi_t; \sigma_t) \equiv \Theta_t(\bar{\omega}_{t+1}, \phi_t)$$

in equation (17) is a sufficient statistic for measuring the difference in equilibrium dynamics between both models. Using $\phi_t = A_t/N_t$ and the definition of the put option value $\pi_t(\bar{\omega}_{t+1})$ (equation 10), we can write

$$\begin{aligned} R_{t+1}^A \Theta_t(\bar{\omega}_{t+1}, \phi_t) &= R_{t+1}^A \left\{ \int^{\bar{\omega}_{t+1}} \omega dF_t(\omega) + \bar{\omega}_{t+1} [1 - F_t(\bar{\omega}_{t+1})] \right\} \frac{A_t}{A_t - N_t} \\ &= \frac{R_{t+1}^A A_t \int^{\bar{\omega}_{t+1}} \omega dF_t(\omega) + \bar{B}_t [1 - F_t(\bar{\omega}_{t+1})]}{B_t}, \end{aligned} \quad (18)$$

where in the second equality we have used $\bar{\omega}_{t+1} = \bar{B}_t / (R_{t+1}^A A_t)$ and $A_t - N_t = B_t$. Expression (18) is just the return on aggregate collateralized debt, B_t . Therefore, $\Theta_t(\bar{\omega}_{t+1}, \phi_t)$ captures the fraction of the total return on capital, R_{t+1}^A , that is received by the household (through the institutional investor). In the absence of financial frictions, households receive the entire return on capital and $\Theta_t(\bar{\omega}_{t+1}, \phi_t)$ is simply 1.¹⁹ With financial frictions, we generally have $\Theta_t(\bar{\omega}_{t+1}, \phi_t) \neq 1$, which drives a wedge between investment decisions in our model and in the RBC model. Importantly, the cross-sectional volatility shock σ_t is absent from the equilibrium conditions in the RBC model, implying that such a shock has no effect whatsoever in the latter model. In our framework, on the contrary, the presence of financial frictions opens a link between cross-sectional volatility in asset returns and aggregate investment dynamics.

4 Quantitative Analysis

4.1 Calibration and steady state

We calibrate our model to the US economy for the period 1984:Q1-2011:Q3. The parameters are shown in Table 3. We may divide the parameters between those that are standard in the real business cycle (RBC) literature, and those that are particular to the financial structure of the model. From now onwards, we let variables without time subscripts denote steady-state values.

We set the RBC parameters to standard values. In particular, we set $\beta = 0.99 = 1/R$, $\alpha = 0.36 = 1 - WL/Y$, $\delta = 0.025 = I/K$, which are broadly consistent with long-run averages for the real interest rate, the labor share, and the investment to capital ratio. For future use, we note that the steady-state return on banks' assets is $R^A = \alpha(Y/K) + 1 - \delta$. We target a capital-output ratio of $K/Y = 8$, which is consistent with a ratio of investment over GDP of 20 percent, roughly in line with the historical evidence. We then have $R^A = 1.02$. Our functional forms for preferences are standard: $u(x) = \log(x)$, $v(L) = L^{1+\varphi}/(1+\varphi)$. We set $\varphi = 1$, in line with other macroeconomic studies (see e.g. Comin and Gertler, 2006). We assume an AR(1) process for the natural log of TFP,

$$\log(Z_t/Z) = \rho_z \log(Z_{t-1}/Z) + \varepsilon_t^z,$$

where $\varepsilon_t^z \stackrel{iid}{\sim} N(0, \sigma_z)$. Our empirical counterpart for $\log(Z_t/Z)$ is the quarterly TFP series constructed by the CSIP at the Federal Reserve Bank of San Francisco, after being logged and linearly detrended.²⁰ We then choose ρ_z and σ_z so as to match their empirical counterparts. Z is chosen such that steady-state output is normalized to one.

Regarding the parameters related to the financial side of the model, our calibration strategy is as follows. Ideally, one would consolidate the balance sheets of the different financial subsectors so

¹⁹ Assuming no financial frictions is equivalent to assuming that households themselves buy the capital and then rent it to firms, as is typically done in the RBC literature.

²⁰ See the data appendix for more information.

as to calibrate the model to the leveraged financial sector as a whole. As explained in section 2, this consolidation is however not feasible, due to the existence of cross-positions among financial subsectors and the need to avoid double-counting. For this reason, we choose to set the steady-state leverage ratio ϕ to match the average leverage ratio of the four type of banks during our sample period, $\phi = 18.3$. The latter implies a repo loan-to-value ratio of $b = B/A = (\phi - 1)/\phi = 0.9454$, or equivalently a repo haircut of 5.46%; the latter is in line with average haircuts for repos backed by corporate debt and private-label ABS, as documented by Krishnamurthy et al. (2012). The same authors show that the spread between the repo rates for the same collateral categories and the Fed funds rate was close to zero in the pre-crisis period. Based on this, we target a spread in short-term collateralized debt contracts of 25 annualized basis points. The repo rate then equals $\bar{R} = R(1.0025)^{1/4} = 1.0107$. The face value of repo debt (normalized by assets) is then $\bar{b} = \bar{R}b = 0.9555$. This implies a default threshold of $\bar{\omega} = \bar{b}/R^A = 0.9368$.

Island-specific shocks are assumed to be lognormally distributed. In particular, the distribution of island-specific shocks to the standard and the substandard firm segment is given by

$$\begin{aligned}\log \omega &\overset{iid}{\sim} N\left(\frac{-\sigma_t^2}{2}, \sigma_t\right), \\ \log \tilde{\omega} &\overset{iid}{\sim} N\left(\frac{-\eta\sigma_t^2 - \psi}{2}, \sqrt{\eta}\sigma_t\right),\end{aligned}$$

respectively. Therefore, $F(\omega; \sigma_t) = \Phi\left(\frac{\log(\omega) + \sigma_t^2/2}{\sigma_t}\right)$, where $\Phi(\cdot)$ is the standard normal cdf. The parameters $\psi > 0$ and $\eta > 1$ control, respectively, the mean and the variance of the substandard technology relative to the standard one. Notice in particular that

$$E[\omega] = 1 > E[\tilde{\omega}] = e^{-\psi/2}.$$

These distributional assumptions imply the following expressions for the values of the unit put options on island-specific risk,²¹

$$\pi(\bar{\omega}_t; \sigma_{t-1}) = \bar{\omega}_t \Phi\left(\frac{\log(\bar{\omega}_t) + \sigma_{t-1}^2/2}{\sigma_{t-1}}\right) - \Phi\left(\frac{\log(\bar{\omega}_t) - \sigma_{t-1}^2/2}{\sigma_{t-1}}\right), \quad (19)$$

$$\tilde{\pi}(\bar{\omega}_t; \sigma_{t-1}) = \bar{\omega}_t \Phi\left(\frac{\log(\bar{\omega}_t) + (\psi + \eta\sigma_{t-1}^2)/2}{\sqrt{\eta}\sigma_{t-1}}\right) - e^{-\psi/2} \Phi\left(\frac{\log(\bar{\omega}_t) + (\psi - \eta\sigma_{t-1}^2)/2}{\sqrt{\eta}\sigma_{t-1}}\right) \quad (20)$$

The standard deviation of island-specific shocks is assumed to follow an AR(1) process in logs,

$$\log(\sigma_t/\sigma) = \rho_\sigma \log(\sigma_{t-1}/\sigma) + \varepsilon_t^\sigma,$$

where $\varepsilon_t^\sigma \overset{iid}{\sim} N(0, \sigma_\sigma)$. In order to calibrate σ , we notice that the participation constraint (eq. 13) in the steady state implies $\pi(\bar{\omega}; \sigma) = \bar{\omega} - (1 - 1/\phi)/\beta R^A = 0.0006$. Using the steady-state counterpart of (19), we can then solve for $\sigma = 0.0373$. The default rate of banks in the steady state then equals $F(\bar{\omega}; \sigma) = 4.17\%$. In order to calibrate the parameters governing the dynamics of island-specific volatility $(\rho_\sigma, \sigma_\sigma)$, we use the TFP series for all 4-digit SIC manufacturing industries constructed by the NBER and the US Census Bureau's Center for Economic Studies (CES). We then construct a time series for σ_t by calculating the cross-sectional standard deviation of the industry-level TFP series (in log deviations from a linear trend) at each point in time. Fitting an autoregressive process to the resulting series, we obtain $\rho_\sigma = 0.9457$ and $\sigma_\sigma = 0.0465$.²²

Regarding the parameters of the substandard technology, ψ and η , we make use of the IC constraint in the steady state,

$$1 - e^{-\psi/2} = \tilde{\pi}(\bar{\omega}; \sigma) - \pi(\bar{\omega}; \sigma),$$

²¹The proof is available upon request.

²²See data appendix for details.

where $\tilde{\pi}(\bar{\omega}; \sigma)$ is given by expression (20) in the steady state. We thus have one equation for two unknowns, ψ and η . We choose to set ψ to 0.001 to replicate the midpoint of the range of standard deviations of equity, and use the IC constraint to solve for $\eta = 1.2691$. This implies that shocks to the substandard firm segment are $\sqrt{\eta} = 1.1$ times more volatile than the standard one.

Finally, the exogenous bank continuation rates θ and the bank equity injection parameter τ are calibrated as follows. In the steady state, the law of motion of bank net worth (eq. 14) becomes

$$\frac{1}{\phi} = \theta R^A \int_{\bar{\omega}} (\omega - \bar{\omega}) dF(\omega; \sigma) + \{1 - \theta [1 - F(\bar{\omega}; \sigma)]\} \tau, \quad (21)$$

where we have normalized by A . Equation (21) implies that τ is a decreasing function of θ , given the other parameters and steady state values. In the choice of θ , we are restricted by the requirement that $\tau \geq 0$, which holds for $\theta \leq 0.84$. We notice that in equilibrium $1/(1-\theta)$ represents the average frequency of dividend payments by banks. We set θ to 0.75, such that banks pay dividends once a year on average. We then use (21) to solve for $\tau = 0.0207$.

4.2 The response to TFP shocks

We follow the lead of the traditional RBC literature by exploring how well a TFP shock can explain the unconditional patterns found in the data. Table 4 displays the second-order moments of interests. They include the standard deviations of GDP, assets, equity and leverage, as well as the correlations of assets, equity and GDP with leverage, and the correlation between assets and GDP. We also show the range of moments across subsectors for the 1984-2011 sample (see Table 2), as the model is calibrated to replicate this period. Model moments are based on simulated series. In order to make the model moments comparable with the empirical ones, we first log the simulated series and filter them using the same bandpass filter as the one applied to the data.²³

As shown by the third column of Table 4, conditional on TFP shocks the model replicates fairly well the standard deviation of GDP, and the correlation between assets and GDP is not outside the empirical range. However, the model fails dramatically at reproducing the volatility of assets, equity and leverage. It also fails to produce any meaningful procyclicality in the leverage ratio or to capture the high negative correlation between leverage and equity.

To understand these results, Figure 3 displays the impulse responses to a negative TFP shock (black dotted line).²⁴ On impact, the fall in TFP produces a sharp fall in the return on assets, which increases the number of bankruptcies in the banking sector. The fall in the profitability of banks' investments reduces their equity. The leverage ratio barely reacts; indeed, the latter responds mainly to *expected* changes in the default threshold (see eq. 13), which is virtually back to baseline after the impact period. This explains the low volatility of leverage and its lack of correlation with output. Since their leverage remains stable, bank assets basically reproduce the response of their net worth; i.e. the effects of TFP shocks on bank credit operate mainly through the equity channel. Since net worth responds relatively little, so do assets, hence their low volatility.

Figure 4 compares the response to the negative TFP shock in our model and in the basic RBC model.²⁵ The financial friction in our model amplifies the percentage decline in investment and the capital stock relative to the RBC framework. This is due to a fall in the *expected* financial wedge $\Theta_t(\bar{\omega}_{t+1}, \phi_t)$ on impact, which discourages investment; it is also due to a stronger fall in the aggregate return on capital, R_{t+1}^A . The percentage decline in consumption is also stronger. However, the percentage fall in output is very similar in both cases, due to the higher investment share of GDP in the RBC model.²⁶

²³In particular, we simulate the model for 55,000 periods and discard the first 5,000 observations to eliminate the effect of initial conditions. The model is solved by means of a second-order perturbation method (in levels). The code has been implemented in Dynare.

²⁴To compute the impulse responses, the model is solved by means of a first-order perturbation method (in levels).

²⁵The parameters of the RBC model ($\alpha, \beta, \delta, \varphi, Z, \rho_z, \sigma_z$) are exactly the same as in our model. The steady state equilibrium differs however in both frameworks. In particular, the absence of a financial wedge in the RBC model ($\Phi = 1$, versus $\Phi = 0.9903$ in our model) implies a higher capital stock (12.23, vs 8), higher output (1.19, vs 1), etc.

²⁶The percentage output change is simply the average of the percentage changes in consumption and investment,

4.3 The volatility-leverage channel

A recent financially oriented literature shows how an increase in the volatility of asset returns reduces borrowers' leverage. For example, Brunnermeier and Pedersen (2009) analyze how an increase in the volatility of asset prices leads investors to demand higher margins, thus forcing borrowers to deleverage. Similarly, Geanakoplos (2010) or Fostel and Geanakoplos (2008), consider shocks that not only decrease the expected asset returns but also their volatility. Such shocks, which the authors refer to as 'scary bad news', lead to tighter margins as lenders protect themselves against increased uncertainty. From a more macro perspective, recent work suggests that exogenous changes in volatility may be an important driving force behind business cycle fluctuations (see e.g. Bloom, 2009; Bloom et al., 2011; Christiano et al., 2010; Gilchrist et al., 2010, Arellano et al., 2012).

In our model, an increase in the standard deviation of island-specific shocks, σ_t , induces a reduction in the leverage of banks, via a mechanism close to the one described in Adrian and Shin (2011a) and sketched in Figure 5. The upper subplot represents the steady-state counterpart of the IC constraint (eq. 12). The upward-sloping curve is the gain in left tail risk from investing in the substandard firm segment, $\Delta\pi(\bar{\omega};\sigma) = \tilde{\pi}(\bar{\omega};\sigma) - \pi(\bar{\omega};\sigma)$, which under our distributional assumptions is an increasing function of the (normalized) face value of debt, $\bar{\omega} = \bar{B}^j / (R^A A^j) = \bar{b} / R^A$. The horizontal line is the loss in mean return, $E(\omega) - \tilde{E}(\omega) = 1 - \int \omega d\tilde{F}(\omega, \sigma)$. The IC constraint requires $\bar{\omega}$ to be such that the gain in left tail risk from investing in the substandard technology does not exceed the loss in mean return. Since the constraint is binding in equilibrium, $\bar{\omega}$ is determined by the intersection of both lines. Consider now an increase in cross-sectional volatility, σ . Provided $\Delta\pi$ is increasing in σ (which holds under our distributional assumptions), then *ceteris paribus* the $\Delta\pi(\bar{\omega}, \cdot)$ schedule shifts upwards and $\bar{\omega}$ goes down. Intuitively, since higher volatility makes it more attractive for the bank to invest inefficiently, the institutional investor reduces the (normalized) face value of debt so as to induce the former to invest efficiently.

The lower subplot of Figure 5 represents the steady-state counterpart of the participation constraint, $\phi = 1 / \{1 - \beta R^A [\bar{\omega} - \pi(\bar{\omega};\sigma)]\}$. The latter represents an upward-sloping relationship between leverage, $\phi = (B^j + N^j) / N^j$, and the normalized face value of debt, $\bar{\omega}$.²⁷ *Ceteris paribus*, the increase in σ has a double effect on leverage. First, the leverage schedule shifts down, which reduces equilibrium leverage for a given $\bar{\omega}$. Intuitively, higher volatility of island-specific shocks increases the downside risk $\pi(\bar{\omega};\sigma)$ of the assets that serve as collateral, which reduces the investor's expected payoff; in order to induce the investor to lend, the bank reduces its demand for funds as a fraction of its net worth. Second, the reduction in $\bar{\omega}$ through the IC constraint produces a leftward movement *along* the leverage schedule, thus further reducing equilibrium leverage. Both effects are mutually reinforcing.

How does this volatility-leverage channel operate in general equilibrium? To analyze this, we simulate the model conditional on shocks to cross-sectional volatility. The results are shown in the fourth column of Table 4. The model generates now large fluctuations in the leverage ratio and equity of banks, comparable to those in the data. It also produces somewhat larger fluctuations in assets than those generated by TFP shocks. The fluctuations in output are however relatively modest. In terms of correlations, volatility shocks produce a strong procyclicality in leverage with respect to GDP, well above the empirical range. It also produces a strong positive comovement between assets and leverage, within the range found in the data.

To understand these results, the red line in Figure 3 displays the responses to an increase in cross-sectional volatility. The shock produces a sharp reduction in the (normalized) face value of

weighted by each component's share of GDP in the steady state. The investment share is higher in the RBC model (25.6%, versus 20% in our model), and thus more weight is put in the percentage investment change, which is much larger than that of consumption.

²⁷The investor's expected payoff is $\beta R^A [\bar{\omega} - \pi(\bar{\omega})]$. That is, the investor's exposure to island-specific risk is equivalent to holding cash in the amount $\bar{\omega}$ and a short position in a put option with strike price $\bar{\omega}$ (Merton, 1974; Adrian and Shin, 2011a). Since $\pi'(\bar{\omega}) = F(\bar{\omega}) < 1$, the investor's expected payoff from lending to the bank *increases* with $\bar{\omega}$. As a result, the bank can borrow more (as a fraction of its net worth) while still persuading the investor to lend the funds.

bank debt, $\bar{\omega}_t = \bar{b}_{t-1}/R_t^A$, after the impact period. This fall in the debt commitment, together with the increase in uncertainty, produce a drastic reduction in the leverage ratio of banks, of about 5%. Banks' net worth increases after the impact period, due to the reduction in the default threshold $\bar{\omega}_t$ and hence the increase in the average return on assets for non-defaulting banks, $R_t^A [1 - \bar{\omega}_t + \pi_{t-1}(\bar{\omega}_t)]$. However, the drop in leverage dominates the increase in net worth, as evidenced by the fall in bank assets. This produces a contraction in the capital stock, investment and aggregate output.²⁸ This volatility-leverage channel provides an alternative mechanism to the ones presented by Bloom et al. (2011) or Gilchrist et al. (2010), through which changes in cross-sectional uncertainty may generate aggregate business cycles.

Finally, the second column in Table 4 shows the combined effects of both TFP and volatility shocks in the model. The existence of two uncorrelated sources of fluctuations reduces the correlation of leverage and GDP to a level comparable to that in the data. Regarding the standard deviations, the unconditional volatility of aggregate output is dominated by TFP shocks, while that of leverage and equity is mostly determined by volatility shocks. The model underpredicts the volatility of banks' assets, while capturing fairly well the size of fluctuations in banks' leverage and equity.

4.4 Sensitivity analysis

We now study how robust these results are to alternative parametrizations. We first consider an alternative value for the steady-state leverage ratio, ϕ . Instead of calibrating it to the average across subsectors for the Great Moderation (18.3), we can calibrate it to match the mean leverage for a specific subsector, such as commercial banks (10.6).²⁹ The fifth column of Table 4 shows that the results are roughly similar to the those of the baseline model. There is a reduction in the volatility of leverage and equity and an increase in those of GDP and assets. In addition, there is an increase in the correlations of leverage with GDP and assets, and in the correlation of assets with GDP.

An alternative exercise involves the considering a different value for the exogenous bank continuation rate θ . As explained in section 4.1, this parameter was calibrated to 0.75, which is equivalent to assuming that banks pay off dividends once a year on average. In the last column of Table 4 we analyze the case when this parameter takes a smaller value (0.5).³⁰ As in the previous case, there is a reduction in the volatilities of leverage and equity with respect to the baseline and an increase in those of GDP and assets. There is also an increase in the correlations of leverage with GDP and assets, in the correlation of assets with GDP and a reduction in the negative correlation of leverage with equity.³¹

An alternative approach for choosing θ is to estimate it. We proceed as follows. We take US chartered commercial banks (the largest subsector in the US financial system) as a proxy for the leveraged financial system as a whole. As we have two shock processes (TFP and volatility), our information set contains two variables: TFP and equity capital for the period 1984Q1:2011Q3, both in log deviations from a linear trend. We employ Bayesian methods to estimate θ . We choose a Beta distribution as a prior, due to the fact that the latter distribution is bounded between 0 and 1. The prior mean is 0.75 and the standard error is set to 0.1. We employ a Metropolis Hasting algorithm with 2 chains and 10,000 replications per chain. The estimated posterior mean is 0.25, with a confidence interval between 0.21 and 0.29.

The low estimated value for θ is the consequence of having estimated the model using data from commercial banks. Their equity volatility was relatively low (3.12 %, see Table 2), thus forcing θ

²⁸Aggregate output falls by less than in the case of TFP shocks, due to a smaller reduction in private consumption (not shown).

²⁹This recalibration modifies the value of three parameters: $\sigma = 0.0564$, $\eta = 1.2471$, $\tau = 0.0568$.

³⁰This recalibration modifies the value of τ to 0.0424.

³¹We have also performed diverse sensitivity analysis to the parameter ψ , concluding that model results are relatively robust to changes in this parameter. The most sensitive moment to the parameter value is the volatility of equity, which is the moment that we use to calibrate it.

to take a low value as we have seen that there is a positive relationship between both objects. It would be natural to take this value as a lower bound.

Given this estimation of θ , we compare the counterfactual evolution of the (unfiltered) model series of leverage and assets with those in the data.³² Results are displayed in Figure 6. In the case of the assets, the model seems to capture reasonably well the expansion in assets with respect to the trend in the mid-80s and the posterior contraction after the ‘savings and loans’ crisis. It also captures, albeit with a steeper trend, the expansion in assets prior to 2008 and the posterior collapse. The model, notwithstanding, fails to reproduce the high frequency movements in assets. In the case of leverage, the model does a better job, capturing both the low and the high-frequency components reasonably well.

4.5 The risk diversification paradox

The exercises presented above seem to indicate that the model is able to roughly replicate the data in a number of dimensions. In particular, it can explain the bank leverage cycles observed in the data as the result of exogenous changes in cross-sectional volatility. Given these results, this section analyzes how different levels of *average* cross-sectional volatility may affect the macroeconomy. We may indeed consider a scenario in which financial innovation allows banks to better diversify their risks. In terms of the model, this amounts to a reduction in the steady-state volatility of island-specific shocks, σ . The question then is: what is the effect of this financial innovation both on the mean level *and* the volatility of output.

To answer this question, we study the behavior of the model as we lower σ from its baseline value of 0.0373 to 0.0253. For the purpose of this exercise, we simulate the model with both TFP and volatility shocks. Figure 7 displays the results. The left panel displays the mean values of leverage (ϕ) and output (Y). The right panel displays the standard deviations of leverage and output. In this case the data have not been filtered, as we need to preserve the means and we do not compare model results with data.

As shown in the figure, a reduction in cross-sectional uncertainty allows banks to increase their leverage on average, through a mechanism very similar to the one explained before. For a given net worth, higher leverage allows banks to expand the size of their balance-sheets. This in turn leads to an increase in the stock of capital, and hence in the average level of output. Therefore, financial innovations that improve risk diversification induce an economic expansion on average via an increase in capital accumulation. This result is not controversial and has been confirmed by historical evidence, as discussed in Kindleberger (1986).

The effects on the volatilities are more striking. A reduction in cross-island volatility generates an *increase* in the volatility of output. For lack of a better name, we have named this effect ‘the risk diversification’ paradox, even though such a paradox is only apparent. A reduction in cross-island volatility increases the mean leverage of the banking sector, which in turn increases the size of fluctuations in leverage. The consequence is that a reduction in cross-island volatility leads to larger fluctuations in total intermediated assets. This in turn results in larger fluctuations in the capital stock, and hence in aggregate output. This unconditional result holds also conditionally on TFP shocks and volatility shocks.³³ This result is similar to the ‘volatility paradox’ of Brunnermeier and Sannikov (2011), although through a different mechanism (cross-section instead of aggregate volatility) and without the need of including any non-linear feedback loop.

The conclusion is that risk diversification has both a positive level effect on economic activity, and a negative effect through an increase in aggregate volatility, where the latter is due to higher leverage. The optimal size of risk diversification will depend on the degree risk aversion of the households, a point that we leave for further research.

³²Using again linearly detrended log-series for commercial banks.

³³Results are available upon request.

5 Conclusions

We have presented empirical evidence regarding the balance sheet dynamics of financial intermediaries in the United States. We have found that fluctuations in leverage, equity and total assets are roughly of the same magnitude, and are all several times more volatile than GDP. Leverage actually contributes more than equity to fluctuations in total assets. We have also found that leverage is positively correlated with total assets and GDP, and negatively correlated with equity. These findings suggest the need to consider endogenous leverage within the context of macroeconomic models with financial intermediaries.

We have then built a general equilibrium model with financial intermediaries and endogenous leverage, and assessed its ability to match the evidence. The model incorporates a financial intermediation sector financed with short-term collateralized debt. The leverage ratio of financial intermediaries is endogenously determined as the result of a contracting problem between the latter and a sector of institutional investors. Due to moral hazard on the part of banks, institutional investors restrict their lending to a certain ratio of the former's net worth. In the model, TFP shocks produce rather small fluctuations in leverage, equity and assets, and fail to produce any meaningful comovement between leverage and GDP. Shocks to cross-sectional volatility do generate large fluctuations in assets and equity, as well as a positive (albeit excessively so) comovement between leverage and GDP. Combining TFP and volatility shocks allows the model to produce a procyclicality of leverage similar to that in the data.

Finally, we have shown that, in the context of our model, an economy with lower average cross-sectional volatility has a higher average stock of capital and higher average output. However, it also has a higher output volatility. This stems from the fact that a lower perception of risk in asset returns leads to an increase in the leverage of financial intermediaries and to larger fluctuations in their lending activity.

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Appendix

A. Data appendix

Data on equity capital and total assets of the four leveraged financial subsectors we consider (US-chartered commercial banks, savings institutions, security brokers and dealers, and finance companies) are from the Z.1 files of the US Flow of Funds.³⁴ The series corresponding to savings institutions are the sum of OTS and FDIC reporters. Data on *levels* in the Z.1 files (denoted by 'FL' in the series identifier) suffer from discontinuities that are caused by changes in the definition of the series. The Flow of Funds accounts correct for such changes by constructing *discontinuities* series (denoted by 'FD').³⁵ In particular, for each series the *flow* (denoted by 'FU') is equal to the change in level outstanding less any discontinuity. That is: $FU_t = FL_t - FL_{t-1} - FD_t$. Therefore, the flow data are free from such discontinuities. In order to construct discontinuity-free level series, we take the value of the level in the first period of the sample and then accumulate the flows onwards.

For each subsector, the leverage ratio is the ratio between total assets and equity capital, both in dollars. In the tables and figures, 'assets' refer to real total assets, which are total assets (in dollars) divided by the GDP Implicit Price Deflator. The latter and Real GDP are both from the Bureau of Economic Analysis. Both series are readily available at the Federal Reserve Bank of St. Louis FRED database.³⁶

In order to obtain an empirical proxy for aggregate TFP, we use the quarterly Business sector TFP growth series (labelled 'dtfp') constructed by the Center for the Study of Income and Productivity (CSIP) at the Federal Reserve Bank of San Francisco.³⁷ We then accumulate growth rates to obtain the level series.

Finally, in order to construct a proxy for island-specific volatility, we use the annual TFP series for all 4-digit SIC manufacturing industries constructed by the National Bureau of Economic Research (NBER) and the US Census Bureau's Center for Economic Studies (CES).³⁸ The data run through 2005, so our sample period in this case is 1984-2005. We discard those industries that exit the sample in the mid-nineties due to the change in industry classification from SIC to NAICS. We then log and linearly detrend each industry TFP series. Our proxy for the time series of (annual) island-specific volatility is the cross-sectional standard deviation of all industry TFP series in each year. We may denote the latter by σ_τ^α , where τ is the year subscript. Assuming that the underlying quarterly process is $\log \sigma_t = (1 - \rho_\sigma) \log \sigma + \rho_\sigma \log \sigma_{t-1} + \varepsilon_t$, with $\varepsilon_t \sim iid(0, \sigma_\sigma)$, and that each annual observation corresponds to the last quarter in the year, then the annual process satisfies $corr(\log \sigma_\tau^\alpha, \log \sigma_{\tau-1}^\alpha) = \rho_\sigma^4$, and $var(\log \sigma_\tau^\alpha) = \frac{1 + \rho_\sigma^2 + \rho_\sigma^4 + \rho_\sigma^6}{1 - \rho_\sigma^8} \sigma_\sigma^2$. The sample autocorrelation and variance of $\log \sigma_\tau^\alpha$ are 0.7997 and 0.0205, respectively, which imply $\rho_\sigma = 0.9457$ and $\sigma_\sigma = 0.0465$.

B. The bank's problem

We start by defining the ratio $\bar{b}_{t-1}^j \equiv \bar{B}_{t-1}^j / A_{t-1}^j$ and using the latter to substitute for $\bar{B}_{t-1}^j = \bar{b}_{t-1}^j A_{t-1}^j$. Given the choice of investment size A_t^j , the bank then chooses the ratio \bar{b}_t^j . With this transformation, and abusing somewhat the notation V_t and \bar{V}_t in the main text, the bank's maximization problem can be expressed as

$$V_t \left(\omega, A_{t-1}^j, \bar{b}_{t-1}^j \right) = \max_{N_t^j} \left\{ \left(\omega - \frac{\bar{b}_{t-1}^j}{R_t^A} \right) R_t^A A_{t-1}^j - N_t^j + \bar{V}_t \left(N_t^j \right) + \mu_t^j \left[\left(\omega - \frac{\bar{b}_{t-1}^j}{R_t^A} \right) R_t^A A_{t-1}^j - N_t^j \right] \right\}, \quad (22)$$

³⁴Website: <http://www.federalreserve.gov/datadownload/Choose.aspx?rel=Z1>

³⁵For instance, changes to regulatory report forms and/or accounting rules typically trigger 'FD' entries for the affected series.

³⁶Website: <http://research.stlouisfed.org/fred2/>

³⁷Website: <http://www.frbsf.org/csip/TFP.php>

³⁸Website: <http://www.nber.org/data/nbprod2005.html>

$$\bar{V}_t(N_t^j) = \max_{A_t^j, \bar{b}_t^j} E_t \Lambda_{t,t+1} \int_{\bar{b}_t^j/R_{t+1}^A} \left[\theta V_{t+1}(\omega, A_t^j, \bar{b}_t^j) + (1-\theta) \left(\omega - \bar{b}_t^j/R_{t+1}^A \right) R_{t+1}^A A_t^j \right] dF_t(\omega)$$

subject to the participation constraint,

$$E_t \Lambda_{t,t+1} R_{t+1}^A A_t^j \left\{ \int_{\bar{b}_t^j/R_{t+1}^A} \omega dF_t(\omega) + \frac{\bar{b}_t^j}{R_{t+1}^A} \left[1 - F_t \left(\frac{\bar{b}_t^j}{R_{t+1}^A} \right) \right] \right\} \geq A_t^j - N_t^j,$$

and the IC constraint

$$\begin{aligned} & E_t \Lambda_{t,t+1} \int_{\bar{b}_t^j/R_{t+1}^A} \left\{ \theta V_{t+1}(\omega, A_t^j, \bar{b}_t^j) + (1-\theta) R_{t+1}^A A_t^j \left(\omega - \frac{\bar{b}_t^j}{R_{t+1}^A} \right) \right\} dF_t(\omega) \\ & \geq E_t \Lambda_{t,t+1} \int_{\bar{b}_t^j/R_{t+1}^A} \left\{ \theta V_{t+1}(\omega, A_t^j, \bar{b}_t^j) + (1-\theta) R_{t+1}^A A_t^j \left(\omega - \frac{\bar{b}_t^j}{R_{t+1}^A} \right) \right\} d\tilde{F}_t(\omega). \end{aligned}$$

The first order condition with respect to N_t^j is given by

$$\mu_t^j = \bar{V}'_t(N_t^j) - 1.$$

We can now guess that $\bar{V}'_t(N_t^j) > 1$. Then $\mu_t^j > 0$ and the non-negativity constraint on dividends is binding, such that a continuing bank optimally decides to retain all earnings,

$$N_t^j = \left(\omega - \frac{\bar{b}_{t-1}^j}{R_t^A} \right) R_t^A A_{t-1}^j. \quad (23)$$

From (22), we then have $V_t(\omega, A_{t-1}^j, \bar{b}_{t-1}^j) = \bar{V}_t((\omega - \bar{b}_{t-1}^j/R_t^A) R_t^A A_{t-1}^j)$. Using the latter, we can express the Bellman equation for $\bar{V}_t(N_t^j)$ as

$$\bar{V}_t(N_t^j) = \max_{A_t^j, \bar{b}_t^j} \left\{ \begin{aligned} & E_t \Lambda_{t,t+1} \int_{\bar{b}_t^j/R_{t+1}^A} \left[\theta \bar{V}_{t+1} \left(\left(\omega - \frac{\bar{b}_t^j}{R_{t+1}^A} \right) R_{t+1}^A A_t^j \right) + (1-\theta) \left(\omega - \frac{\bar{b}_t^j}{R_{t+1}^A} \right) R_{t+1}^A A_t^j \right] dF_t(\omega) \\ & + \lambda_t^j \left\{ E_t \Lambda_{t,t+1} R_{t+1}^A A_t^j \left[\int_{\bar{b}_t^j/R_{t+1}^A} \omega dF_t(\omega) + \frac{\bar{b}_t^j}{R_{t+1}^A} \left[1 - F_t \left(\frac{\bar{b}_t^j}{R_{t+1}^A} \right) \right] \right] - \left(A_t^j - N_t^j \right) \right\} \\ & + \xi_t^j E_t \Lambda_{t,t+1} \int_{\bar{b}_t^j/R_{t+1}^A} \left\{ \theta \bar{V}_{t+1} \left(\left(\omega - \frac{\bar{b}_t^j}{R_{t+1}^A} \right) R_{t+1}^A A_t^j \right) + (1-\theta) R_{t+1}^A A_t^j \left(\omega - \frac{\bar{b}_t^j}{R_{t+1}^A} \right) \right\} dF_t(\omega) \\ & - \xi_t^j E_t \Lambda_{t,t+1} \int_{\bar{b}_t^j/R_{t+1}^A} \left\{ \theta \bar{V}_{t+1} \left(\left(\omega - \frac{\bar{b}_t^j}{R_{t+1}^A} \right) R_{t+1}^A A_t^j \right) + (1-\theta) R_{t+1}^A A_t^j \left(\omega - \frac{\bar{b}_t^j}{R_{t+1}^A} \right) \right\} d\tilde{F}_t(\omega) \end{aligned} \right\}$$

where λ_t^j and ξ_t^j are the Lagrange multipliers associated to the participation and IC constraints, respectively. The first order conditions with respect to A_t^j and \bar{b}_t^j are given by

$$\begin{aligned} 0 & = E_t \Lambda_{t,t+1} R_{t+1}^A \int_{\bar{\omega}_{t+1}^j} \left[\theta \bar{V}'_{t+1}(N_{t+1}^j) + 1 - \theta \right] \left(\omega - \bar{\omega}_{t+1}^j \right) dF_t(\omega) \\ & + \lambda_t^j \left\{ E_t \Lambda_{t,t+1} R_{t+1}^A \left[\int_{\bar{\omega}_{t+1}^j} \omega dF_t(\omega) + \bar{\omega}_{t+1}^j \left[1 - F_t(\bar{\omega}_{t+1}^j) \right] \right] - 1 \right\} \\ & + \xi_t^j E_t \Lambda_{t,t+1} R_{t+1}^A \int_{\bar{\omega}_{t+1}^j} \left\{ \theta \bar{V}'_{t+1}(N_{t+1}^j) + 1 - \theta \right\} \left(\omega - \bar{\omega}_{t+1}^j \right) dF_t(\omega) \\ & - \xi_t^j E_t \Lambda_{t,t+1} R_{t+1}^A \int_{\bar{\omega}_{t+1}^j} \left\{ \theta \bar{V}'_{t+1}(N_{t+1}^j) + 1 - \theta \right\} \left(\omega - \bar{\omega}_{t+1}^j \right) d\tilde{F}_t(\omega), \end{aligned}$$

$$\begin{aligned}
0 &= -E_t \Lambda_{t,t+1} \int_{\bar{\omega}_{t+1}^j} \left[\theta \bar{V}'_{t+1} \left(N_{t+1}^j \right) + (1 - \theta) \right] dF_t(\omega) - E_t \Lambda_{t,t+1} \theta \frac{\bar{V}_{t+1}(0)}{R_{t+1}^A A_t^j} f_t \left(\bar{\omega}_{t+1}^j \right) \\
&\quad + \lambda_t^j E_t \Lambda_{t,t+1} \left[1 - F_t \left(\bar{\omega}_{t+1}^j \right) \right] \\
&\quad - \xi_t^j E_t \Lambda_{t,t+1} \int_{\bar{\omega}_{t+1}^j} \left\{ \theta \bar{V}'_{t+1} \left(N_{t+1}^j \right) + (1 - \theta) \right\} dF_t(\omega) - \xi_t^j E_t \Lambda_{t,t+1} \theta \frac{\bar{V}_{t+1}(0)}{R_{t+1}^A A_t^j} f_t \left(\bar{\omega}_{t+1}^j \right) \\
&\quad + \xi_t^j E_t \Lambda_{t,t+1} \int_{\bar{\omega}_{t+1}^j} \left\{ \theta \bar{V}'_{t+1} \left(N_{t+1}^j \right) + (1 - \theta) \right\} d\tilde{F}_t(\omega) + \xi_t^j E_t \Lambda_{t,t+1} \theta \frac{\bar{V}_{t+1}(0)}{R_{t+1}^A A_t^j} \tilde{f}_t \left(\bar{\omega}_{t+1}^j \right),
\end{aligned}$$

respectively, where we have used $\bar{b}_t^j / R_{t+1}^A = \bar{\omega}_{t+1}^j$. We also have the envelope condition

$$\bar{V}_t' \left(N_t^j \right) = \lambda_t^j.$$

At this point, we guess that in equilibrium $\bar{V}_t(N_t^j) = \lambda_t^j N_t^j$, and that the multipliers λ_t^j and ξ_t^j are equalized across islands: $\lambda_t^j = \lambda_t$ and $\xi_t^j = \xi_t$ for all j . Using this, the IC constraint simplifies to

$$E_t \Lambda_{t,t+1} R_{t+1}^A \left\{ \theta \lambda_{t+1} + (1 - \theta) \right\} \left[\int_{\bar{\omega}_{t+1}^j} \left(\omega - \bar{\omega}_{t+1}^j \right) dF_t(\omega) - \int_{\bar{\omega}_{t+1}^j} \left(\omega - \bar{\omega}_{t+1}^j \right) d\tilde{F}_t(\omega) \right] \geq 0. \quad (24)$$

The first order conditions then become

$$\begin{aligned}
0 &= E_t \Lambda_{t,t+1} R_{t+1}^A \left[\theta \lambda_{t+1} + 1 - \theta \right] \int_{\bar{\omega}_{t+1}^j} \left(\omega - \bar{\omega}_{t+1}^j \right) dF_t(\omega) \\
&\quad + \lambda_t \left\{ E_t \Lambda_{t,t+1} R_{t+1}^A \left[\int_{\bar{\omega}_{t+1}^j} \omega dF_t(\omega) + \bar{\omega}_{t+1}^j \left[1 - F_t \left(\bar{\omega}_{t+1}^j \right) \right] \right] - 1 \right\},
\end{aligned} \quad (25)$$

$$\begin{aligned}
0 &= \lambda_t E_t \Lambda_{t,t+1} \left[1 - F_t \left(\bar{\omega}_{t+1}^j \right) \right] - E_t \Lambda_{t,t+1} \left[\theta \lambda_{t+1} + 1 - \theta \right] \left[1 - F_t \left(\bar{\omega}_{t+1}^j \right) \right] \\
&\quad + \xi_t E_t \Lambda_{t,t+1} \left\{ \theta \lambda_{t+1} + 1 - \theta \right\} \left[F_t \left(\bar{\omega}_{t+1}^j \right) - \tilde{F}_t \left(\bar{\omega}_{t+1}^j \right) \right],
\end{aligned} \quad (26)$$

where in (25) we have used the fact that ξ_t^j times the left-hand side of (24) must be zero as required by the Kuhn-Tucker conditions, and in (26) we have used the fact that, according to our guess, $\bar{V}_{t+1}(0) = 0$. Solving for the Lagrange multipliers, we obtain

$$\lambda_t = \frac{E_t \Lambda_{t,t+1} R_{t+1}^A \left[\theta \lambda_{t+1} + 1 - \theta \right] \int_{\bar{\omega}_{t+1}^j} \left(\omega - \bar{\omega}_{t+1}^j \right) dF_t(\omega)}{1 - E_t \Lambda_{t,t+1} R_{t+1}^A \left[\int_{\bar{\omega}_{t+1}^j} \omega dF_t(\omega) + \bar{\omega}_{t+1}^j \left[1 - F_t \left(\bar{\omega}_{t+1}^j \right) \right] \right]}, \quad (27)$$

$$\xi_t = \frac{\lambda_t E_t \Lambda_{t,t+1} \left[1 - F_t \left(\bar{\omega}_{t+1}^j \right) \right] - E_t \Lambda_{t,t+1} \left[\theta \lambda_{t+1} + 1 - \theta \right] \left[1 - F_t \left(\bar{\omega}_{t+1}^j \right) \right]}{E_t \Lambda_{t,t+1} \left\{ \theta \lambda_{t+1} + 1 - \theta \right\} \left[\tilde{F}_t \left(\bar{\omega}_{t+1}^j \right) - F_t \left(\bar{\omega}_{t+1}^j \right) \right]}. \quad (28)$$

In the steady state, the Lagrange multipliers are

$$\begin{aligned}
\lambda &= \frac{\beta R^A (1 - \theta) \int_{\bar{\omega}^j} \left(\omega - \bar{\omega}^j \right) dF(\omega)}{1 - \beta R^A + (1 - \theta) \beta R^A \int_{\bar{\omega}^j} \left(\omega - \bar{\omega}^j \right) dF(\omega)}, \\
\xi &= \frac{(\lambda - 1) (1 - \theta) \left[1 - F \left(\bar{\omega}^j \right) \right]}{\theta \lambda + 1 - \theta \left[\tilde{F} \left(\bar{\omega}^j \right) - F \left(\bar{\omega}^j \right) \right]},
\end{aligned}$$

where we have used $\int (\omega - \bar{\omega}^j) dF(\omega) = 1 - \bar{\omega}^j$. Provided the parameter values are such that

$$0 < \beta R^A - 1 < (1 - \theta) \beta R^A \int_{\bar{\omega}^j} (\omega - \bar{\omega}^j) dF(\omega),$$

then $\lambda > 1$, which in turn implies $\xi > 0$. That is, both the participation and IC constraints hold in the steady state.³⁹ Provided aggregate shocks are sufficiently small, we will also have $\lambda_t > 1$ and $\xi_t > 0$ along the cycle. But if $\lambda_t > 1$, then our guess that $\bar{V}'_t(N_t^j) > 1$ is verified. Also, given that $\bar{\omega}_{t+1}^j = \bar{b}_t^j/R_{t+1}$, the ratio \bar{b}_t^j is then pinned down by the IC constraint (equation 24) holding with equality. Since we have guessed that the multiplier λ_t is equalized across islands, so are $\bar{b}_t^j = \bar{b}_t$ and $\bar{\omega}_{t+1}^j = \bar{\omega}_{t+1} = \bar{b}_t/R_{t+1}$. But if $\bar{\omega}_{t+1}$ is equalized, then from (27) and (28) our guess that λ_t and ξ_t are symmetric across islands is verified too.

The participation constraint (holding with equality) is given by

$$E_t \Lambda_{t,t+1} R_{t+1}^A A_t^j \left\{ \int_{\bar{\omega}_{t+1}}^{\bar{\omega}_{t+1}} \omega dF_t(\omega) + \bar{\omega}_{t+1} [1 - F_t(\bar{\omega}_{t+1})] \right\} = A_t^j - N_t^j.$$

Using the latter to solve for A_t^j , we obtain

$$A_t^j = \frac{1}{1 - E_t \Lambda_{t,t+1} R_{t+1}^A \{\bar{\omega}_{t+1} - \pi_{t+1}(\bar{\omega}_{t+1})\}} N_t^j \equiv \phi_t N_t^j,$$

where we have also used the definition of the put option value, $\pi_t(\bar{\omega}_{t+1}) = \int_{\bar{\omega}_{t+1}}^{\bar{\omega}_{t+1}} (\bar{\omega}_{t+1} - \omega) dF_t(\omega)$. Therefore, the leverage ratio $A_t^j/N_t^j = \phi_t$ is equalized across firms too. Finally, using $\bar{V}_{t+1}(N_{t+1}^j) = \lambda_{t+1} N_{t+1}^j$, $N_{t+1}^j = (\omega - \bar{\omega}_{t+1}) R_{t+1}^A A_t^j$ and $A_t^j = \phi_t N_t^j$, the value function $\bar{V}_t(N_t^j)$ can be expressed as

$$\bar{V}_t(N_t^j) = \phi_t N_t^j E_t \Lambda_{t,t+1} R_{t+1}^A [\theta \lambda_{t+1} + 1 - \theta] \int_{\bar{\omega}_{t+1}} (\omega - \bar{\omega}_{t+1}) dF_t(\omega),$$

which is consistent with our guess that $\bar{V}_t(N_t^j) = \lambda_t N_t^j$ only if

$$\begin{aligned} \lambda_t &= \phi_t E_t \Lambda_{t,t+1} R_{t+1}^A [\theta \lambda_{t+1} + 1 - \theta] \int_{\bar{\omega}_{t+1}} (\omega - \bar{\omega}_{t+1}) dF_t(\omega) \\ &= \frac{E_t \Lambda_{t,t+1} R_{t+1}^A [\theta \lambda_{t+1} + 1 - \theta] \{1 - \bar{\omega}_{t+1} + \pi_t(\bar{\omega}_{t+1})\}}{1 - E_t \Lambda_{t,t+1} R_{t+1}^A \{\bar{\omega}_{t+1} - \pi_t(\bar{\omega}_{t+1})\}}. \end{aligned}$$

But the latter corresponds exactly with (27) without j subscripts, once we use the definition of $\pi_t(\bar{\omega}_{t+1})$. Our guess is therefore verified.

³⁹Our calibration in Table 3 implies $\lambda = 2.5528$ and $\xi = 7.6371$.

C. Model summary and comparison to standard RBC model

Our model can be reduced to the following 11-equation system,

$$\frac{v'(L_t)}{u'(C_t)} = (1 - \alpha) \frac{Y_t}{L_t}, \quad (\text{S1})$$

$$Y_t = Z_t L_t^{1-\alpha} K_t^\alpha, \quad (\text{S2})$$

$$K_{t+1} = I_t + (1 - \delta) K_t, \quad (\text{S3})$$

$$Y_t = C_t + I_t \quad (\text{S4})$$

$$R_t^A = (1 - \delta) + \alpha \frac{Y_t}{K_t}, \quad (\text{S5})$$

$$1 = \beta E_t \left\{ \frac{u'(C_{t+1})}{u'(C_t)} R_{t+1}^A [\bar{\omega}_{t+1} - \pi(\bar{\omega}_{t+1}; \sigma_t)] \frac{\phi_t}{\phi_t - 1} \right\}, \quad (\text{S6})$$

$$1 - \int \omega d\tilde{F}_t(\omega) = E_t \left\{ \frac{u'(C_{t+1}) R_{t+1}^A (\theta \lambda_{t+1} + 1 - \theta)}{E_t u'(C_{t+1}) R_{t+1}^A (\theta \lambda_{t+1} + 1 - \theta)} \left[\tilde{\pi}_t \left(\frac{\bar{b}_t}{R_{t+1}^A} \right) - \pi_t \left(\frac{\bar{b}_t}{R_{t+1}^A} \right) \right] \right\}, \quad (\text{S7})$$

$$K_{t+1} = \phi_t N_t, \quad (\text{S8})$$

$$\bar{\omega}_t = \bar{b}_{t-1} / R_t^A, \quad (\text{S9})$$

$$N_t = \theta R_t^A [1 - \bar{\omega}_t + \pi_{t-1}(\bar{\omega}_t)] K_{t-1} + \{1 - \theta [1 - F_{t-1}(\bar{\omega}_t)]\} \tau K_{t-1}, \quad (\text{S10})$$

$$\lambda_t = \frac{E_t \beta u'(C_{t+1}) R_{t+1}^A [\theta \lambda_{t+1} + 1 - \theta] \{1 - \bar{\omega}_{t+1} + \pi_t(\bar{\omega}_{t+1})\}}{u'(C_t) - E_t \beta u'(C_{t+1}) R_{t+1}^A \{\bar{\omega}_{t+1} - \pi_t(\bar{\omega}_{t+1})\}}, \quad (\text{S11})$$

which jointly determine the dynamics of 11 endogenous variables: $C_t, L_t, K_t, I_t, Y_t, R_t^A, N_t, \bar{\omega}_t, \bar{b}_t, \lambda_t, \phi_t$.

The standard RBC model is given by equations (S1) to (S5), *plus* the following investment Euler equation,

$$1 = \beta E_t \left\{ \frac{u'(C_{t+1})}{u'(C_t)} R_{t+1}^A \right\}, \quad (\text{S6}')$$

which jointly determine the path of 6 endogenous variables: $C_t, L_t, K_t, I_t, Y_t, R_t^A$.

Tables and Figures

Table 1. Business cycle statistics, full sample, US data

| | Commercial banks | Savings institutions | Security broker/dealers | Finance companies | <i>Range</i> |
|--------------------------------|-----------------------|-------------------------|----------------------------|-----------------------|---------------|
| <i>Standard deviations (%)</i> | | | | | |
| Assets | 1.88 | 4.00 | 8.93 | 3.85 | 1.88 : 8.83 |
| Leverage | 2.84 | 8.16 | 10.21 | 4.71 | 2.84 : 10.21 |
| Equity | 2.55 | 8.13 | 8.08 | 3.64 | 2.55 : 8.08 |
| GDP: 1.48 | | | | | |
| <i>Correlations</i> | | | | | |
| Leverage - Assets | 0.48 *** (0.0000) | 0.25 *** (0.0008) | 0.65 *** (0.0000) | 0.65 *** (0.0000) | 0.25 : 0.48 |
| Leverage - Equity | -0.76 *** (0.0000) | -0.88 *** (0.0000) | -0.54 *** (0.0000) | -0.60 *** (0.0000) | -0.88 : -0.54 |
| Leverage - GDP | 0.18 ** (0.0188) | 0.12 (0.1055) | 0.26 *** (0.0006) | 0.32 *** (0.0000) | 0.12 : 0.32 |
| Assets - GDP | 0.63 *** (0.0000) | 0.71 *** (0.0000) | 0.42 *** (0.0000) | 0.52 *** (0.0000) | 0.42 : 0.71 |

Note: Leverage is total assets divided by equity capital (both in dollars). 'Assets' in the table refer to real total assets, which are total assets (in dollars) divided by the GDP deflator. All series are from the US Flow of Funds, except real GDP and the GDP deflator which are from the Bureau of Economic Analysis. The sample period is 1963:Q1-2011:Q3. See Data Appendix for details. Leverage, real total assets and real GDP have been logged and detrended with a band-pass filter that preserves cycles of 6 to 32 quarters (lag length $K = 12$). P-values of the test of no correlation against the alternative of non-zero correlation are reported in parenthesis. Asterisks denote statistical significance of non-zero correlation at the 1% (***) and 5% (**) confidence level.

Table 2. Business cycle statistics, 1984:Q1-2011:Q3, US data

| | Commercial banks | Savings institutions | Security broker/dealers | Finance companies | <i>Range</i> |
|--------------------------------|-----------------------|-------------------------|----------------------------|-----------------------|---------------|
| <i>Standard deviations (%)</i> | | | | | |
| Assets | 1.30 | 4.59 | 7.57 | 3.05 | 1.30 : 7.57 |
| Leverage | 3.12 | 8.61 | 7.62 | 5.34 | 3.12 : 8.61 |
| Equity | 3.12 | 8.35 | 5.27 | 4.58 | 3.12 : 8.35 |
| GDP: 1.03 | | | | | |
| <i>Correlations</i> | | | | | |
| Leverage - Assets | 0.21 (0.0518) | 0.32 *** (0.0023) | 0.76 *** (0.0000) | 0.52 *** (0.0000) | 0.21 : 0.76 |
| Leverage - Equity | -0.91 *** (0.0000) | -0.85 *** (0.0000) | -0.35 *** (0.0008) | -0.82 *** (0.0000) | -0.91 : -0.35 |
| Leverage - GDP | -0.06 (0.5942) | 0.34 *** (0.0014) | 0.22 ** (0.0444) | 0.24 ** (0.0252) | -0.06 : 0.34 |
| Assets - GDP | 0.46 *** (0.0000) | 0.73 *** (0.0000) | 0.47 *** (0.0000) | 0.41 *** (0.0001) | 0.41 : 0.73 |

Note: Leverage is total assets divided by equity capital (both in dollars). 'Total assets' in the table refer to real total assets, which are total assets (in dollars) divided by the GDP deflator. All series are from the US Flow of Funds, except real GDP and the GDP deflator which are from the Bureau of Economic Analysis. The sample period is 1984:Q1-2011:Q3. See Data Appendix for details. Leverage, real total assets and real GDP have been logged and detrended with a band-pass filter that preserves cycles of 6 to 32 quarters (lag length $K = 12$). P-values of the test of no correlation against the alternative of non-zero correlation are reported in parenthesis. Asterisks denote statistical significance of non-zero correlation at the 1% (***) and 5% (**) confidence level.

Table 3. Model parameters

| Parameter | Value | Description | Source/Target |
|-------------------------|--------|--|-------------------------------------|
| RBC parameters | | | |
| β | 0.99 | discount factor | $R^4 = 1.04$ |
| α | 0.36 | capital share | $WL/Y = 0.64$ |
| δ | 0.025 | depreciation rate | $I/K = 0.025$ |
| φ | 1 | inverse labor supply elasticity | macro literature |
| \bar{Z} | 0.5080 | steady-state TFP | $Y = 1$ |
| ρ_z | 0.9297 | autocorrelation TFP | FRB San Francisco-CSIP TFP series |
| σ_z | 0.0067 | standard deviation TFP | FRB San Francisco-CSIP TFP series |
| Non-standard parameters | | | |
| σ | 0.0373 | steady-state island-specific volatility | average leverage ($\phi = 18.3$) |
| η | 1.2691 | variance substandard technology | $(\bar{R}/R)^4 - 1 = 0.25\%$ |
| ψ | 0.001 | mean substandard technology | volatility of equity 5.7% |
| τ | 0.0207 | equity injections new banks | $I/Y = 0.2$ |
| θ | 0.75 | continuation prob. banks | annual dividends ($\tau > 0$), |
| ρ_σ | 0.9457 | autocorr. island-specific volatility | NBER-CES manufacturing industry TFP |
| σ_σ | 0.0465 | standard dev. island-specific volatility | NBER-CES manufacturing industry TFP |

Table 4. Business cycle statistics: data and model

| | Data | | Model | | Sensitivity analysis | |
|--------------------------------|---------------|-------------|-------|------------|----------------------|----------------|
| | 1984-2011 | Both shocks | TFP | Volatility | $\phi = 10.6$ | $\theta = 0.5$ |
| <i>Standard deviations (%)</i> | | | | | | |
| GDP | 1.03 | 1.08 | 1.00 | 0.38 | 1.17 | 1.31 |
| Assets | 1.30 : 7.57 | 0.58 | 0.36 | 0.45 | 0.78 | 1.07 |
| Leverage | 3.12 : 8.61 | 6.09 | 0.21 | 6.09 | 5.39 | 4.16 |
| Equity | 3.12 : 8.35 | 5.83 | 0.32 | 5.82 | 5.00 | 3.71 |
| <i>Correlations</i> | | | | | | |
| Leverage - Assets | 0.21 : 0.76 | 0.49 | 0.49 | 0.63 | 0.55 | 0.53 |
| Leverage - Equity | -0.91 : -0.35 | -0.996 | -0.12 | -0.998 | -0.991 | -0.97 |
| Leverage - GDP | -0.06 : 0.34 | 0.31 | -0.10 | 0.87 | 0.45 | 0.59 |
| Assets - GDP | 0.41 : 0.73 | 0.41 | 0.41 | 0.59 | 0.43 | 0.47 |

Note: Model statistics are obtained by simulating the model for 55,000 periods and discarding the first 5,000 observations. The model is solved using a second-order perturbation method. Both data and model-simulated series have been logged and detrended with a band-pass filter that preserves cycles of 6 to 32 quarters (lag length $K = 12$).

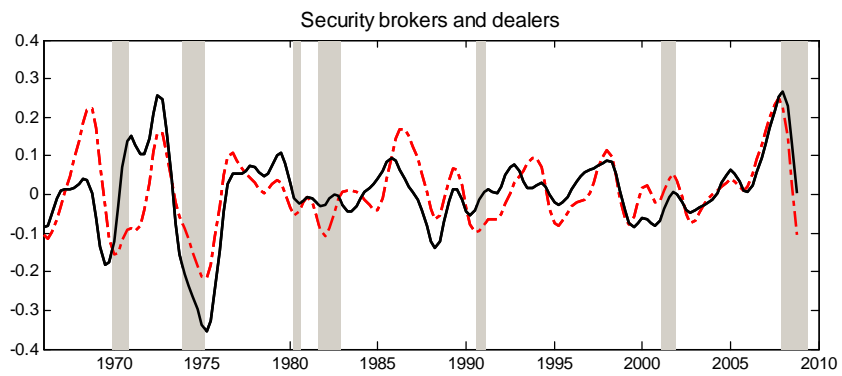
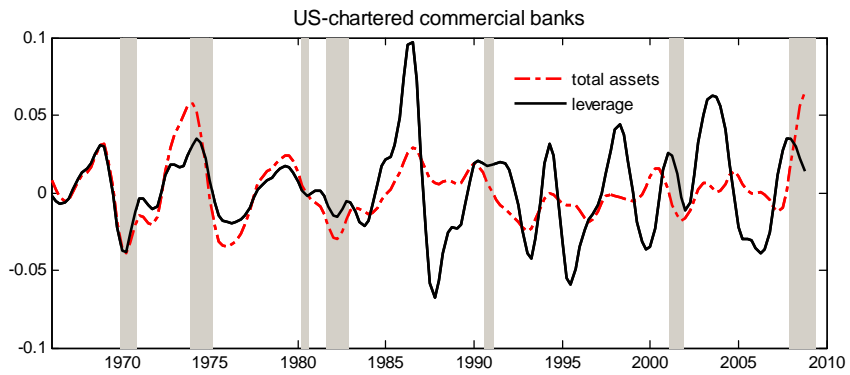


Figure 1: Cyclical components of total assets and leverage

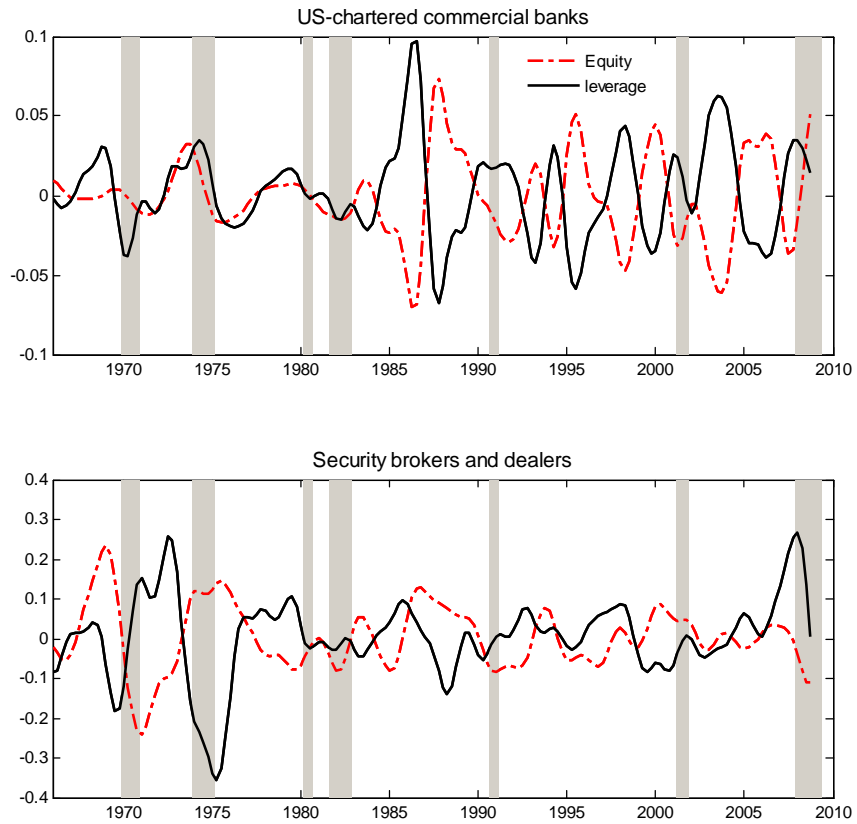


Figure 2: Cyclical components of equity and leverage

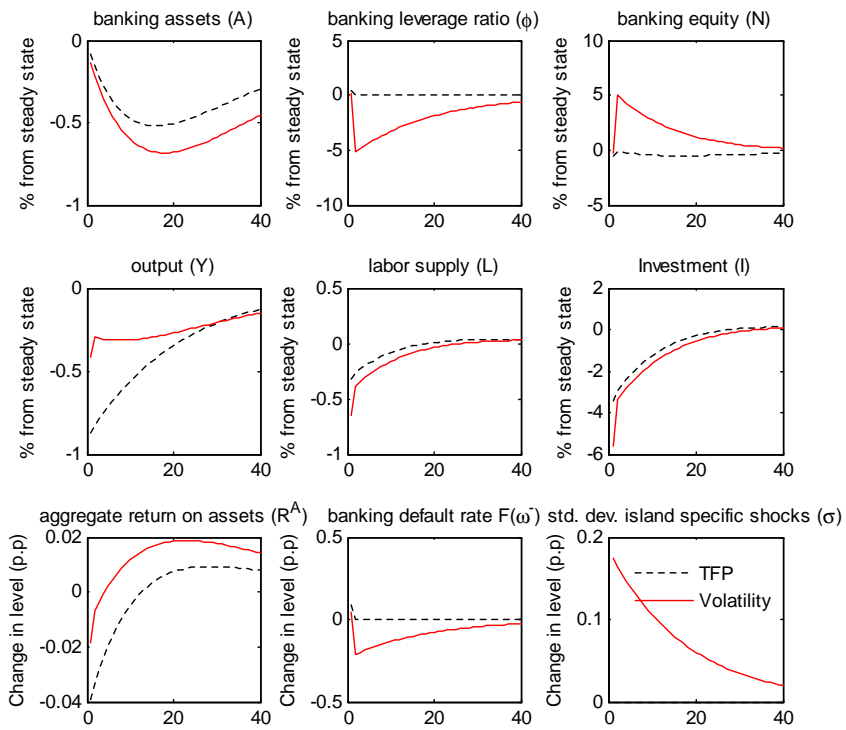


Figure 3: Impulse responses: TFP and volatility shock

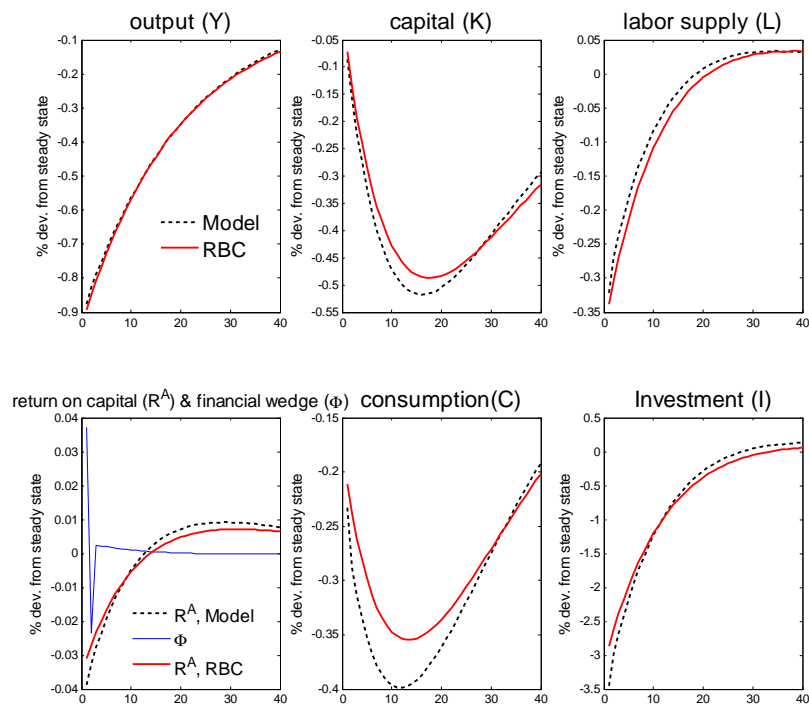


Figure 4: Impulse-responses to TFP shock: model vs. standard RBC

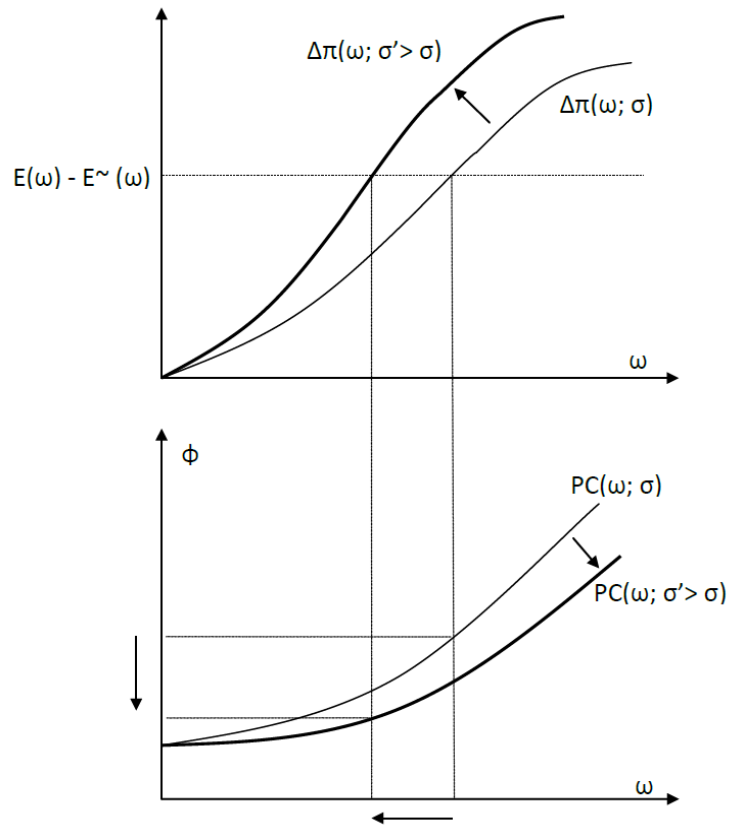


Figure 5: The volatility-leverage channel

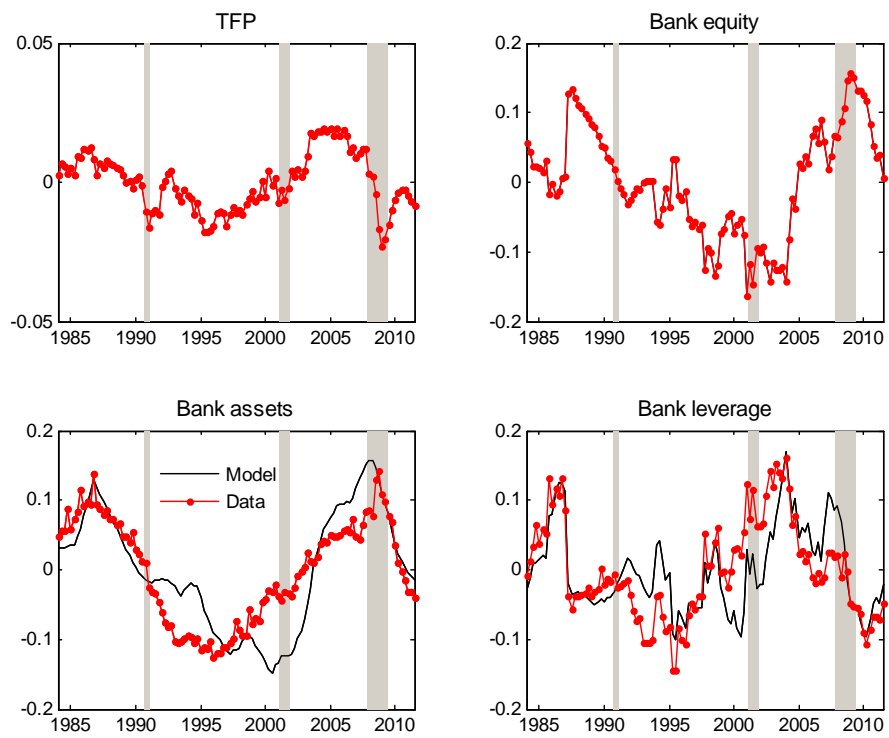


Figure 6: Counterfactual evolution of assets and leverage (US - chartered commercial banks)

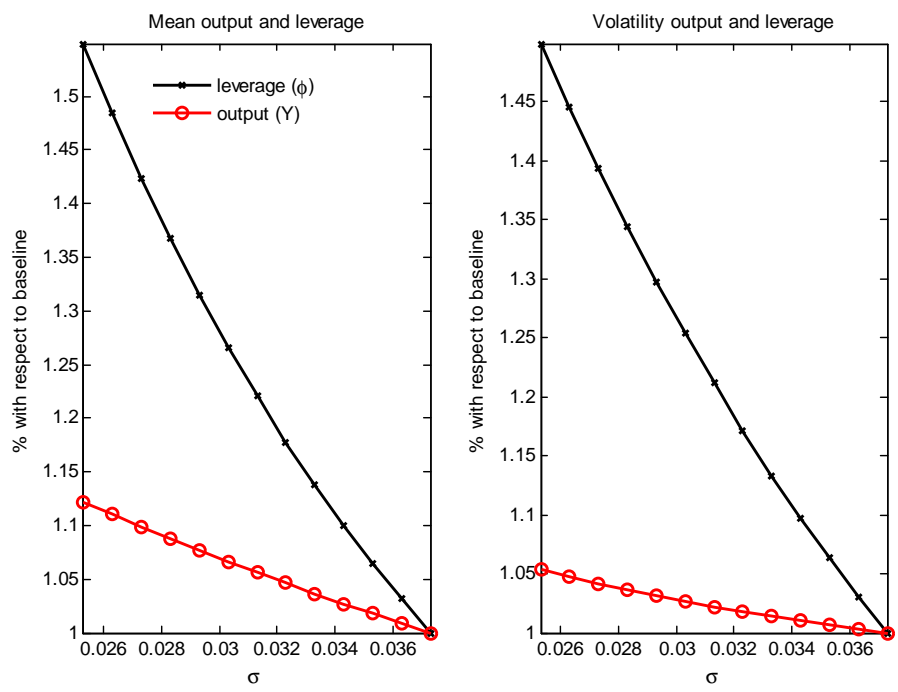


Figure 7: The effect of changes in average cross-sectional volatility