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Leveraged property cycles

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Abstract

This paper studies the effects of imperfect risk-sharing between lenders and borrowers on commercial property prices and leverage. The key friction is that agents use different discount rates to evaluate future flows. Eliminating this pecuniary externality generates large reductions in the volatility of real estate prices and credit. Therefore, policies that enhance risk-sharing between lenders and borrowers reduce the magnitude of boom-bust cycles in real estate prices. We also introduce health shocks to study the effect of the COVID-19 crisis on the commercial property market.

- **JEL:** E32, E44, G10, E23.
- **Keywords:** Leverage Cycle; Pecuniary Externalities; Asset Pricing; Incomplete Markets.

Executive Summary

The new consensus among central banks is that booms in property prices are dangerous phenomena, especially when they are fuelled by credit. Corrective policy measures are therefore necessary because markets can, at times, fail to allocate resources efficiently.

From a policy perspective, the key question is therefore to identify the distortions at the origin of this excessive volatility. This paper explores one potential cause of excess volatility and discusses its policy implications.

According to our theory, the root cause of the problem is that debtors and financiers have conflicting interests. The reason is that borrowing and lending decisions are made by different types of agents, who often have diverging objectives. This coordination problem in turn creates an externality that justifies policy intervention.

This theory implies that the high volatility of property prices is caused by an inefficient allocation of risk. Too much risk is borne by debtors, whereas financiers are excessively insured against business cycle fluctuations. Consequently, the optimal policy consists of correcting this misallocation of risk.

We find that the optimal policy is prudential in the sense that borrowing should be curbed during booms and stimulated during recessions. This paper therefore provides a novel rationale for introducing a policy instrument that attenuates fluctuations in real estate prices and leverage.

This externality amplifies real estate cycles by generating excessive fluctuations in property demand. In good times, the source of the misallocation is that the economy as whole saves too much. In other words, agents' desire to build precautionary buffers to protect themselves against future shocks is excessive. Since real estate is a good hedge, the amounts saved during periods of expansion are channelled to the property market. Consequently, this precautionary behavior exacerbates volatility by amplifying the demand for real estate in boom periods.

Because it achieves a better allocation of risk across agents, the policy attenuates this precautionary motive. The measure works by slowing the accumulation of real estate, and hence the increase in leverage, in boom periods. During periods of recession, the policy attenuates the decline in property prices by providing support to borrowers. Over the business cycle, property demand becomes more stable, an effect which in turn dampens the volatility of the financial cycle.

1 Introduction

In the United States, the commercial real estate market offers one of the most striking examples of a boom in asset prices that ended in a bust. From 2003 to 2007, commercial property prices increased by around 70%. Equally striking is the magnitude of the bust, which in 2010 brought commercial property prices back to a level last seen in the late 1990's. These large fluctuations in commercial real estate prices are difficult to explain by standard measures of fundamental value, such as construction costs for instance. Indeed, commercial real estate prices rise above construction costs during booms. As illustrated in Figure A.1, these large procyclical deviations in turn give rise to persistent fluctuations in the price to construction cost ratio. Besides its damaging effects on the global economy, The COVID-19 crisis also triggered a decline in this ratio that was particularly abrupt.

Deviations between property prices and fundamentals are closely monitored because they are typically interpreted as a sign of market malfunctioning (e.g. Glaeser and Gyourko, 2018). Indeed, one main lesson from the 2007-2009 global financial crisis is that booms in property prices are dangerous phenomena, especially when they are fuelled by credit (e.g. Jordà, Schularick, and Taylor, 2015).[1] This explains why the commercial property market came under particular scrutiny in recent years.[2] In Europe, corrective policy measures that were aimed at addressing potential imbalances were even explicitly recommended.[3]

If deviations between real estate prices and fundamentals are considered excessive, the implicit assumption is that part of these fluctuations are inefficient. Identifying the type of frictions that could give rise to excessive fluctuations in real estate prices is therefore an important question. Indeed, if corrective policy measures are deemed necessary, they need to be tailored to the root cause of the problem.

In this paper, we ask whether imperfect risk-sharing between lenders and borrowers could contribute to the volatility of real estate prices. Lending and borrowing decisions are made by different types of agents. Consequently, one potential source of inefficiency is that agents in the economy use different discount rates to form expectations about the future.

[1] Lowe and Borio (2002) already alerted on the risk posed by rapid credit growth combined with large increases in asset prices.

[2] High valuations in the commercial real estate markets were mentioned by J. Yellen in 2018. See also Federal Reserve Board (2018). See also "Monetary Policy Report", Feb. 2018. In Europe, concerns about commercial property prices were also voiced. See ECB Financial Stability Review, May 2018.

[3] In 2018, the European Systemic Risk Board (ESRB) advised to address risks and vulnerabilities in the European commercial real estate market. See ESRB (2018). See "Report on vulnerabilities in the EU commercial real estate sector", November 2018.

Under complete markets, differences in discount rates or stochastic discount factors (SDF) across agents are irrelevant. When markets are incomplete, however, this misalignment creates a distortion, which is typically referred to in the literature as distributive externality (e.g. Dávila and Korinek, 2018). This misalignment in turn implies that the allocation of resources is inefficient, since a social planner could improve welfare by eliminating the distortion.

In this study, this question is analyzed in a production economy composed of two types of agents: risk averse households and risk neutral bankers. We start by developing a model that is able to match a set of moments of interest, which includes the high volatility of commercial property prices observed in the data, and we then analyze the policy implications of this mechanism. This second step is achieved by comparing the fluctuations in macroeconomic aggregates and asset prices observed in the laissez-faire economy, which corresponds to the incomplete market model, with those obtained under the optimal policy. The optimal policy corresponds to the case in which the externality is corrected by introducing a tax in the decentralized equilibrium.

Our first main result is that distributive externalities can have sizeable asset pricing and business cycle implications. Under incomplete markets, our laissez-faire economy replicates the volatility of real estate prices and construction costs that are observed in the data. It also generates large and persistent fluctuations in the price to construction cost ratio. We find that removing the distributive externality leads to a large reduction in the volatility of commercial real estate prices. Deviations between prices and construction costs also essentially disappear if the distortion caused by imperfect risk-sharing can be eliminated.

The main implication of this result is therefore that boom-bust cycles in property prices can be amplified by imperfect risk-sharing. The intuition is that agents use the real estate market as a hedge against business cycle fluctuations. From the perspective of a risk averse household, consumption is too volatile and the amount of risk that needs to be borne is excessive under a laissez-faire economy. Since buildings are good stores of value, the key is that agents use the property market to protect themselves against adverse shocks. In good times, this is achieved by increasing investment in commercial structures. Accumulating large stocks of real estate then allows agents to sharply reduce new constructions in recessions, when marginal utility of consumption is high. The large fluctuations in investment that are induced by this consumption smoothing motive in turn amplify the volatility of property prices and leverage.

Completing the markets by removing the distributive externality lowers the volatility of real estate prices by reducing the amount of risk borne by households. The key is that the insurance provided by the policy reduces households' precautionary saving motives. This lower precautionary motive attenuates the procyclical fluctuations in the demand for properties, an effect which in turn reduces the volatility of prices. The policy is prudential in the sense that it is optimal to tax borrowers in boom times and provides subsidy in periods of stress. Our results therefore provide a novel rationale for introducing a policy instrument that attenuates the volatility of property prices and leverage.

Our second main result is that distributive externalities can introduce a wedge between commercial real estate prices and construction costs. We illustrate this point in a modified version of Tobin's Q theory (e.g. Hayashi, 1982). Commercial real estate prices can be divided into three components. Firstly, prices are determined by construction costs. Relative to the textbook model, this term appears because new commercial structures are produced by a construction sector that uses capital, labor and land as inputs. Secondly, adjusting the stock of commercial real estate is costly. Consequently, prices also depend on the investment to real estate stock ratio, as in the textbook case. The third component captures the effect of the borrowing constraint faced by developers. It is determined by the discounted sum of future interest rates and debt repayments that developers expect to disburse over the duration of a loan. We refer to this novel term in this otherwise standard Q theory as the distributive wedge.

Depending on the source of the shock, we find that the distributive wedge can cause substantial deviations between commercial real estate prices and construction costs. The distributive wedge plays a particularly important role if the economy is hit by what we refer to as health shocks, which are meant to capture the effect of the COVID crisis on the commercial real estate market. Following the seminal contribution of Grossman (1972), we assume that households not only derive utility from consumption and leisure but also from their health stock. Agents control the evolution of their health stock by choosing the amount of time dedicated to health-related activities. A health shock forces agents to increase the time allocated to their health, which reduces hours worked in the final good and construction sectors, and hence their labor income.

Although prices and wages are fully flexible, our model also generates monetary non-neutralities. In particular, a monetary policy expansion leads to an increase in commercial property prices and stimulates new construction. Whereas the real effects of monetary policy shocks remain small in magnitude, monetary policy generates fluctuations in leverage that can be sizeable and persistent. Introducing monetary policy shocks therefore helps to explain the higher amplitude and longer duration of financial cycles that are documented in empirical studies (e.g. Hiebert et al., 2018).

Our paper is firstly related to a body of literature on the pecuniary externalities created by financial constraints. In Dávila and Korinek (2018), distributive externalities arise when intertemporal rates of marginal substitution (IRMS) or SDF differ across agents. Since they are zero sum across agents, distributive externalities are eliminated if markets can be completed. Relative to their article, we study this externality from a quantitative perspective within a model that reproduces the volatility of property prices that is observed in the data. Danthine and Donaldson (2003) and Carceles-Poveda (2003) are probably the first papers that studied the macroeconomic implications of misalignments in IRMS from a quantitative perspective.

Lorenzoni (2008) shows that pecuniary externalities can explain why credit booms can be inefficient from an ex ante perspective. Building on Mendoza (2002), Bianchi (2011) studies the role of credit externalities in generating overborrowing in a two-sector dynamic stochastic general equilibrium (DSGE) model.

This paper is also related to a recent strand of literature that studies the macroeconomic and asset pricing implications of imperfect risk-sharing. Carceles-Poveda (2009) finds that the presence of market incompleteness has a negligible impact on the behavior of asset returns. Krueger and Lustig (2010) derive the conditions under which idiosyncratic risk has no impact on the price of aggregate risk. Berger, Bocola, and Dovis (2019) study the role of imperfect risk-sharing over the business cycle and find that the risk-sharing wedge played an important role during the global financial crisis. Di Tella (2017) studies the link between leverage and aggregate risk-sharing. Without uncertainty shocks, the balance sheet channel disappears if agents can write contracts on the aggregate state of the economy. Krishnamurthy (2003) highlights the importance of incomplete hedging in amplifying shocks in models with collateral constraints.

Tuzel (2010) is probably one of the first papers that studies the asset pricing implications of commercial real estate in a production economy. Commercial real estate represents a large share of the capital stock held by firms. In this article, she shows that the composition of firms' capital stock has significant asset pricing implications if investment decisions are costly to reverse.

Our approach is also related to a strand of literature that studies the role of real estate in amplifying business cycle fluctuations. Following the seminal contribution of Kiyotaki and Moore (1997), we study the link between asset prices and credit in a model with different types of agents. Households and bankers are different but we do not consider the case of within-agents heterogeneity. Iacoviello (2005) and Iacoviello and Neri (2010) study house prices in a model in which loans are collateralized by the future value of the housing stock that is expected by agents. Following a related approach, Liu, Wang, and Zha (2013) develop and estimate a model in which the value of land affect firms' borrowing constraints. Kiyotaki, Michaelides, and Nikolov (2011) study commercial and residential real estate in a model in which households face uninsurable earning shocks, borrowing constraints, and where the role of land is explicitly modelled.

Relative to this latter strand of the literature, we study financial frictions that are akin to earning-based constraints. Our approach is motivated by the empirical findings of Lian and Ma (2020). Those authors document the central role of cash flows for corporate borrowing in the US. Indeed, 80% of corporate debt is based on the value of cash flows, whereas only 20% is based on the liquidation value of physical assets. Using data for Spain and Peru, Ivashina, Laeven, and Moral-Benito (2020) find that the effects of the financial crisis on banks' balance sheets were essentially driven by cash flow-based lending.

Gourio (2013) studies the macroeconomic implications of leverage from a quantitative perspective. Realistic asset pricing implications are obtained by combining disaster risk (e.g. Gourio, 2012) with Epstein-Zin-Weil preferences (Epstein and Zin, 1989; Weil, 1989; Weil, 1990). Debt amplifies macroeconomic fluctuations and reduces welfare.

In Kydland, Rupert, and Šustek (2016) and Garriga, Kydland, and Šustek (2017), loans are used to finance new construction in the current period. Long-term debt is modelled by introducing a law of motion that relates the change in the stock of debt to new loans that are extended within the period.

Reproducing the volatility of real estate prices remains an outstanding puzzle, especially for models with rational expectations (Piazzesi and Schneider, 2016). Considering the case of commercial real estate adds to the challenge, since prices in this market segment are even more volatile than their residential counterparts. In this paper, the model's ability to jointly reproduce the volatility of prices and investment is due to the combination of two factors. Firstly, habit formation reduces the elasticity of intertemporal substitution of the representative borrower. Secondly, consumption smoothing is altered by introducing adjustment costs. As initially shown by Jermann (1998), combining habit formation with capital adjustment costs helps to reproduce realistic risk premiums in a real business cycle model within which labor is fixed.

Jaccard (2011) introduces this mechanism into the model developed by Davis and Heathcote (2005). Relative to this latter study, a specification of habits in the composite of consumption, housing and leisure is combined with adjustment costs. This modification helps to generate sizeable risk premiums in a model in which labor supply is endogenously determined.

Relative to Jermann (1998), we use an autoregressive specification of habit formation. As anticipated by Cochrane (2006), slow-moving habits help to generate more stable risk-free rate variations within this class of models.[4] Moreover, the facts documented by Jordà, Knoll, Kuvshinov, Schularick, and Taylor (2019) suggest that safe short-term real rates are more volatile than often assumed in the asset pricing literature.

Our article is also related to a body of literature that analyzes the implications of housing supply restrictions for house price dynamics. Glaeser, Gyourko, and Saiz (2008) show that boom-bust cycles are more likely to occur in cities where housing supply is more inelastic. Van Nieuwerburgh and Weill (2010) find that the combination of housing supply restrictions and wage differences across regions can reproduce the dynamics as well as the dispersion in house prices observed in the United States. Stokey (2009) studies the asset pricing implications of housing adjustment costs.

Since the financial crisis, several major improvements were proposed to account for the puzzling dynamics of house prices that have been observed in recent years. Landvoigt, Piazzesi, and Schneider (2015) highlight the importance of credit for the cross section of capital gains. Justiniano, Primiceri, and Tambalotti (2019) emphasize the importance of supply side constraints. In particular, they show that a progressive relaxation of lending constraints can rationalize the increase in leverage and house prices that were observed before the financial crisis. Favilukis, Ludvigson, and Van Nieuwerburgh (2017) develop a quantitative general equilibrium asset pricing model of housing with heterogeneous agents and production. One main finding of their study is that the boom in house prices cannot be explained by interest rates but rather by a decline in the housing risk premium. Garriga, Manuelli, and Peralta-Alva (2019) include shocks to expectations about housing finance to explain the apparent disconnect between house prices and rents. Garriga and Hedlund (2020) study house price dynamics and credit by introducing search frictions to capture the role played by illiquidity during the crisis. Their framework can also be used to study the effects of a decline in mortgage rates in crisis times.

Gorton and Ordoñez (2020) show that not all credit booms are alike. Whereas credit booms sometime end in a financial crisis, others do not. Bad credit booms occur when the dynamics of credit is disconnected from that of productivity growth. Bad booms can be explained by a lack of information production during the boom phase. Asriyan, Laeven and Martin (2021) develop a theory in which credit booms driven by high collateral values end in crises and slow recoveries.[5]

[4] See for instance Jaccard (2014).

[5] A comprehensive review of the literature on housing in macroeconomic models is provided by Piazzesi

Except for a few exceptions (e.g. Gyourko, 2009; Van Nieuwerburgh, 2017), the recent dynamics of commercial property prices attracted very little attention. Despite the unprecedented magnitude of this boom-bust cycle, most of the literature focussed instead on the residential market (e.g. Mian and Sufi, 2009; 2011). Recent evidence however suggest that pre-crisis exposures to non-household borrowers, such as developers of commercial properties, could have played a major role (e.g. Antoniades, 2019). Moreover, commercial real estate represents a significant fraction of investment. Indeed, currently, non-residential construction accounts for around 47% of private construction.[6]

Commercial real estate is also a major asset class for the non-financial and non-corporate sectors, the cumulated holdings of which exceeds those of households (e.g. Duca and Ling, 2019). The facts documented by Ghent, Torous, and Valkanov (2019) also demonstrate that commercial real estate plays a major role in the US economy. In the 1980's, together with residential real estate, commercial real estate represented the largest asset class relative to GDP. At the current juncture, the amount of commercial real estate outstanding is comparable to that of Treasuries. Overall, this suggests that the role of non-residential real estate, which mainly affects the supply side of the economy, is probably understudied.

Section 2 describes the decentralized equilibrium under incomplete markets. The incomplete market model is compared with the data and its ability to match a series of asset pricing and business cycle facts is evaluated. The model calibration is discussed in Section 3. The first main results are discussed in Section 4, which provides a quantitative assessment of the distributive externality. The complete market allocation is discussed in Section 5 and compared with that obtained under imperfect risk-sharing. Section 6 concludes.

2 Real Estate Cycles under Incomplete Markets

The economy is populated by two types of agents: households and bankers, which in terms of notation, are referred to as agents A and B, respectively. All major economic decisions are made by households. These agents decide how much to consume, invest, and how to allocate their time endowment between hours worked in the different sectors, leisure and health-related activities. Households' physical health is modelled as a stock that affects utility. This stock depreciates and its evolution depends on the fraction of time that households spend on health-related activities.

and Schneider (2016). See also Davis and Van Nieuwerburgh (2014).

[6] Source: U.S. Bureau of Census. https://www.census.gov/construction/c30/c30index.html.

For expositional purposes, we assume that households own (i) the non-financial corporate sector, (ii) the construction sector, and (iii) the real estate developers. Since decisions made by these firms are fully consistent with households' utility maximization, it would be equivalent to treating this part of the economy as a single consolidated sector. Financial frictions are introduced by assuming that real estate developers choose to finance some projects through leverage.

Credit is supplied by a banker who finances consumption using revenues from the bank's activity. Since developers are owned by households, the two types of agents are linked by the credit market. Markets are incomplete in the sense that the SDF of the two agents are not aligned.

2.1 Households and the Real Economy

Households maximize lifetime utility, which is given as follows:

$$U_{At} = E_0 \sum_{t=0}^{\infty} \widehat{\beta}_A^t \frac{(c_{At} S_t^{\kappa} (\psi + L_t^{\upsilon}) - x_t)^{1-\sigma}}{1-\sigma} \tag{1}$$

where E is the expectation operator, $\widehat{\beta}_A$ is the subjective discount factor, which is adjusted for growth (e.g. Kocherlakota, 1990)[7], and σ is the curvature coefficient. Agents firstly derive utility from consuming a market consumption good c_A and from enjoying leisure L. ψ and υ are two labor supply parameters that determine the steady state share of hours worked as well as the Frisch elasticity of labor supply, respectively. Households' health stock is denoted by S. The parameter κ determines the share of time spent on health-enhancing activities.

The habit stock, which is denoted by x, evolves according to the following law of motion:

$$\gamma x_{t+1} = m x_t + (1-m) c_{At} S_t^{\kappa} (\psi + L_t^{\upsilon}) \tag{2}$$

This preference specification is consistent with balanced growth and the deterministic trend at which the economy is growing is denoted by γ. The parameter m, where $1 \leq m \leq 0$, is a memory parameter that measures the weight of past decisions on the current habit stock. This autoregressive specification implies slow movements in the habit stock (e.g. Campbell and Cochrane, 1999). To restrict the number of degrees of freedom, the impact of current choices on the habit stock is given by $1-m$. Note that the habit stock x is

[7] The adjustment for growth implies that $\widehat{\beta}_A = \widetilde{\beta}_A \gamma^{1-\sigma}$, where γ is the deterministic growth rate of the economy.

equal to zero when m is set to 1. This particular case therefore corresponds to a model without habits. Lowering the parameter m below 1 increases the habit stock as well as the curvature of the utility function.

The time t budget constraint of the representative household is given as follows:

$$w_t (N_{Ft} + N_{Ct}) + r_t (k_{Ft} + k_{Ct}) + d_t t_{Ct} + prof_{Tt} = c_{At} + i_{Tt} \tag{3}$$

Households can choose to work for firms in the non-financial corporate and construction sectors and hours worked in these two sector are denoted by N_F and N_C, respectively. Since labor is perfectly mobile, the wage rate common to both sectors is denoted by w. Households allocate their time endowment, which is normalized to 1, between leisure L, hours worked in the corporate and construction sectors, and time spent taking care of their health, which we denote by N_S :

$$1 = L_t + N_{Ft} + N_{Ct} + N_{St} \tag{4}$$

The evolution of households' health stock is given by the following law of motion:

$$S_{t+1} = (1 - \delta_S) S_t + N_{St} - \log \chi_t \tag{5}$$

where δ_S is the rate at which the health stock depreciates.[8] Households can invest in their health by increasing the time spent on health-enhancing activities. This law of motion is also affected by an exogenous health shock, which we denote by χ_t. The exogenous shock follows an autoregressive process of order 1:

$$\log \chi_t = \rho_\chi \log \chi_{t-1} + \varepsilon_{\chi t}$$

where $\varepsilon_{\chi t}$ is the innovation, which is normally distributed with mean 0 and standard deviation $std(\varepsilon_{\chi t})$.

Households own the stock of capital, which is denoted by k_T. They then choose how to allocate capital between the two sectors. Capital allocated to firms and construction is denoted by k_F and k_C, respectively, where:

$$k_{Tt} = k_{Ft} + k_{Ct} \tag{6}$$

Since capital is perfectly mobile, the return on capital, which is denoted by r, is equalized across sectors. Households also receive a revenue from the construction and corporate

[8] With labor augmenting technological progress, hours worked are stationary. This therefore implies that the health stock is also not growing along the balanced-growth path.

sectors as well as from developers in the real estate sector. This dividend income is denoted by $prof_T$. There is an exogenous endowment of land, which is denoted by t_C, owned by households. For simplicity, we assume that the land endowment grows at the rate γ, which ensures the existence of a balance-growth path.[9] The rate at which land is rented to firms in the construction sector is denoted by d.

On the expenditure side, households divide their total income between consumption and investment, which is denoted by i_T. The accumulation of physical capital k_T is subject to adjustment costs and the law of motion takes the following form:

$$\gamma k_{Tt+1} = (1-\delta_K)k_{Tt} + \left[\frac{\theta_1^K}{1-\epsilon_K}\left(\frac{i_{Tt}}{k_{Tt}}\right)^{1-\epsilon_K} + \theta_2^K\right] k_t \tag{7}$$

This is the specification used in Jermann (1998) where δ_K is the rate at which capital depreciates, and where ϵ_K is the elasticity of Tobin's Q with respect to a change in the investment to capital ratio i_T/k_T.[10] The problem of households consists of choosing consumption, investment, hours worked in the two sectors, time spent on health-enhancing activities, the allocation of capital across sectors, the economy's capital stock, the evolution of their health stock and the habit stock such as to maximize lifetime utility, which is given by equation (1), subject to constraints (2), (3), (4), (5), (6) and (7).

Real Estate Developers

The business model of developers consists of accumulating commercial buildings that are then rented to the corporate sector. The stock of commercial real estate that they own is denoted by h, whereas z represents the rental rate. Their stock of commercial buildings in turn depends on new acquisitions. Each period, developers therefore need to decide on the quantity of new structures to purchase from a construction sector. Newly acquired commercial structures are denoted by y_C. The price at which developers purchase these structures is denoted by p_C. The price of new structures p_C can thus be interpreted as the construction cost.

[9] This assumption captures the idea that the quantity of land which is available for production may increase over time, depending on economic development. An increase in the quantity available for production could be due to a loosening of regulation or to other actions such as deforestation.

[10] The two parameters θ_1^K and θ_2^K are calibrated to ensure that capital adjustment costs do not affect the deterministic steady state of the economy. This is achieved by imposing the following restrictions:

$$\theta_1^K = (\gamma - (1-\delta_K))^{\epsilon_K} \text{ and } \theta_2^K = -(\gamma - (1-\delta_K))^{\frac{\epsilon_K}{1-\epsilon_K}}$$

The key is that developers resort to leverage to finance a fraction of these newly built structures. This implies that a fraction μ of the cost of purchasing new structures needs to be financed via credit. One simplification is that we do not endogenize the capital structure of developers. The parameter μ can also be interpreted as a reduced form for the advantage provided by debt over equity (e.g. Jermann and Quadrini, 2012; Gourio 2013).

The remaining fraction, i.e. $(1-\mu)p_C y_C$, is financed via retained earnings. This capital structure implies that profits in the real estate sector at time t, which are denoted by $prof_D$, are given as follows:

$$prof_{Dt} = z_t h_t - (1-\mu)p_{Ct}y_{Ct} - \varkappa \frac{b_t}{1+\pi_t} - (1-\varkappa)i_{Bt-1}\frac{b_t}{1+\pi_t} \tag{8}$$

Net operating profits are given by the rental income minus the cost of new structures financed via retained earnings, i.e. $zh-(1-\mu)p_C y_C$. The stock of debt due to bankers at the beginning of period t is denoted by b. Each period, developers reimburse a fraction $\varkappa$ of the debt owed to financiers. An interest rate, which is denoted by i_B, is then paid on the outstanding amount. Debt is a nominal asset and the real value of debt is obtained by deflating the current stock using the inflation rate, which we denote by π. The term $(1-\varkappa)\frac{b}{1+\pi}$ therefore stands for the amount of debt that remains due to bankers in period t. Since a fraction $\varkappa$ is reimbursed each period, debt is completely amortized after $1/\varkappa$ periods.

At the end of period t, the amount of debt accumulated by developers must be superior or equal to the the sum of two components. Firstly, as new acquisitions are in part financed via leverage, the increase in new debt in period t depends on the fraction of current expenses $\mu p_C y_C$ that requires external financing. Secondly, since this is a model with long-term debt, end of the period debt cannot be smaller than the amount at the beginning of the period minus the fraction reimbursed, i.e. $(1-\varkappa)\frac{b}{1+\pi}$. This therefore implies the following inequality constraint:

$$\gamma b_{t+1} \geq \mu p_{Ct} y_{Ct} + (1-\varkappa)\frac{b_t}{1+\pi_t} \tag{9}$$

To our knowledge, this specification was first introduced into the neoclassical growth model by Kydland, Rupert, and Šustek (2016) and Garriga, Kydland, and Šustek (2017).

An earnings-based constraint interpretation

In our context, it can also be interpreted as a debt-service-to-income limit. Indeed, combining equations (9) and (8), and after rearranging terms, we obtain that:

$$(1-\varkappa)i_{Bt-1}\frac{b_t}{1+\pi_t} \leq \gamma b_{t+1} - \frac{b_t}{1+\pi_t} + z_t h_t - p_{Ct}y_{Ct} - prof_{Dt} \tag{10}$$

This constraint therefore states that the net interest expense on the outstanding amount cannot exceed a certain limit. This limit, which is given by the right-hand side, firstly depends on the amount of new liquidity received in period t, i.e. $\gamma b_{t+1} - \frac{b_t}{1+\pi_t}$. The second term, i.e. $z_t h_t - p_{Ct}y_{Ct}$, represents the difference between the rental income and the cost of inputs. Finally, since developers are owned by households, dividends distributed to shareholders $prof_{Dt}$ also reduce developers' ability to service debt.

Real estate adjustment costs

The accumulation of commercial buildings is subject to adjustment costs and evolves according to a law of motion which is similar to that of the capital stock:

$$\gamma h_{t+1} = (1-\delta_H)h_t + \left[\frac{\theta_1^H}{1-\epsilon_H}\left(\frac{y_{Ct}}{h_t}\right)^{1-\epsilon_H} + \theta_2^H\right]h_t \tag{11}$$

where the depreciation rate of the stock of commercial buildings is denoted by δ_H, and where ϵ_H represents the elasticity of commercial real estate prices, which we denote by q_H, to a change in the amount of new structures to real estate stock ratio y_C/h.[11] With this specification, the model without adjustment costs is obtained by setting the elasticity parameter ϵ_H to zero.

Since they own real estate developers, managers in this sector use the stochastic discount factor of households to evaluate future flows. Their problem consists of maximizing the expected discounted sum of future profits:

$$E_0 \sum_{t=0}^{\infty} \widehat{\beta}_A^t \frac{\lambda_{At}}{\lambda_{A0}} prof_{Dt}$$

where λ_A denotes the marginal utility of consumption of households, by deriving the optimal trajectories for the stock of commercial real estate, the quantity of new structures purchased and the stock of debt, subject to constraints (8), (9) and (11).

[11] As in the case of capital, we have that:

$$\theta_1^H = (\gamma - (1-\delta_H))^{\epsilon_H} \text{ and } \theta_2^H = -(\gamma - (1-\delta_H))^{\frac{\epsilon_H}{1-\epsilon_H}}$$

The Construction Sector

Firms in the construction sector produce new commercial structures y_C using capital k_C and labor N_C. The share of capital in the production function is denoted by ϑ. New commercial buildings are produced by combining capital and labor with land parcels, which are denoted by t_C. The share of land in the production process is represented by the parameter η. The production function for new buildings is of a Cobb-Douglas form:

$$y_{Ct} = \left(k_{Ct}^{\vartheta} N_{Ct}^{1-\vartheta}\right)^{1-\eta} t_{Ct}^{\eta} \tag{12}$$

Profits in the construction sector are given as follows:

$$prof_{Ct} = p_{Ct} \left(k_{Ct}^{\vartheta} N_{Ct}^{1-\vartheta}\right)^{1-\eta} t_{Ct}^{\eta} - w_t n_{Ct} - r_t k_{Ct} - d_t t_{Ct}$$

The optimization problem of managers consists of choosing the quantity of capital, the number of hours worked and the quantity of land that maximizes profits.

The Corporate Non-financial Sector

Firms in the corporate sector produce the final output good y_F by renting labor N_F and capital k_F from the representative household. A stock of commercial buildings h is required to produce the final output good and commercial buildings are rented from real estate developers. The production function for the final output good takes a Cobb-Douglas form and is given as follows:

$$y_{Ft} = A_t k_{Ft}^{\alpha} N_{Ft}^{\xi} h_t^{1-\alpha-\xi} \tag{13}$$

where A is a random technology process that is governed by an autoregressive process of order one:

$$\log A_t = \rho_A \log A_{t-1} + \varepsilon_{At}$$

and where the random disturbance ε_A is normally distributed with mean zero and standard deviation $std(\varepsilon_{At})$. The autoregressive parameter is denoted by ρ_A, where $0 \leq \rho_A \leq 1$.

The capital and labor share parameters in the production function are denoted by α and ξ, respectively. Profits in the corporate sector, which are denoted by $prof_F$, are given as follows:

$$prof_{Ft} = A_t k_{Ft}^{\alpha} N_{Ft}^{\xi} h_t^{1-\alpha-\xi} - w_t N_{Ft} - r_t k_{Ft} - z_t h_t$$

Each period, the problem for managers in the corporate sector consists of choosing the combination of production inputs that would maximize profits.

The Monetary Policy Authorities

The model is closed by introducing a monetary policy rule that relates the interest rate i_B to the inflation rate π :

$$i_{Bt} = \overline{i_B} + \phi_\pi(\pi_t - \pi^*) + \log \iota_t \tag{14}$$

where $\overline{i_B}$ denotes the steady state value of the policy rate. The inflation target set by the central bank is denoted by π^*. The parameter ϕ_π measures the sensitivity of the instrument i_B to deviations of the actual inflation rate π from its target. The monetary policy innovation, which is denoted by ι_t, is determined by the following exogenous process:

$$\log \iota_t = \rho_\iota \log \iota_{t-1} + \varepsilon_{\iota t}$$

where ρ_ι is a persistence parameter and $\varepsilon_{\iota t}$ a random variable that is normally distributed with mean 0 and standard deviation $std(\varepsilon_{\iota t})$.

2.2 Bankers

In this economy, credit is supplied by a different type of agent. Extending credit to real estate developers requires a particular expertise, which is only possessed by risk neutral financiers. These agents derive utility from consumption, which we denote by c_B :

$$U_{Bt} = E_0 \sum_{t=0}^{\infty} \widehat{\beta}_B^t c_{Bt} \tag{15}$$

where the subjective discount factor of bankers is denoted by $\widehat{\beta}_B$.[12]

The role of banks is to supply credit to the economy and the revenue generated from their activity is used to finance consumption and extend new loans. This implies the following time t budget constraint:

$$(\varkappa + (1-\varkappa)i_{Bt-1})\frac{b_t}{1+\pi_t} = \gamma b_{t+1} - (1-\varkappa)\frac{b_t}{1+\pi_t} + c_{Bt} \tag{16}$$

The left-hand side of equation (16) represents the income received from real estate developers. As explained above, the term $\varkappa\frac{b}{1+\pi}$ stands for the amount that developers reimburse in period t, whereas $(1-\varkappa)i_B\frac{b}{1+\pi}$ is the interest payment on the outstanding amount. This income is used to finance new loans, the amount of which is denoted by $\gamma b_{t+1} - (1-\varkappa)\frac{b_t}{1+\pi_t}$, and consumption.

[12] Since bankers are risk neutral, the modified discount factor is given by $\widehat{\beta}_B = \widetilde{\beta}_B\gamma$.

In every period, the dynamic optimization problem of bankers therefore consists of choosing the stock of credit and consumption that maximize lifetime utility in equation (15) subject to constraint (16).

2.3 Equilibrium Definition

Next, let us formally define the incomplete market competitive equilibrium and then analyze how leverage affects the dynamics of commercial real estate prices. All corresponding efficiency conditions are derived in the appendix.

Definition 1 *The incomplete market competitive equilibrium (IMCE) in the economy is a sequence of prices* $w_t, r_t, d_t, q_{Kt}, q_{Ht}, p_{Ct}, \varphi_t, \varpi_t, i_{Bt}, \omega_t, \lambda_{At}, \lambda_{Bt}$ *where* q_{Kt} *is Tobin's Q,* q_{Ht} *the price of commercial real estate,* φ_t *the Lagrange multiplier on constraint (2),* ϖ_t *the Lagrange multiplier on constraint (9),* λ_{Bt} *marginal utility of consumption of bankers and* ω_t *the Lagrange multiplier on constraint (5), and quantities* $c_{At}, c_{Bt}, i_{Tt}, N_{Ft}, N_{Ct}, N_{St}, L_t, y_{Ct}, y_{Ft}, b_t, k_{Ft}, k_{Ct}, k_{Tt}, h_t, x_t, S_t$ *that satisfy households' and firms' optimality conditions as well as the aggregate resource constraint:*

$$y_{Ft} = c_{At} + c_{Bt} + i_{Tt}$$

for all states, for $t = 1...\infty$*, and given initial values* k_{T0}, x_0, h_0, S_0 *for the four endogenous state variables.*

2.4 Real Estate Prices under Incomplete Markets

The dynamics of commercial real estate prices, which we denote by q_H, can be derived by analyzing the efficiency conditions of real estate developers. Optimal accumulation of commercial real estate gives rise to the following asset pricing formula:

$$q_{Ht} = \beta_A E_t \frac{\lambda_{At+1}}{\lambda_{At}} \left\{ q_{Ht+1} \left[(1-\delta_H) + \frac{\theta_1^H}{1-\epsilon_H} \left(\frac{y_{Ct+1}}{h_{t+1}} \right)^{1-\epsilon_H} + \theta_2^H - \theta_1^H \left(\frac{y_{Ct+1}}{h_{t+1}} \right)^{1-\epsilon_H} \right] + z_{t+1} \right\} \tag{17}$$

Non-arbitrage implies that the price of a commercial building today needs to be equal to the expected discounted sum of two components.[13] The first term inside the curly bracket is the capital gain component of the valuation. In the next period, the stock of commercial buildings owned today will have depreciated by a factor δ_H. The value of the remaining stock

[13] $\beta_A \frac{\lambda_{Ht+1}}{\lambda_{Ht}}$ represents the stochastic discount of factor of households, who own the real estate developers.

therefore needs to be adjusted for depreciation. Relative to a financial asset, the difference is that adjusting the stock of real estate is costly. The term $\frac{\theta_1^H}{1-\epsilon_H}\left(\frac{y_{Ct+1}}{h_{t+1}}\right)^{1-\epsilon_H}+\theta_2^H-\theta_1^H\left(\frac{y_{Ct+1}}{h_{t+1}}\right)^{1-\epsilon_H}$ captures the effect of capital adjustment costs on the capital gain component of the valuation. The magnitude of this term depends on the elasticity parameter ϵ_H. When this parameter is set to zero, adjustment costs disappear from this formula. This is the case of a perfectly elastic supply of real estate. Relative to this extreme case, increasing ϵ_H above zero reduces the supply elasticity of real estate.

The second term inside this bracket, which denotes rents, is the payoff component of the valuation. Since commercial real estate is used as a production input in the corporate sector, commercial rents are determined by the marginal productivity of buildings, i.e. $z_t = (1-\alpha-\xi)\frac{y_{Ft}}{h_t}$.

Agents must therefore be indifferent between purchasing a building at the price q_{Ht} today or investing in a construction project that will be worth $q_{Ht+1}\left((1-\delta_H)+\frac{\theta_1^H}{1-\epsilon_H}\left(\frac{y_{Ct+1}}{h_{t+1}}\right)^{1-\epsilon_H}+\theta_2^H-\theta_1^H\left(\frac{y_{Ct+1}}{h_{t+1}}\right)^{1-\epsilon_H}\right)$ in the next period, and which will bring a rental income of z_{t+1}.

We next discuss how the presence of financing frictions affects the relationship between commercial real estate prices q_H and construction costs p_C in a model with adjustment costs. Our first main result can be stated as follows:

Proposition 1 *In the IMCE, financial leverage introduces a gap between fundamentals and the price of a commercial building.*

In other words, real estate prices can deviate from the value implied by Tobin's Q theory in a model with leverage in which markets are incomplete. This result can be demonstrated by deriving the optimality condition with respect to y_C :

$$q_{Ht} = p_{Ct}\frac{\left(\frac{y_{Ct}}{h_t}\right)^{\epsilon_H}}{\theta_1^H}\left(1-\mu+\mu\frac{\varpi_t}{\lambda_{At}}\right) \tag{18}$$

This condition firstly relates the price of a building to what we refer to as the adjusted construction cost. The reason is that in this formula construction costs not only depend on the price at which developers purchase new buildings, which is given by p_C, they are also affected by adjustment costs. Indeed, the term $\left(\frac{y_{Ct}}{h_t}\right)^{\epsilon_H}/\theta_1^H$ captures the additional cost encountered by developers when adjusting their real estate stock. Without adjustment costs, which corresponds to the case $\epsilon_H = 0$, this latter term is equal to 1, and construction costs are solely determined by the price of new buildings p_C.

The term $1 - \mu + \mu \frac{\varpi_t}{\lambda_{At}}$ captures the contribution of leverage to the dynamics of real estate prices. Note that this term reduces to 1 if developers choose to entirely finance new acquisitions via retained earnings. Indeed, the model without leverage can be obtained by setting the parameter μ to 0. In this special case, this model reduces to a real estate version of Tobin's Q theory and leverage plays no role.

This second term contains a time-varying component which is firstly given by ϖ, and which represents the Lagrange multiplier that is associated with constraint (9). It is important to note that this time-varying component in fact depends on the ratio between this Lagrange multiplier and marginal utility λ_A. This ratio measures the tightness of the earnings-based constraint (10).

We refer to this term as the distributive wedge. As we explain in the next section, the time-variation in this wedge is caused entirely by the difference in SDF across lenders and borrowers and disappears if markets can be completed. Intuitively, this wedge captures a redistribution of resources. Under perfect risk-sharing, agents are able to completely insure against this type of risk.

How is the distributive wedge determined? Using the optimal choice of debt accumulation by real estate developers, the following expression for this ratio can be derived as:

$$\frac{\varpi_t}{\lambda_{At}} = \beta_A E_t \frac{\lambda_{At+1}}{\lambda_{At}} \frac{1}{1+\pi_{t+1}} \left(\varkappa + (1-\varkappa) i_{Bt} + (1-\varkappa) \frac{\varpi_{t+1}}{\lambda_{At+1}} \right) \tag{19}$$

From the developer perspective, this ratio can be interpreted as the tightness of the financing constraint, which can be interpreted as in equation (10). The debt capacity of developers depends on the amount of debt they can service. Consequently, the tightness of the constraint is determined by the discounted value of future repayment obligations. Since developers are owned by households, these future repayments are discounted using the stochastic discount factor of households.

In the next period, a fraction $\varkappa$ will need to be repaid and an interest payment on the outstanding amount $(1-\varkappa)$ will have to be made. The last term of this formula $(1-\varkappa)\frac{\varpi_{t+1}}{\lambda_{At+1}}$ is then the continuation cost from contracting a loan that will only be fully repaid in $1/\varkappa$ periods.

Note that the cost of credit not only depends on the short-term lending rate i_B, but also on the entire term structure of interest rates. The importance of interest rate duration risk on the distributive wedge is in turn determined by the maturity parameter $\varkappa$.

3 Calibration

Whenever possible, parameter values are selected using external evidence. A second set of parameters is chosen by selecting values that are considered as standard in the literature. The 10 remaining parameters are then selected to maximize the model's ability to match 10 moments of interest, which include the volatility of commercial real estate prices.

The model is solved by using perturbation methods via the toolbox developed by Adjemian et al. (2014). The effect of risk is captured by solving the model using a second- and a fourth-order approximation to the policy function.

The model is calibrated using quarterly data for the United States for the period from the first quarter of 1984 to the first quarter of 2020. Since the COVID shock hit during the first quarter of 2020, we then simulate a health shock using the calibration corresponding to the pre-pandemic period. This implicitly assumes that the COVID shock took agents by surprise and was not reflected in expectations about future possible shocks. The model is calibrated using monetary policy and technology shocks.

Standard Values

With internal habits, long-term risk aversion increases with respect to the curvature coefficient but is independent of the habit parameter (e.g. Constantinides, 1990; Swanson, 2012). To minimize the role played by this parameter, the curvature coefficient is set to 1. The subjective discount factor of households β_A is set to 0.984, which is a standard value in the real business cycle literature (e.g. King and Rebelo, 1999).

With this preference specification, the curvature parameter υ controls the Frisch elasticity of labor supply (e.g. Jaccard, 2014). Given recent findings that are documented in the literature (e.g. Hall, 2009; Chetty, Guren, Manoli, and Weber, 2011), we choose a value for υ that implies a Frisch elasticity of 0.7. The second labor supply parameter ψ is calibrated to reproduce the fact that agents in the United States spend on average twenty percent of their time on work-related activities. Given these values, the allocation between hours worked in the construction and final good sectors is then endogenously determined.

Using External Evidence

In this environment, the parameter κ determines the steady state fraction of time that agents spend taking care of their health. This parameter can thus be calibrated to ensure that the steady state value of N_S is consistent with available evidence.

According to the 2019 American Time of Use Survey, households allocate on average 0.08 hour per day to health-related self care. Moreover, they spend on average 0.06 hour per day obtaining medical and care services from professionals. This therefore implies that households spend about 10 minutes per day taking care of their health. To be consistent with this empirical evidence, we set κ to 0.008, which implies that households will on average spend 1% of their time on health-related activities.

The parameter γ denotes the deterministic growth rate of the economy. Its value can be estimated using data on real output. For the period from 1984 to 2018, the average quarterly growth rate of the US economy stood at 0.67%. This therefore implies a value for γ of 1.0067.

In the United States, the annual depreciation rate of capital for the period from 1950 to 2017 is on average 4% (Feenstra, Inklaar, and Timmer (2015)). In this quarterly model, this implies a value for δ_K of 0.01. An important difference between equipment and structures is that the latter depreciates more slowly. Following Tuzel (2010), we therefore set the rate of depreciation of commercial buildings to 0.005, which implies an annual depreciation rate of around 2%.

In this model, the maturity structure of debt is determined by the parameter $\varkappa$. Since $\varkappa$ represents the fraction of debt amortized in period t, a loan has a duration of $1/\varkappa$ periods. This parameter can thus be calibrated using evidence on the maturity structure of debt in the real estate sector, which is available form the National Association of Real Estate Investment Trusts (NAREIT). As an average from the first quarter of 2000 to the fourth quarter of 2018, the time to maturity of debt for publicly traded real estate companies stood at 69.1 months. In our quarterly model, an average loan duration of around 17 quarters can be reproduced by setting $\varkappa$ to 0.0588.

To calibrate the share of commercial real estate in the production function, we use data on value added by industries. In the model, the main activity of real estate developers consists of developing and renting commercial buildings. In the data, the closest counterpart is the real estate and rental and leasing sector. A major portion of this sector comprises establishments that rent or lease their own assets to others. As an average over the period from 2005 to 2020, value added in this sector amounted to 12% of total GDP. This evidence can be used to calibrate the share of commercial real estate in the production function, which is denoted by $1 - \alpha - \xi$.

Over the period from 1984 to 2020, estimates provided by the Bureau of Labor Statistics (BLS) suggest that the labor share represented 60% of output.[14] This evidence can be used to set the labor share parameter ξ to 0.6. Along with the value found for $1 - \alpha - \xi$, this

[14]Source: BLS (2017). "Estimating the U.S. labor share", Monthly Labor Review, February 2017 for a description.

therefore implies a value for the capital share parameter of 0.28.

In this environment with risk neutral financiers, fluctuations in the interest rate i_B are entirely driven by the innovation to the monetary policy rule (14). The autoregressive parameter ρ_ι can therefore be calibrated to reproduce the high persistence of bank lending rates observed in the data. For the period from 1984 to 2020, the second-order autocorrelation coefficient for i_B is 0.90. The persistence of interest rates observed in the data can be matched by setting ρ_ι to 0.95.

Households' Health Stock

Since the model is calibrated for the pre-pandemic period, the standard deviation of the health shock $std(\varepsilon_\chi)$ is set to zero. To our knowledge, there is no consensus in the literature on the rate at which health depreciates. Given this lack of a priori information, we set δ_S to 0.1. This value implies that the economy's health stock loses around 80 percent of its value after about 4 years if households completely stop allocating time to health-related activities.

Moment Matching Procedure

We are left with 10 structural parameters: the habit formation parameter m, the real estate adjustment cost coefficient ϵ_H, the capital adjustment costs coefficient ϵ_K, the land share in the production function η, the fraction of new buildings financed through leverage μ, the inflation coefficient parameter in the monetary policy rule ϕ_π, the subjective discount factor of bankers β_B, the monetary policy shock standard deviation $std(\varepsilon_\iota)$, and the two technology shock process parameters ρ_A and $std(\varepsilon_A)$.

This set of parameters is selected to maximize the model's ability to reproduce 10 empirical moments that characterize the link between the commercial real estate market and the macroeconomy. Firstly, as in a standard business cycle model, we ask whether the model can match the volatility of the growth rates of macroeconomic aggregates. In our case, this first set of variables includes the standard deviations of output $std(\Delta y_F)$, aggregate consumption $std(\Delta c)$, where in our model aggregate consumption is the sum of consumption of the two agents, business investment $std(\Delta x)$, and investment in structures $std(\Delta y_H)$. This first set of moments is particularly sensitive to values for the two adjustment costs parameters, as well as the two shock process parameters.

Secondly, since our objective is to shed light on the determinants of the volatility of real estate prices, we use data on commercial real estate prices $std(\Delta q_H)$ and construction costs $std(\Delta p_H)$. As we discuss in the next section, the habit formation parameter has a particularly strong impact on the volatility of real estate prices. The share of land, which is denoted by η, is akin to an adjustment cost (e.g. Davis and Heathcote, 2005). The volatility of construction costs is therefore sensitive to this parameter value.

Finally, we use NAREIT data to compute the average interest rate expense to net operating income ratio for the period from the first quarter of 2000 to the fourth quarter of 2018. In our model, this financial ratio, which we denote by $\mathcal{F}$, can be computed as follows:

$$E(\mathcal{F}) = E\left(\frac{\varkappa b_t + (1-\varkappa) i_{Bt} b_t}{z_t h_t - (1-\mu) p_{Ht} y_{Ht}}\right)$$

where the total cost of servicing debt in the numerator consists of both principal and interest repayment. In the denominator, the net operating income is given by the rental revenue minus the amount spent on new acquisitions, which is financed via retained earnings. This ratio essentially serves to identify μ, a parameter which has no direct empirical counterpart in the data.

The coefficient ϕ_π in the monetary policy rule can be identified by adding the volatility of inflation, which we denoted by $std(\pi)$, into the loss function. Finally, the subjective discount factor of bankers and the monetary policy shock standard deviation mainly affect the mean as well as the volatility of bank lending rates. These two parameters can thus be identified by including the mean $E(i_B)$ and volatility of bank lending rates $std(i_B)$, expressed in annualized terms, into the set of moments to be matched.

The combination of parameter values that minimizes the distance between the model implied moments and the data is reported in Table 1 below. Table 2 compares the theoretical moments with their empirical counterparts, where the estimated confidence intervals for the estimated means and standard deviations are reported in the first column.

Table 1: Moment matching

m	ϵ_H	ϵ_K	η	μ	β_B	ϕ_π	ρ_A	$std(\varepsilon_\iota)$	$std(\varepsilon_A)$
0.88	0.46	1.68	0.11	0.56	0.998	2.0	0.982	0.0019	0.0096

Is the Financial Constraint Always Binding?

If we abstract from the inequality constraint in equation (9), the model reduces to a representative agent model without bankers. Indeed, if μ is set to zero, developers do not need to borrow to operate their businesses. The demand for credit therefore goes to zero in this case.

Since the model is solved using perturbation methods it is important to check whether the inequality constraint is always satisfied with equality. Given that credit is a state variable, it is difficult to derive results analytically. Therefore, we check the tightness of the constraint by analyzing the behavior of the Lagrange multiplier ϖ over the business cycle. For the calibration summarized in Table 1, ϖ always remains far from the zero line. This result is illustrated in Figure B.1. As illustrated by the red continuous line, we have also checked that increasing the order of approximation from 2 to 4 does not alter this conclusion. This illustrates that the case of occasionally binding constraints is extremely unlikely to occur in this model.

Another potential concern is that we do not consider the case of collateral constraints. Indeed, following the seminal contribution of Kiyotaki and Moore (1997), many models include a constraint of the form:

$$b_t \leq \varrho E_t \frac{\lambda_{Ht+1}}{\lambda_{Ht}} q_{t+1} h_{t+1} \tag{20}$$

where ϱ is, for instance, set to 0.85 in the case of residential real estate (e.g. Iacoviello and Neri (2010)).

To check whether collateral constraints could play a role, we simulate values for b_t as well as the discounted collateral value $\varrho E_t \frac{\lambda_{Ht+1}}{\lambda_{Ht}} q_{t+1} h_{t+1}$, where ϱ is set to 0.85, in the version of the model in which this constraint is absent. We always observe values for b that are significantly lower than the collateral value given by the right-hand side of equation (20). This is illustrated in Figure B.2, which also shows the result obtained when a fourth-order approximation is used to solve the model. For the calibration that reproduces the facts reported in Table 2, collateral constraints are therefore very unlikely to play a role in our environment.

4 Results

Overall, and as shown by Table 2, it is fair to say that the model does well at reproducing the moments that were targeted. Relative to the literature, one major improvement is that the dynamics of macroeconomic aggregates can be reproduced in a model that is able to jointly match the volatility of commercial real estate prices. As in Jermann (1998), this improvement is essentially due to the combination of habit formation and adjustment costs. Indeed, the volatility of commercial real estate prices declines from 8.8% to 2.0% if we remove habits by setting m to 1. Relative to the model without habits, the volatility of commercial real estate prices further declines from 2.0% down to 1.5% if we also remove adjustment costs by setting ϵ_K and ϵ_H to zero.

Table 2: Model vs. Data

	Data		Model
	95% confidence interval	Estimated empirical moments	Theoretical moments
$std(\Delta y_F)$	[1.4, 1.8]	1.6	1.6
$std(\Delta c)$	[1.0, 1.3]	1.1	1.0
$std(\Delta x)$	[5.2, 6.6]	5.8	5.8
$std(\Delta y_C)$	[8.2, 10.4]	9.2	9.2
$std(\Delta q_H)$	[7.9, 10.0]	8.9	8.8
$std(\Delta p_C)$	[2.7, 3.5]	3.1	3.1
$std(\pi)$	[0.4, 0.5]	0.5	0.6
$std(i_B)$	[2.3, 2.9]	2.6	2.6
$E(i_B)$	[6.1, 7.0]	6.5	6.5
$E(\mathcal{F})$	[0.29, 0.32]	0.31	0.31

The model not only reproduces the volatility of investment in commercial structures $std(\Delta y_C)$ but also that of construction costs $std(\Delta p_C)$. It can therefore be used to assess the importance of leverage for the dynamics of property prices. To isolate the effect of leverage, let us divide the formula derived in equation (18) into two components. Firstly, a component reflecting the contribution of construction costs, corrected for the effect of adjustment costs q^F_{Ht}, which we interpret as the fundamental value of a commercial building. Secondly, a component reflecting the impact of the distributive wedge on the dynamics of prices, which we denote by q^W_H :

$$q_{Ht} \quad = \quad \underbrace{p_{Ht}\frac{\left(\frac{y_{Ht}}{h_t}\right)^{\epsilon_H}}{\theta^H_1}}_{q^F_{Ht}} \underbrace{\left(1-\mu+\mu\frac{\varpi_t}{\lambda_{Ht}}\right)}_{q^W_{Ht}} \tag{21}$$

Next, to illustrate how leverage affects commercial real estate prices, the left panel of Figure 1 shows how these different components respond to a positive technology shock. The black dotted line denotes the response of commercial real estate prices q_H. The fundamental value q^F_H is denoted by the blue continuous line, whereas the red dashed line represents the contribution of the wedge q^W_H.

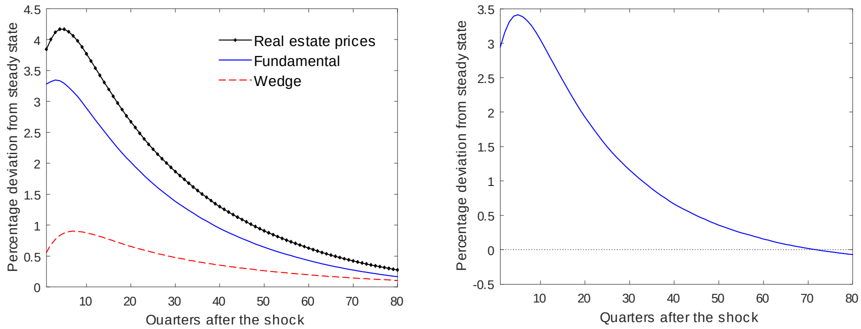

Figure 1. Impulse response to a technology shock of commercial real estate prices and its components (left panel) and the price to construction costs ratio q_H/p_H (right panel).

The first main takeaway from Figure 1 is that leverage causes persistent deviations between real estate prices and fundamentals. At the same time, the distributive wedge only has a moderate effect on the volatility of commercial real estate prices if technology shocks are the main driving force. Indeed, the standard deviation of q_H^F, which represents fluctuations in constructions costs plus adjustment costs, is 7.2% whereas the volatility of commercial real estate prices q_H reaches 8.8%. Consequently, in a model in which technology shocks are the dominant source of fluctuations, the contribution of the distributive wedge in amplifying shocks is limited.

The right panel of Figure 1 shows the response of the commercial real estate price to construction cost ratio q_H/p_H to a technology shock. The model not only matches the standard deviation of each variable, but it also generates large procyclical variations in this ratio, as observed in the data (see Figure A.1). Since leverage only has a moderate impact on the dynamics of commercial real estate prices, most of these fluctuations reflect changes in the investment to real estate ratio, which is the standard Tobin's Q effect.

Rents

As illustrated in Figure A.2, this mechanism also generates a disconnect between rents, the response of which is denoted by the red dashed line, and commercial real estate prices, which are represented by the blue continuous line. Recall from equation (17) that commercial real estate prices can also be expressed as the discounted sum of two components: (i) a capital gain component, which includes a correction for the effect of adjustment costs, and (ii) a payoff component that reflects the contribution of rents. The large difference in dynamics illustrates that the volatility of real estate prices is not due to excessive fluctuations in rents.

Table 3: Volatility of Rents
vs. Commercial Real Estate Prices

	Data	Model
$\frac{std(\Delta q_H)}{std(\Delta z)}$	5.6	5.6

This difference in dynamics can also be illustrated by the much greater volatility of commercial real estate prices. Whereas the standard deviation of real estate prices reaches 8.8 percent, that of rents is only 1.6 percent. Rents are therefore around 5.6 times less volatile than commercial real estate prices. This result illustrates that small variations in rents can be consistent with large equilibrium fluctuations in real estate prices.

Is this difference in dynamics consistent with the data? To the best of our knowledge, data on commercial rents are only available starting from 2005. It is, nevertheless, possible to compare the standard deviation of rents, which we denote by $std(\Delta z)$, with that of commercial real estate prices using a shorter sample than that used in Table 2. Using data on commercial rents by REIS/Moody's, we find that commercial real estate prices are around 5.6 times as volatile as rents. As shown in Table 3, although this moment was not targeted, the model is able to account for this difference in relative volatility.

Real Estate Returns, the Risk-free Rate and the Real Estate Premium

With adjustment costs, real estate returns can be defined as follows:

$$r_t^H = \frac{q_H\left((1-\delta_H) + \frac{\theta_1^H}{1-\epsilon_H}\left(\frac{y_{Ht}}{h_t}\right)^{1-\epsilon_H} + \theta_2^H - \theta_1^H\left(\frac{y_{Ht}}{h_t}\right)^{1-\epsilon_H}\right) + z_t}{q_{Ht-1}} - 1 \qquad (22)$$

The SDF of households can then be used to derive the risk-free return that corresponds to the asset pricing formula for commercial real estate prices shown in equation (17):

$$r_t^F = \frac{1}{\beta_A E_t \frac{\lambda_{At+1}}{\lambda_{At}}} - 1 \qquad (23)$$

The model generates small but persistent variations in stochastic discounting. These cyclical properties are in turn reflected in the risk-free rate dynamics. Expressed in annualized terms, for the calibration described in Table 1, we obtain a risk-free rate standard deviation of 3.9 percent, a value that is well within the range of estimates available in the literature. Introducing time-variation in stochastic discounting, which in turn generates small but persistent risk-free rate variations, therefore provides an explanation for this apparent disconnect between real estate prices and rents.

Finally, equations (22) and (23) can be used to define a model-implied measure of real estate excess return. For the benchmark calibration that reproduces the moments shown in Table 2, we obtain a real estate premium of 1.3 percent. The magnitude that we obtain is, for instance, broadly in line with the facts documented by Piazzesi, Schneider, and Tuzel (2007) in the case of residential real estate. At the same time, it is significantly lower than that documented by Jordà, Schularick, and Taylor (2019).

Leverage

In this environment, the joint dynamics of real estate prices and leverage can in part be explained by an inefficient allocation of risk. Relative to the perfect risk-sharing benchmark, the amount of consumption risk borne by households is excessive. The real estate market therefore serves as an adjustment margin that households use to facilitate consumption smoothing. In boom times, this is achieved by pouring resources into the construction sector.

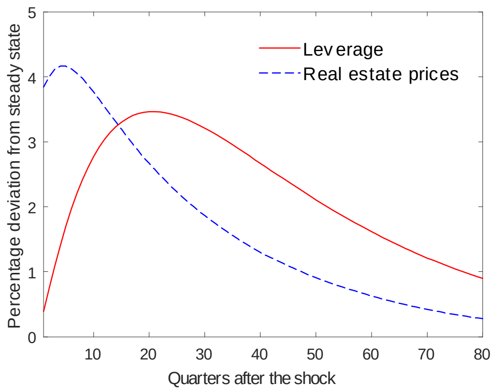

Figure 2. Impulse response to a technology shock.

Since real estate developers are financially constrained, the investment boom that occurs during expansions needs to be financed by debt. Consequently, the construction boom is accompanied by a large increase in leverage. As stated in Proposition 1, this boom in leverage in turn causes real estate prices to deviate from fundamentals. This dynamics is illustrated in Figure 2, where the response of real estate prices q_H, and leverage b are depicted by the blue dashed line and the red continuous line, respectively.

4.1 Monetary Policy Shocks

Whereas monetary policy shocks only have a marginal impact on the real economy, their effect on the leverage cycle is sizeable. For the parameter values discussed in Section 4, a one standard deviation monetary policy shock reduces the interest rate by about 0.8% on impact. The response of the policy rate to the shock, which is expressed in annualized terms, is shown in the upper left panel of Figure 3.

As illustrated by the upper right panel of Figure 3, an expansionary monetary policy shock increases leverage. This hump-shaped response illustrates that monetary policy can have a very persistent effect on the leverage cycle. In a model with long-term debt, monetary policy also affects the leverage cycle with a considerable lag. Whereas the shock generates an immediate decline in the policy rate, the maximum impact on leverage is only reached after about 4 years.

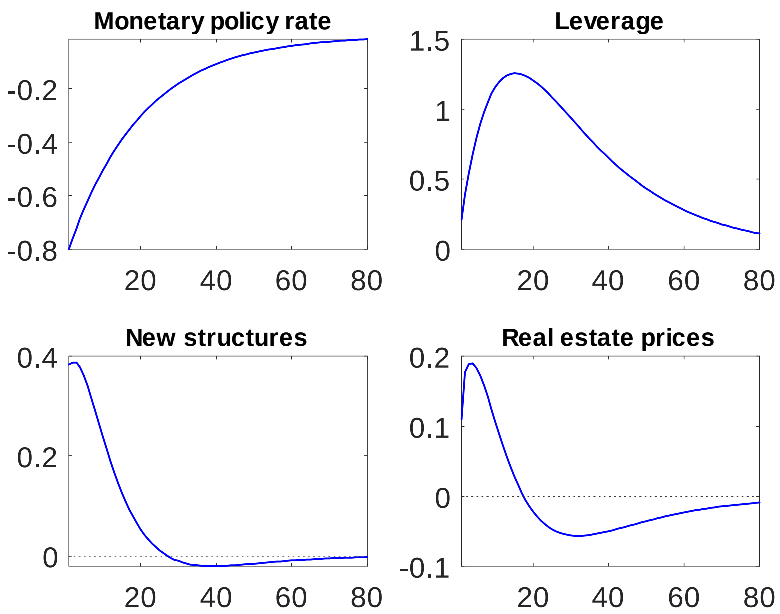

Figure 3. Impulse response to a monetary policy shock. Horizontal axis: Vertical axis: Percentage deviation from steady state.

The lower left panel of Figure 3 shows that the monetary policy shock affects the real economy by stimulating investment in commercial real estate structures y_C. A decline in i_B relaxes the constraint (9) by reducing the net interest rate expense of real estate developers. They find it optimal to use the additional revenue generated by the interest rate reduction by increasing the stock of commercial real estate that they own.

Finally, as illustrated by the lower right panel of Figure 3, the initial response of commercial real estate prices to the monetary policy expansion is positive. In terms of the decomposition shown in equation (21), this positive effect is driven by the fundamental component of the valuation $p_{Ht}\left(\frac{y_{Ht}}{h_t}\right)^{\epsilon_H}/\theta_1^H$.

In contrast to the case of a positive technology shock, an expansionary monetary policy shock reduces the distributive wedge. Indeed, the reduction in interest rate expense makes it easier for real estate developers to service the cost of debt. The decline in the net present discounted value of repayment declines, an effect which relaxes the cash-flow constraint faced by developers.

In sum, the first main takeaway is that monetary policy shocks have a sizeable effect on leverage. Moreover, whereas the distributive wedge amplifies the effects of shocks affecting the demand for credit, it attenuates those affecting credit supply, such as monetary policy shocks.

4.2 The COVID-19 Shock

We capture the effects of the COVID-19 crisis by introducing a shock that reduces the economy's health stock. The health shock propagates to the economy via its impact on equation (5). The persistence of the health shock ρ_χ is set to 0.25. This low persistence ensures that the effect of the shock completely dissipates after 1 year. The shock standard deviation is calibrated to increase the time spent on health-related activities N_S by 10 percent on impact. According to the American use of time survey, households spend on average 10 minutes per day on health-related activities. The magnitude of the shock therefore implies an increase in the time spent recovering from the health shock of 1 minute in the quarter in which the pandemic hits.

The main objective of this section is to document the model's ability to amplify and propagate the effects of small and highly transitory health shocks. The strength of this amplification mechanism is illustrated in Figure A.3, which compares the magnitude of the shock, which is depicted by the red dotted line, with the endogenous response of hours spent on health-related activities. The main takeaway is therefore that it only takes a small exogenous health shock to generate a large endogenous response in N_S.

As can be seen in Figure A.1, the COVID-19 shock had a large effect on the real estate price to construction cost ratio. As shown in Figure 4, which shows the response of q_H/p_H to a health shock, this calibration implies a 2% decline on impact. Moreover, it takes this ratio about 16 quarters or 4 years to fully recover from the shock.

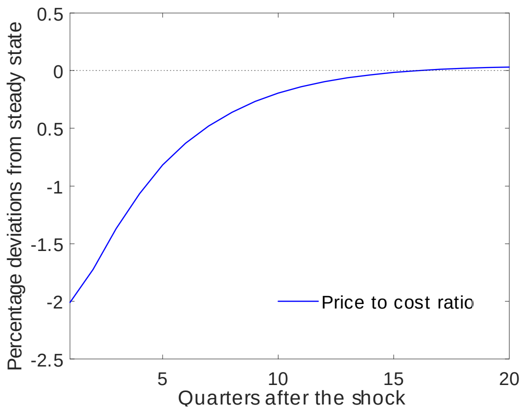

Figure 4. Effect of the COVID shock on the real estate price to cost ratio q_H/p_H.

To gain intuition into how the shock propagates to the real economy, the response of hours spent on the different activities and financial variables are reported in Figure A.4. The four panels show the response of commercial property prices q_H, leverage b, and hours worked in the construction and final good sectors.

As illustrated by the two lower panels, the shock propagation mechanism works via the labor market. As agents are forced to allocate more time to their health, they have to reduce the number of hours worked in the construction and final good sectors. The reduction in hours in the final good sector lowers the marginal productivity of commercial real estate. Consequently, commercial rents fall, which in turn explains the decline in property prices shown in the upper left panel. On impact, commercial real estate prices fall by more than 2%.

The decline in the number of hours worked in the construction sector triggered by the shock also reduces new construction. Combined with the lower demand for commercial real estate, this reduction in construction in turn reduces the need for external financing. As a result, and as illustrated by the upper right panel, the shock also generates a persistent decline in leverage.

The role of the distributive wedge

As shown by the left panel of Figure 1, in the case of technology shocks, the distributive wedge has only a moderate effect on the dynamics of commercial real estate prices. In contrast, and as illustrated by Figure 5, it plays a much bigger role when the economy is hit by a health shock. Since this wedge is essentially a measure of market incompleteness, it has a much stronger effect on the dynamics of real estate prices when borrowers are hit by an asymmetric shock.

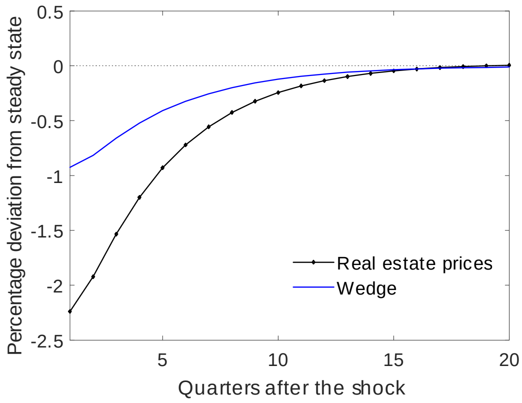

Figure 5. Contribution of the distributive wedge.

In terms of the decomposition performed in equation (21), a much larger share of the decline in commercial real estate prices can therefore be attributed to the wedge. Indeed, on impact real estate prices decline by around 2.2% and almost half of this decline can be accounted for by the distributive wedge.

5 The Complete Markets Allocation

The incomplete markets economy is inefficient because of the misalignments in SDF or IRMS across lenders and borrowers. This distortion can be corrected by introducing transfers that equalize marginal utilities across agents, a case which corresponds to the complete markets allocation.

To ensure comparability with the incomplete markets model, we assume that transfers between households and bankers are equal to zero on average. Consequently, the policy does not affect the deterministic steady state of the economy, only its dynamics. The advantage of this approach is that it allows us to isolate the effect of risk-sharing from that of redistribution. From a political economy perspective, the case of transitory transfers also seems more realistic. Indeed, imposing permanent transfers changes the distribution of wealth across agents. With transitory transfers, the policy only corrects the inefficiency caused by imperfect risk-sharing without generating any permanent redistribution of resources.

Relative to the incomplete market allocation described in Section 2, the risk-sharing equilibrium is achieved by introducing a policy instrument. The role of the government is to find the sequence of transfers tr_t, where $E(tr) = 0$, ensuring that $\lambda_{At} = \lambda_{Bt} = \lambda_t$ for all states and for $t = 1 ... \infty$. We refer to this case as the risk-sharing competitive equilibrium, which can be formally defined as follows:

Definition 2 *The risk-sharing competitive equilibrium (RSCE) is a sequence of prices* $w_t, r_t, d_t, q_{Kt}, q_{Ht}, p_{Ct}, \varphi_t, \varpi_t, i_{Bt}, \omega_t, \lambda_t$ *where* λ_t *denotes the marginal utility of consumption that is common across agents, quantities* $c_{At}, c_{Bt}, i_{Tt}, N_{Ft}, N_{Ct}, N_{St}, L_t, y_{Ct}, y_{Ft}, b_t, k_{Ft}, k_{Ct}, k_{Tt}, h_t, x_t, S_t,$ *and transfers between bankers and households* tr_t*, such that* $E(tr) = 0$*, that satisfy households' and firms' optimality conditions as well as the aggregate resource constraint for all states, for* $t = 1...\infty$*, and given initial values* k_{T0}, x_0, h_0, S_0 *for the four endogenous state variables.*

5.1 Real Estate Prices under Risk-sharing

Our second main result is that the distributive wedge only plays a marginal role in the complete markets equilibrium. In the special case in which agents have the same subjective discount factors, i.e. $\beta_A = \beta_B$, it can be shown that the distributive wedge is constant and equal to 1. This result is formally derived under the form of a Proposition:

Proposition 2 *In the risk-sharing equilibrium, real estate prices are exactly equal to fundamentals when* $\beta_A = \beta_B = \beta$*.*

The difference, relative to the result obtained in Proposition 1, is therefore that the gap between prices and fundamentals disappears under complete markets. This result can be demonstrated by analyzing how risk-sharing affects the decomposition shown in equation (18). Under complete markets, the only difference is that this condition no longer depends on the marginal utility of households λ_A. Since marginal utilities are equalized across agents, it now depends on the common value λ:

$$q_{Ht} = p_{Ht} \frac{\left(\frac{y_{Ht}}{h_t}\right)^{\epsilon_H}}{\theta_1^H} \left(1 - \mu + \mu \frac{\varpi_t}{\lambda_t}\right) \tag{24}$$

Under perfect risk-sharing, the term ϖ_t / λ_t is determined by the following optimality condition:

$$\frac{\varpi_t}{\lambda_t} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{1}{1 + \pi_{t+1}} \left[\varkappa + (1 - \varkappa) \left(\frac{\varpi_{t+1}}{\lambda_{t+1}} + i_{Bt}\right)\right] \tag{25}$$

where the only difference relative to equation (19) is that marginal utility of consumption is now the same across agents.

Equilibrium in the debt market is not only determined by the demand from developers but also by the supply from bankers. Indeed, bankers optimally decide how to divide their income between new debt issuance and consumption. This dynamic trade-off implies the following first-order condition:

$$\lambda_t = \beta E_t \lambda_{t+1} \frac{1}{1+\pi_{t+1}} \left[1 + (1-\varkappa) i_{Bt}\right] \tag{26}$$

Relative to the IMCE, the difference is that this condition now depends on the common marginal utility λ_t, and no longer on the marginal utility of bankers λ_B. To understand the result stated in Proposition 2, first combine equations (25) and (26) to obtain the following expression for the distributive wedge:

$$\frac{\varpi_t}{\lambda_t} - 1 = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{1}{1+\pi_{t+1}} (1-\varkappa) \left\{ \frac{\varpi_{t+1}}{\lambda_{t+1}} - 1 \right\}$$

Solving this expression forward, we then have that:

$$\frac{\varpi_t}{\lambda_t} - 1 = (\beta(1-\varkappa))^T \left\{ E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{1}{1+\pi_{t+1}} E_{t+1} \frac{\lambda_{t+2}}{\lambda_{t+1}} \frac{1}{1+\pi_{t+1}} \ldots E_{t+T} \frac{\lambda_{t+T}}{\lambda_{t+T-1}} \frac{1}{1+\pi_{t+T}} \left(\varpi_{t+T} - 1\right) \right\}$$

Since this is an infinite horizon model, and since $\beta(1-\varkappa)$ is strictly smaller than 1, the distributive wedge is constant and equal to unity once a risk-sharing mechanism is introduced:

$$\frac{\varpi_t}{\lambda_t} = 1$$

Equation (24) becomes:

$$q_{Ht} = p_{Ht} \frac{\left(\frac{y_{Ht}}{h_t}\right)^{\epsilon_H}}{\theta_1^H} \tag{27}$$

which thus implies that real estate prices are exactly equal to fundamentals under complete markets if $\beta_A = \beta_B = \beta$.

In the more general case that we consider, i.e. $\beta_A \neq \beta_B$, the distributive wedge is constant if technology and health shocks are the only source of fluctuations. Under complete markets, monetary policy shocks can still induce some time-variation in ϖ/λ if $\beta_A \neq \beta_B$ but these fluctuations are negligible in this model.

5.2 The Simulated Economy under Complete Markets

The inefficiency due to imperfect risk-sharing not only generates deviations between real estate prices and fundamentals through the distributive wedge. It also affects the volatility of real estate and business cycles. This result can be illustrated by comparing the simulated economy under perfect risk-sharing with the incomplete markets benchmark. To facilitate comparisons, the results obtained in Section 3 are presented again in Table 3 in the column "Incomplete markets equilibrium". The results, corresponding to the risk-sharing case, are reported in the column entitled "Complete markets equilibrium".

Under complete markets, risk-sharing is achieved by implementing transfers tr from households to bankers in boom periods. From the perspective of households, who are more risk averse than bankers, these transfers facilitate consumption smoothing. Households need to transfer funds to bankers during expansions and are net beneficiary in recessions. Consequently, relative to the incomplete market allocation, the volatility of household consumption decreases from 1 to 0.7 percent. The increase in the volatility of total consumption reported in Table 3 can be explained by the fact that transfers give rise to very large fluctuations in the consumption of bankers. Since these agents are risk neutral, it is optimal to let them absorb as much consumption risk as possible.

The large decline in the volatility of output obtained in the complete markets model can be explained by a labor supply effect. The insurance provided by the policy reduces consumption volatility and hence fluctuations in marginal utility of consumption. From the perspective of households, these lower fluctuations in marginal utility in turn reduce the wealth effect on labor supply. Relative to incomplete markets, this lower wealth effect increases the volatility of hours worked. The resulting larger fluctuations in hours worked then explain the increase in output volatility reported in Table 3.

The significant decline in the amount of consumption risk borne by households lowers the volatility of investment. Indeed, the volatility of investment in business capital x and commercial structures y_C, decline sharply in the complete markets economy. Since transfers facilitate consumption smoothing, households no longer need to resort to the investment margins so intensively. In boom times, the need to accumulate business capital and commercial buildings is therefore less pressing. By reducing consumption risk, eliminating the inefficiency caused by imperfect risk-sharing therefore lowers the volatility of investment in business capital and commercial structures.

Another striking difference across the two cases is the decline in the volatility of real estate prices. Relative to the incomplete market allocation, $std(\Delta q_H)$ declines from 8.8 to 1.7 percent. This decline in volatility is not driven by the dynamics of rents. Indeed, relative to the incomplete markets model, the increase in the volatility of output also implies that the volatility of rents is higher under risk-sharing. The reduction in the volatility of commercial real estate prices is therefore due to the effect of transfers on the stochastic discount factor and investment.

Table 3: Complete vs. Incomplete Markets

	Data	Models	
	Estimated empirical moments	Incomplete Markets	Complete Markets
$std(\Delta y_F)$	1.6	1.6	2.6
$std(\Delta c)$	1.1	1.0	2.8
$std(\Delta x)$	5.8	5.8	1.2
$std(\Delta y_C)$	9.2	9.2	0.5
$std(\Delta q_H)$	8.9	8.8	1.7
$std(\Delta p_C)$	3.1	3.1	1.6
$std(\pi)$	0.5	0.6	0.6
$std(i_B)$	2.6	2.6	2.6
$E(i_B)$	6.5	6.5	6.5
$E(\mathcal{F})$	0.31	0.31	0.29

The effect of the policy on risk premiums is reported in Table 4. Since the policy affects the stochastic discount factor, it also has a large impact on risk premiums. As discussed in Section 4, we obtain an average risk premium $E(r^H - r^F)$ of 1.3 percent in the incomplete markets model. Under complete markets, the commercial real estate premium declines to 0 percent.

Table 4: Effect on the Risk Premium

	Incomplete markets	Complete markets
$E(r^H - r^F)$	1.3	0.0

Leveraged Real Estate Cycles under Complete Markets

The large reduction in the volatility of investment in new structures is also reflected in the amplitude of the leverage cycle. The left panel of Figure 6 shows the response of b under risk-sharing and compares it with the incomplete markets model. The right panel performs this comparison for the case of commercial real estate prices.

The very large difference that is obtained under complete markets demonstrates that the distribution of risk has a critical impact on the joint dynamics of commercial real estate prices and leverage. Under incomplete markets, the volatility in the real estate market is caused by the large fluctuations in investment that are needed to stabilize the consumption of risk averse investors. Since building precautionary buffers facilitates consumption smoothing, this inefficient allocation of risk is a source of procyclicality. In boom times, consumption smoothing is achieved by pouring resources into the construction sector. In contrast, during recessions, households manage to avoid a larger decline in consumption by reducing investment in new construction. Since leverage is needed to finance investment in commercial structures, the volatility induced by imperfect risk-sharing in turn leads to more volatile fluctuations in credit.

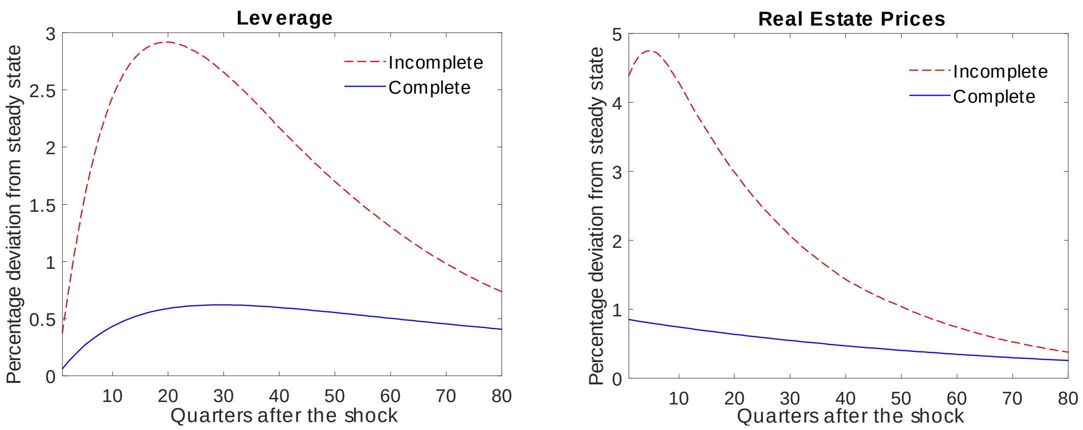

Figure 6. Leveraged real estate cycles: complete vs. incomplete markets

This comparison illustrates that boom-bust cycles in leverage and real estate prices can be amplified by imperfect risk-sharing. Under incomplete markets, the allocation of risk is inefficient in the sense that borrowers bear too much consumption risk. Introducing risk-sharing schemes therefore addresses the root cause of the problem. The difference in magnitude that we obtain also suggests that this source of inefficiency can be sizeable, hence it is relevant from a policy perspective.

This exercise is performed under the assumption that bankers are risk neutral. This assumption implies that the banking sector is willing to support as much risk as possible. The quantitative effect that we obtain therefore represents an upper bound. If bankers are risk averse, optimal risk-sharing implies that a larger share of consumption risk needs to be supported by households. Relative to the results illustrated in Figure 6, a larger share of consumption risk in the households sector would in turn imply larger fluctuations in new construction, and hence in real estate prices and leverage under complete markets.

We have checked that our conclusions are robust to the introduction of concavity in the utility function of bankers. In the case of log utility for example, the effect of the policy remains very large. At the same time, the magnitude of the effect critically depends on the initial degree of cross-agent heterogeneity that we assume. If agents are identical, the effect of risk-sharing will be negligible.

Deviations between Prices and Construction Costs under Complete Markets

As discussed in the introduction, over the business cycle, we observe large deviations between commercial real estate prices and construction costs. These fluctuations are procyclical and persistent. To illustrate the effect of market structures on this ratio, Figure 7 compares the response of the ratio q_H over p_C to a positive technology shock in the two cases. Whereas large and persistent deviations are obtained under incomplete markets, real estate prices essentially follow construction costs in the complete markets model.

In terms of the decomposition that is done in equation (21), this result firstly reflects that the time-variation in the distributive wedge essentially disappears under complete markets. The strong reduction in the volatility of new construction $std(\Delta y_C)$ then implies that the adjustment cost component of the formula, i.e. $\left(\frac{y_{Ht}}{h_t}\right)^{\epsilon_H}/\theta_1^H$ becomes very stable. Consequently, and as illustrated by the results reported in Table 3, the standard deviations of construction costs and commercial real estate prices are very similar once the inefficiency stemming from imperfect risk-sharing is removed.

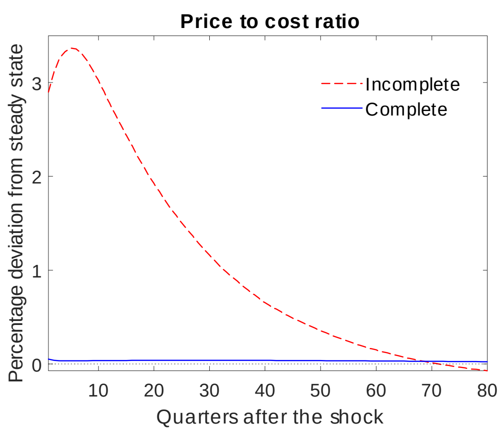

Figure 7. Price to cost ratio. Incomplete vs. complete markets model.

5.3 The COVID Shock under Complete Markets

The policy also significantly alters the transmission of asymmetric shocks, such as health shocks. This point is illustrated in Figure 8, which compares the response to the COVID shock analyzed in Section 4.1 with that obtained in the complete markets economy.

Relative to the incomplete markets benchmark, and as illustrated by the left panel of Figure 8, the first key difference is that the decline in aggregate welfare is significantly lower under risk-sharing. Aggregate welfare is determined by the sum of lifetime utilities of households and bankers, which are given by equations (1) and (15), respectively. Under the optimal policy, the effect of the shock on welfare fully dissipates after about 10 quarters. In contrast, the welfare loss is considerably more persistent in the incomplete markets model. Consequently, this comparison confirms that the economy as a whole is better off once a risk-sharing mechanism can be implemented.

The right panel illustrates that risk-sharing is obtained by implementing a transfer from bankers to households that amounts to around 1 percent of GDP on impact. Since bankers are risk neutral, it is optimal to let them absorb the effect of the shock by reducing their consumption. As transfers are equal to zero on average, this policy does not distort the steady state distribution of consumption. The policy only removes the inefficiency caused by market incompleteness.

Figure A.5 illustrates how the policy affects the adjustment to a COVID shock. As depicted by the upper left panel, the key difference is that agents can afford to dedicate more time to health-related activities under the optimal policy. Consequently, and as depicted by the upper right panel, the economy's health stock recovers much faster under risk-sharing.

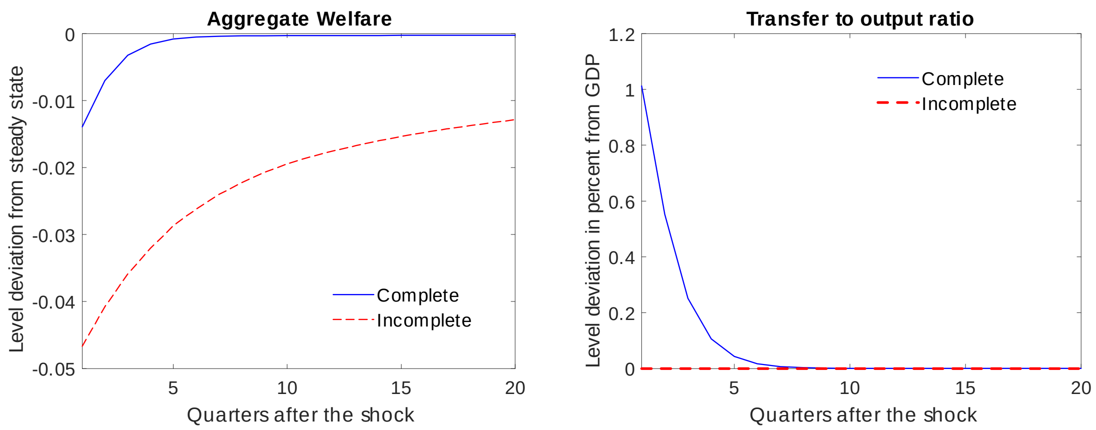

Figure 8. Response to a COVID shock. Incomplete vs. complete market economies.

The bottom left panel of Figure A.5 shows that this stronger recovery is made possible by the transfer that households receive from bankers. Indeed, the difference with the incomplete markets case is that household consumption increases despite the negative shock. Households can therefore afford to reduce hours worked in the final good and construction sectors and allocate more time to health-related activities, such as self-care or visits to the doctor. This in turn explains why the effect of the shock is less persistent once the optimal policy is in place.

Finally, and as illustrated by the bottom right panel, removing the distortion also has a large effect on the response of the price to construction costs ratio q_H/p_H. Whereas this ratio falls by around 2% on impact in the incomplete markets model, the decline obtained under complete markets is of an order of magnitude that is considerably smaller.

6 Conclusion

Lending and borrowing decisions are made by different types of agents with diverging interests. One potential source of pecuniary externality is therefore that IMRS or SDF across agents are not aligned.

In a model with risk averse households and risk neutral bankers, we find that correcting this externality can have a large effect on the joint dynamics of property prices and leverage. The optimal policy significantly reduces the procyclicality in real estate demand. These lower fluctuations in demand in turn reduce the volatility of new construction, which then attenuate the amplitude of the leverage cycle.

In the context of our model, the inefficiency stems from the fact that borrowers bear too much risk. This underinsurance problem amplifies real estate cycles because borrowers use the property market as a hedge against business cycle fluctuations. Providing insurance reduces the need to build precautionary buffers in good times, and hence the procyclicality of property markets.

One main limitation is that the model remains too stylized to fully characterize the complex nexus between the commercial property market and leverage. Introducing default into the analysis would for instance be a natural extension. At the same time, since default increases procyclicality, introducing this additional source of risk should in principle not overturn the main message of the paper.

Another simplification is that technology and monetary policy shocks are the only source of disturbance in the pre-COVID period. Introducing financial shocks, or other types of shocks, would be necessary to capture the effect of the 2007-2009 financial crisis.

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7 Data Appendix

Variable	Description	Source	Sample
y	Real gross domestic product	Bureau of Economic Analysis	1984q1-2020q1
c	Services, non-durables goods, non-defense government consumption	Bureau of Economic Analysis	1984q1-2020q1
x	Non-residential investment	Bureau of Economic Analysis	1984q1-2020q1
y_C	Investment in structures	Bureau of Economic Analysis	1984q1-2020q1
q_H	Index of commercial real estate prices	Federal Reserve Board	1984q1-2020q1
p_C	Index of construction prices non-financial corporate businesses	Census Bureau	1984q1-2020q1
$\mathcal{F}$	Interest rate expense to net operating income	National Association of Real Estate Investment Trusts	2000q1-2020q1
z	Commercial rents	REIS/Moody's	2005q1-2020q1
i_B	Bank prime loan rate	Federal Reserve Board	1984q1-2020q1
π	CPI inflation	Federal Reserve Board	1984q1-2020q1

8 Annex A

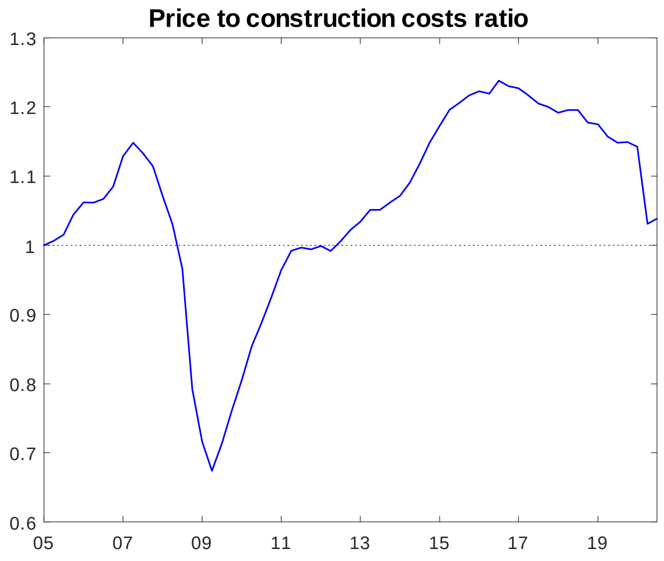

Figure A.1. Commercial real estate prices to construction costs ratio. Source: New warehouse building construction, Bureau of Labor Statistics, and Green Street Commercial Property Price Index. Normalized data, Q1 2005=1.

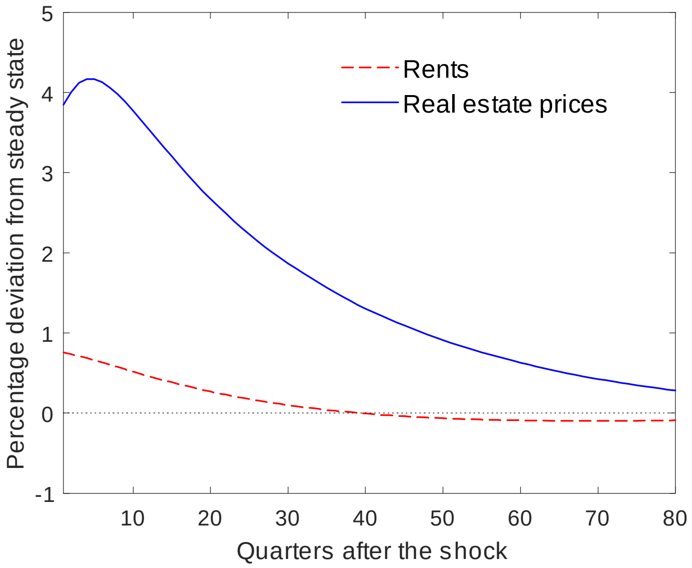

Figure A.2. Impulse response to a technology shock of commercial real estate prices and rents.

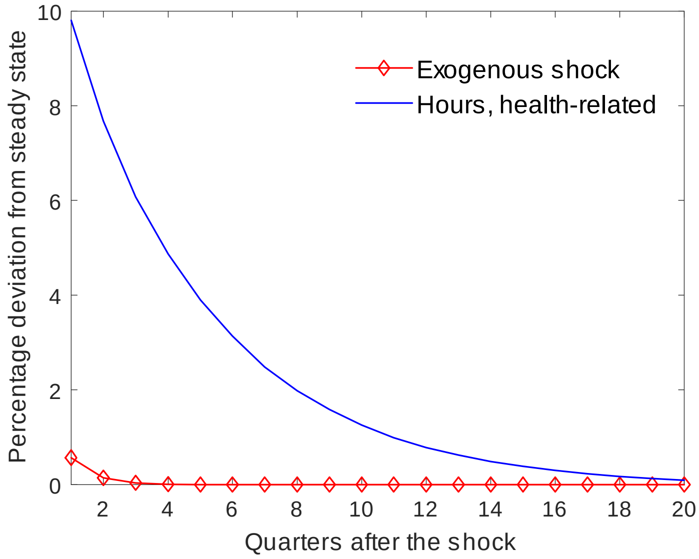

Figure A.3. Calibration of the health shock.

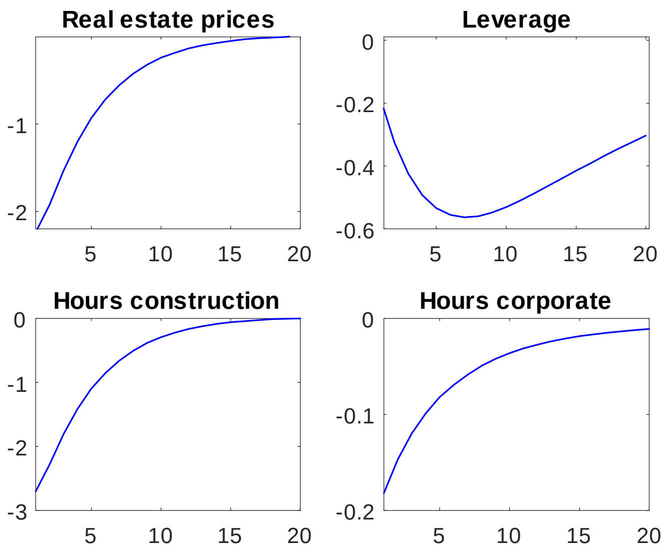

Figure A.4. Effect of the COVID shock on the real economy.

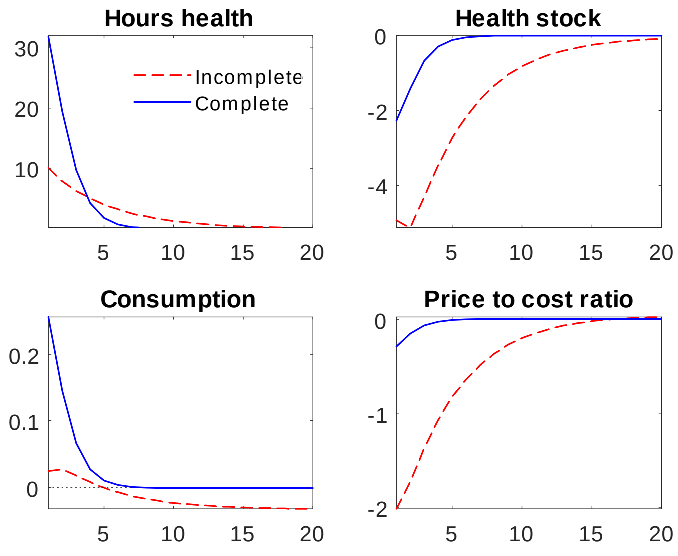

Figure A.5. Response to a COVID shock. Incomplete vs. complete market economies.

9 Annex B

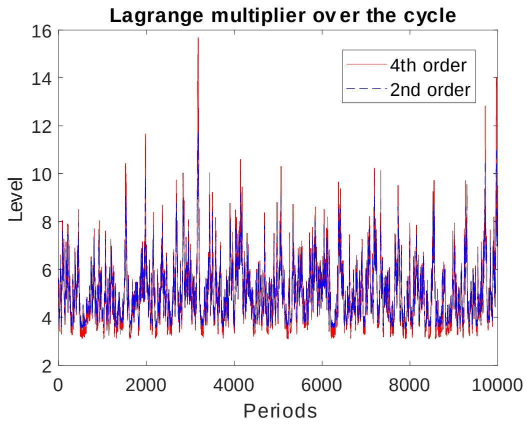

Figure B.1. Lagrange multiplier ϖ over the business cycle.

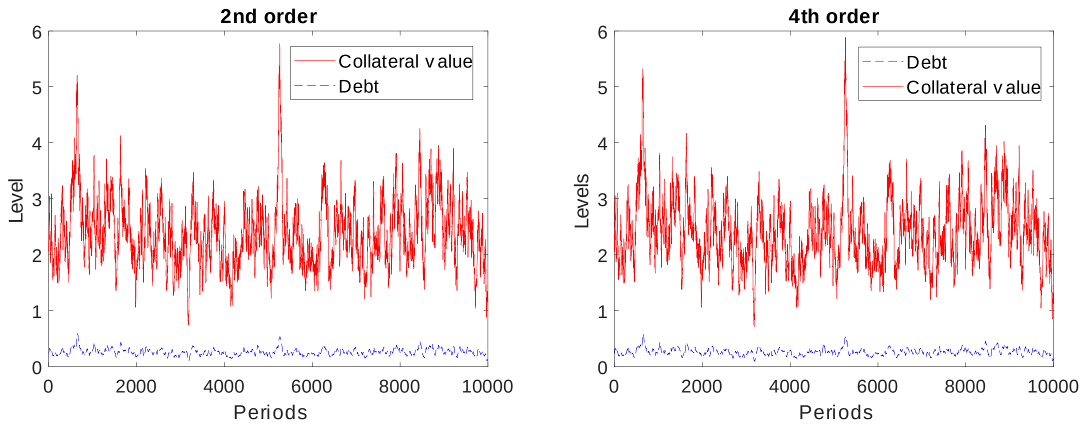

Figure B.2. Tightness of collateral constraint over the business cycle.

10 Appendix C

The model is solved using perturbation methods. We use the codes developed by Adjemian et al. (2014) and solve the model up to a fourth-order approximation. The deterministic steady state of the model can be solved analytically. Additional documentation concerning the derivation of closed-form solutions for the deterministic version of the model is available upon request.

10.1 The Incomplete Markets Equilibrium

Consumers

$$E_0 \sum_{t=0}^{\infty} \widehat{\beta}_A^t \frac{\left[c_{At} S_t^{\kappa} (\psi + L_t^{\upsilon}) - x_t\right]^{1-\sigma}}{1-\sigma}$$

where:

$$\widehat{\beta}_A = \widetilde{\beta}_A \gamma^{1-\sigma}$$

such that:

$$prof_{Tt} + w_t \left(N_{Ft} + N_{Ct} + N_{St}\right) + r_{Ct} k_{Ct} + r_{Ft} k_{Ft} + d_t t_{Ct} = c_{At} + i_{Tt}$$

$$\gamma k_{Tt+1} = (1-\delta_K) k_{Tt} + \left[\frac{\theta_1^K}{1-\epsilon_K} \left(\frac{i_{Tt}}{k_{Tt}}\right)^{1-\epsilon_K} + \theta_2^K\right] k_{Tt}$$

$$k_{Tt} = k_{Ft} + k_{Ct}$$

$$S_{t+1} = (1-\delta_S) S_t + N_{St} - \log \chi_t$$

$$L_t = 1 - N_{St} - N_{Ct} - N_{Ft}$$

Lagrangian:

$$\mathcal{L} = E_0 \left\{ \sum_{t=0}^{\infty} \widehat{\beta}_A^t \frac{\left[c_{At} S_t^{\kappa} (\psi + L_t^{\upsilon}) - x_t\right]^{1-\sigma}}{1-\sigma} \right.$$

$$+ \sum_{t=0}^{\infty} \widehat{\beta}_A^t \lambda_{At} \left[w_t \left(N_{Ft} + N_{Ct} + N_{St}\right) + r_{Ft} k_{Ft} + r_{Ct} \left(k_{Tt} - k_{Ft}\right) + prof_{Tt} - c_{At} - i_{Tt}\right]$$

$$+ \sum_{t=0}^{\infty} \widehat{\beta}_A^t q_{Kt} \lambda_{At} \left[(1-\delta_K) k_{Tt} + \left(\frac{\theta_1^K}{1-\epsilon_K} \left(\frac{i_{Tt}}{k_{Tt}}\right)^{1-\epsilon_K} + \theta_2^K\right) k_{Tt} - \gamma k_{Tt+1}\right]$$

$$+ \sum_{t=0}^{\infty} \widehat{\beta}_A^t \omega_t \lambda_{At} \left[(1-\delta_S) S_t + N_{St} - \log \chi_t - S_{t+1}\right]$$

$$+\sum_{t=0} \widehat{\beta}_A^t \varphi_t \left[m x_t + (1-m)c_{At}(\psi + L_t^{\upsilon}) - \gamma x_{t+1}\right] \Bigg\}$$

First-order conditions:

k_{Ft} :

$$r_{Ft} = r_{Ct} = r_t$$

c_{At} :

$$\left[(c_{At}S_t^{\kappa}(\psi + L_t^{\upsilon}) - x_t)^{-\sigma} + \varphi_t(1-m)\right] S_t^{\kappa}(\psi + L_t^{\upsilon}) = \lambda_{At}$$

N_{Ft} :

$$\left[(c_{At}S_t^{\kappa}(\psi + L_t^{\upsilon}) - x_t)^{-\sigma} + \varphi_t(1-m)\right] c_{At}S_t^{\kappa}\upsilon L_t^{\upsilon-1} = \lambda_{At}w_t$$

N_{Ct} :

$$w_{Ct} = w_{Ft} = w_t$$

x_{t+1} :

$$\varphi_t = \beta_A E_t \varphi_{t+1} m - \beta_A E_t \left[c_{At+1}S_{t+1}^{\kappa}(\psi + L_{t+1}^{\upsilon}) - x_{t+1}\right]^{-\sigma}$$

where:

$$\beta_A = \widehat{\beta}_A / \gamma$$

φ_t :

$$m x_t + (1-m)c_{At}S_t^{\kappa}(\psi + L_t^{\upsilon}) - \gamma x_{t+1} = 0$$

i_{Tt} :

$$1 = q_{Kt}\theta_1^K \left(\frac{i_{Tt}}{k_{Tt}}\right)^{-\epsilon_K}$$

k_{Tt+1} :

$$\lambda_{At}q_{Kt} = \beta_A E_t \lambda_{At+1} q_{Kt+1} \left[(1-\delta_K) + \frac{\theta_1^K}{1-\epsilon_K}\left(\frac{i_{Tt+1}}{k_{Tt+1}}\right)^{1-\epsilon_K} + \theta_2^K - \theta_1^K \left(\frac{i_{Tt+1}}{k_{Tt+1}}\right)^{1-\epsilon_K}\right]$$

$$+\beta_A E_t \lambda_{At+1} r_{t+1}$$

N_{St} :

$$\left[(c_{At} S_t^{\kappa}(\psi + L_t^{\upsilon}) - x_t)^{-\sigma} + \varphi_t(1-m)\right] c_{At} \upsilon L_t^{\upsilon-1} = \omega_t \lambda_{At}$$

S_{t+1} :

$$\omega_t \lambda_{At} = \widehat{\beta}_A E_t \lambda_{At+1}(1-\delta_S)\omega_{t+1} + \widehat{\beta}_A E_t \left[c_{At+1} S_{t+1}^{\kappa}\left(\psi + (L_{t+1})^{\upsilon}\right) - x_{t+1}\right]^{-\sigma} c_{At+1} \kappa S_{t+1}^{\kappa-1}\left(\psi + L_{t+1}{}^{\upsilon}\right)$$

ω_t :

$$(1-\delta_H) S_t + N_{St} - \log \chi_t - S_{t+1} = 0$$

λ_{At} :

$$w_t \left(N_{Ft} + N_{St} + N_{Ct}\right) + r_{Ft} k_{Ft} + r_{Ct}\left(k_{Tt} - k_{Ft}\right) + prof_{Tt} - c_{At} - i_{Tt} = 0$$

φ_t :

$$m x_t + (1-m) c_{At}(\psi + L_t^{\upsilon}) - \gamma x_{t+1} = 0$$

q_{Kt} :

$$(1-\delta_K) k_{Tt} + \left[\frac{\theta_1^K}{1-\epsilon_K}\left(\frac{i_{Tt}}{k_{Tt}}\right)^{1-\epsilon_K} + \theta_2^K\right] k_{Tt} - \gamma k_{Tt+1} = 0$$

Bankers

$$E_0 \sum_{t=0}^{\infty} \widehat{\beta}_B^t c_{Bt}$$

where:

$$\widehat{\beta}_B = \beta_B \gamma$$

such that:

$$\gamma b_{t+1} - (1-\varkappa)\frac{b_t}{1+\pi_t} + c_{Bt} = \left[\varkappa + (1-\varkappa) i_{Bt-1}\right] \frac{b_t}{1+\pi_t}$$

$$
\begin{aligned}
\mathcal{L} &= E_0 \left\{ \sum_{t=0}^{\infty} \widehat{\beta}_B^t c_{Bt} \right. \\
&\left. + \sum_{t=0}^{\infty} \widehat{\beta}_B^t \lambda_{Bt} \left[\left[\varkappa + \left(1 + i_{Bt-1}\right)(1-\varkappa)\right] \frac{b_t}{1+\pi_t} - \gamma b_{t+1} - c_{Bt} \right] \right\}
\end{aligned}
$$

First-order conditions:

c_{Bt} :

$$1 = \lambda_{Bt}$$

b_{t+1} :

$$\lambda_{Bt} = \beta_B E_t \lambda_{Bt+1} \frac{1}{1+\pi_{t+1}} \left[\varkappa + \left(1 + i_{Bt}\right)(1-\varkappa)\right]$$

λ_{Bt} :

$$\varkappa \frac{b_t}{1+\pi_t} + (1-\varkappa) i_{Bt-1} \frac{b_t}{1+\pi_t} - \left(\gamma b_{t+1} - (1-\varkappa)\frac{b_t}{1+\pi_t}\right) - c_{Bt} = 0$$

Real estate developers

$$prof_{Dt} = z_t h_t - (1-\mu) p_{Ct} y_{Ct} - \varkappa \frac{b_t}{1+\pi_t} - (1-\varkappa) i_{Bt-1} \frac{b_t}{1+\pi_t}$$

such that:

$$\gamma b_{t+1} - (1-\varkappa)\frac{b_t}{1+\pi_t} \geq \mu p_{Ct} y_{Ct}$$

and:

$$(1-\delta_H) h_t + \left[\frac{\theta_1^H}{1-\epsilon_H} \left(\frac{y_{Ct}}{h_t}\right)^{1-\epsilon_H} + \theta_2^H \right] h_t - \gamma h_{t+1} \geq 0$$

The problem:

$$
\mathcal{L} = E_0 \sum_{t=0}^{\infty} \widehat{\beta}_A^t \frac{\lambda_{At}}{\lambda_{A0}} \left\{ \begin{array}{c}
\left[z_t h_t - (1-\mu) p_{Ct} y_{Ct} - \varkappa \frac{b_t}{1+\pi_t} - (1-\varkappa) i_{Bt-1} \frac{b_t}{1+\pi_t} \right] \\
+ \frac{\varpi_t}{\lambda_{At}} \left[\left(\gamma b_{t+1} - (1-\varkappa) \frac{b_t}{1+\pi_t} \right) - \mu p_{Ct} y_{Ct} \right] \\
+ q_{Ht} \left[(1-\delta_H) h_t + \left(\frac{\theta_1^H}{1-\epsilon_H} \left(\frac{y_{Ct}}{h_t}\right)^{1-\epsilon_H} + \theta_2^H \right) h_t - \gamma h_{t+1} \right]
\end{array} \right\}
$$

First-order conditions:

b_{t+1} :

$$\frac{\varpi_t}{\lambda_{At}} = \beta_A E_t \frac{\lambda_{At+1}}{\lambda_{At}} \left[\varkappa + (1-\varkappa) \left(\frac{\varpi_{t+1}}{\lambda_{At+1}} + i_{Bt} \right) \right]$$

y_{Ct} :

$$q_{Ht} \theta_1^H \left(\frac{y_{Ct}}{h_t} \right)^{-\epsilon_H} = p_{Ct} \left(1 - \mu + \mu \frac{\varpi_t}{\lambda_{At}} \right)$$

ϖ_t :

$$\gamma b_{t+1} - (1-\varkappa) \frac{b_t}{1+\pi_t} = \mu p_{Ct} y_{Ct}$$

h_{t+1} :

$$q_{Ht} = \beta_A E_t \frac{\lambda_{At+1}}{\lambda_{At}} q_{Ht+1} \left[(1-\delta_H) + \frac{\theta_1^H}{1-\epsilon_H} \left(\frac{y_{Ct+1}}{h_{t+1}} \right)^{1-\epsilon_H} + \theta_2^H - \theta_1^H \left(\frac{y_{Ct+1}}{h_{t+1}} \right)^{1-\epsilon_H} \right] + \beta_A E_t \frac{\lambda_{At+1}}{\lambda_{At}} z_{t+1}$$

where:

$$\beta_A = \widetilde{\beta}_A / \gamma$$

q_{Ht} :

$$(1-\delta_H) h_t + \left[\frac{\theta_1^H}{1-\epsilon_H} \left(\frac{y_{Ct}}{h_t} \right)^{1-\epsilon_H} + \theta_2^H \right] h_t - \gamma h_{t+1} = 0$$

Construction

$$prof_{Ct} = p_{Ct} \left(k_{Ct}^{\vartheta} N_{Ct}^{1-\vartheta} \right)^{1-\eta} t_{Ct}^{\eta} - w_t N_{Ct} - r_{Ct} k_{Ct} - d_t t_{Ct}$$

First-order conditions:

N_{Ct} :

$$p_{Ct}(1-\vartheta)(1-\eta) \frac{y_{Ct}}{N_{Ct}} = w_{Ct}$$

k_{Ct} :

$$p_{Ct} \vartheta (1-\eta) \frac{y_{Ct}}{k_{Ct}} = r_{Ct}$$

t_{Ct} :

$$p_{Ct}\eta\frac{y_{Ct}}{t_{Ct}} = d_t$$

Firms

$$prof_{Ft} = A_t k_{Ft}^{\alpha} N_{Ft}^{\xi} h_t^{1-\alpha-\xi} - w_t N_{Ft} - r_{Ft} k_{Ft} - z_t h_t$$

First-order conditions:

k_{Ft} :

$$r_{Ft} = \alpha\frac{y_{Ft}}{k_{Ft}}$$

N_{Ft} :

$$w_{Ft} = \xi\frac{y_{Ft}}{N_{Ft}}$$

h_t :

$$z_t = (1-\alpha-\xi)\frac{y_{Ft}}{h_t}$$

Central bank

$$i_{Bt} = \overline{i_B} + \phi_\pi(\pi_t - \pi^*) + \log\iota_t$$

Resource constraint

$$A_t k_{Ft}^{\alpha} N_{Ft}^{\xi} h_t^{1-\alpha-\xi} = c_{At} + i_{Tt} + c_{Bt}$$

10.2 The Dynamic System

26 Endogenous variables: $c_A, c_B, N_F, N_C, N_H, L, k_T, k_C, k_F, b, h, x, S, p_C, q_K, q_H,$

$\lambda_A, \lambda_B, \varpi, \omega, \varphi, i_B, \pi, i_T, y_C, y_F$.

3 exogenous variables: A_t, χ_t, ι_t.

$$\left[(c_{At}S_t^{\kappa}(\psi + L_t^{\upsilon}) - x_t)^{-\sigma} + \varphi_t(1-m)\right] S_t^{\kappa}(\psi + L_t^{\upsilon}) = \lambda_{At}$$

$$\left[(c_{At}S_t^{\kappa}(\psi+L_t^{\upsilon})-x_t)^{-\sigma}+\varphi_t(1-m)\right]c_{At}S_t^{\kappa}\upsilon L_t^{\upsilon-1}=\lambda_{At}\xi\frac{y_{Ft}}{N_{Ft}}$$

$$mx_t+(1-m)c_{At}S_t^{\kappa}(\psi+L_t^{\upsilon})-\gamma x_{t+1}=0$$

$$\varphi_t=\beta_A E_t\varphi_{t+1}m-\beta_A E_t\left[c_{At+1}S_{t+1}^{\kappa}(\psi+L_{t+1}^{\upsilon})-x_{t+1}\right]^{-\sigma}$$

$$p_{Ct}(1-\vartheta)(1-\eta)\frac{y_{Ct}}{N_{Ct}}=\xi\frac{y_{Ft}}{N_{Ft}}$$

$$p_{Ct}\vartheta(1-\eta)\frac{y_{Ct}}{k_{Ct}}=\alpha\frac{y_{Ft}}{k_{Ft}}$$

$$\begin{aligned}\lambda_{At}q_{Kt} \quad = \quad & \beta_A E_t\lambda_{At+1}q_{Kt+1}\left[(1-\delta_K)+\frac{\theta_1^K}{1-\epsilon_K}\left(\frac{i_{Tt+1}}{k_{Tt+1}}\right)^{1-\epsilon_K}+\theta_2^K-\theta_1^K\left(\frac{i_{Tt+1}}{k_{Tt+1}}\right)^{1-\epsilon_K}\right] \\ & +\beta_A E_t\lambda_{At+1}\alpha\frac{y_{Ft+1}}{k_{Ft+1}}\end{aligned}$$

$$1=q_{Kt}\theta_1^K\left(\frac{i_{Tt}}{k_{Tt}}\right)^{-\epsilon_K}$$

$$(1-\delta_K)k_{Tt}+\left[\frac{\theta_1^K}{1-\epsilon_K}\left(\frac{i_{Tt}}{k_{Tt}}\right)^{1-\epsilon_K}+\theta_2^K\right]k_{Tt}-\gamma k_{Tt+1}=0$$

$$k_{Tt}=k_{Ct}+k_{Ft}$$

$$L_t=1-N_{St}-N_{Ft}-N_{Ct}$$

$$\omega_t=\xi\frac{y_{Ft}}{N_{Ft}}$$

$$\omega_t\lambda_{At}=\widehat{\beta}_A E_t\lambda_{At+1}(1-\delta_S)\omega_{t+1}+\widehat{\beta}_A E_t\left[c_{At+1}S_{t+1}^{\kappa}\left(\psi+{L_{t+1}}^{\upsilon}\right)-x_{t+1}\right]^{-\sigma}c_{At+1}\kappa S_{t+1}^{\kappa-1}\left(\psi+{L_{t+1}}^{\upsilon}\right)$$

$$S_{t+1}=(1-\delta_S)S_t+N_{St}-\log\chi_t$$

$$1 = \lambda_{Bt}$$

$$\lambda_{Bt} = \beta_B E_t \lambda_{Bt+1} \frac{1}{1+\pi_{t+1}} \left[\varkappa + (1-\varkappa) i_{Bt} \right]$$

$$\varkappa \frac{b_t}{1+\pi_t} + (1-\varkappa) i_{Bt-1} \frac{b_t}{1+\pi_t} - \left(\gamma b_{t+1} - (1-\varkappa) \frac{b_t}{1+\pi_t} \right) - c_{Bt} = 0$$

$$\frac{\varpi_t}{\lambda_{At}} = \beta_A E_t \frac{\lambda_{At+1}}{\lambda_{At}} \frac{1}{1+\pi_{t+1}} \left[(1-\varkappa) \left(\frac{\varpi_{t+1}}{\lambda_{At+1}} + i_{Bt} \right) \right]$$

$$q_{Ht} \theta_1^H \left(\frac{y_{Ct}}{h_t} \right)^{-\epsilon_H} = p_{Ct} \left(1 - \mu + \mu \frac{\varpi_t}{\lambda_{At}} \right)$$

$$\gamma b_{t+1} - (1-\varkappa) \frac{b_t}{1+\pi_t} = \mu p_{Ct} y_{Ct}$$

$$\begin{aligned} q_{Ht} \quad = \quad & \beta_A E_t \frac{\lambda_{At+1}}{\lambda_{At}} q_{Ht+1} \left[(1-\delta_H) + \frac{\theta_1^H}{1-\epsilon_H} \left(\frac{y_{Ct+1}}{h_{t+1}} \right)^{1-\epsilon_H} + \theta_2^H - \theta_1^H \left(\frac{y_{Ct+1}}{h_{t+1}} \right)^{1-\epsilon_H} \right] \\ & + \beta_A E_t \frac{\lambda_{At+1}}{\lambda_{At}} (1-\alpha-\xi) \frac{y_{Ft+1}}{h_{t+1}} \end{aligned}$$

$$(1-\delta_H) h_t + \left[\frac{\theta_1^H}{1-\epsilon_H} \left(\frac{y_{Ct}}{h_t} \right)^{1-\epsilon_H} + \theta_2^H \right] h_t - \gamma h_{t+1} = 0$$

$$i_{Bt} = \overline{i_B} + \phi_\pi (\pi_t - \pi^*) + \log \iota_t$$

$$y_{Ft} = c_{At} + c_{Bt} + i_{Tt}$$

$$y_{Ft} = A_t k_{Ft}^{\alpha} N_{Ft}^{\xi} h_t^{1-\alpha-\xi}$$

$$y_{Ct} = \left(k_{Ct}^{\vartheta} N_{Ct}^{1-\vartheta} \right)^{1-\eta} t_{Ct}^{\eta}$$

Exogenous shock processes:

$$\log \chi_t = \rho_\chi \log \chi_{t-1} + \varepsilon_{\chi t}$$

$$\log A_t = \rho_A \log A_{t-1} + \varepsilon_{At}$$

$$\log \iota_t = \rho_\iota \log \iota_{t-1} + \varepsilon_{\iota t}$$

10.3 The Complete Markets Equilibrium

The complete markets equilibrium corresponds to the case $\lambda_{At} = \lambda_{Bt} = \lambda_t$. Relative to the incomplete market equilibrium, the difference is that we add the instrument tr_t. To preserve the initial allocation of consumption across agents, we further impose the steady state restriction $tr = 0$. This restriction in turn implies a steady state value for marginal utility that ensures that the initial allocation of consumption prevailing under incomplete markets is preserved.

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