Inflation Variability and the Level of Inflation

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– *Work-in-Progress* –

9th ECB Workshop on Forecasting Techniques

June 3 2016

Does the variability of inflation increase when the level of inflation increases?

Old Question:

Okun (1971), Friedman (1977), Logue and Willett (1976), Fischer (1981), Taylor (1981), Engle (1983), … Devereux (1989), Ball (1992), … Garcia and Perron (1996) …

Issues:

(i) What variability? (Anticipated/Unanticipated, Long-run/Short-run, etc.)

(ii) What variation in level? (Cross-section, time-series, etc.)

Ball and Cecchetti (BPEA 1990) "Inflation and Uncertainty at Short and Long Horizons" …

(1) $$\pi_t = \hat{\pi}_t + \eta_t,$$

(2) $$\hat{\pi}_t = \hat{\pi}_{t-1} + \epsilon_t,$$

(10) $$\sigma_\epsilon^2(t) = \beta_0 + \beta_1 \hat{\pi}_{t-1};$$

(11) $$\sigma_\eta^2(t) = \delta_0 + \delta_1 \hat{\pi}_{t-1}.$$

Their findings:

- US: $\beta_1 > 0$, $\delta_1 \approx 0$
- Different for some other countries

Our contribution: Revisit, augmenting their analysis with (a) more data and (b) improved filtering tools.

Framework: Univariate UCSV model (with outlier adjustment: SW(2015))

$$\pi_t = \tau_t + \varepsilon_t$$

$$\tau_t = \tau_{t\text{-}1} + \sigma_{\Delta\tau,t} \times \eta_{\tau,t}$$

$$\varepsilon_t = \sigma_{\varepsilon,t} \times s_t \times \eta_{\varepsilon,t}$$

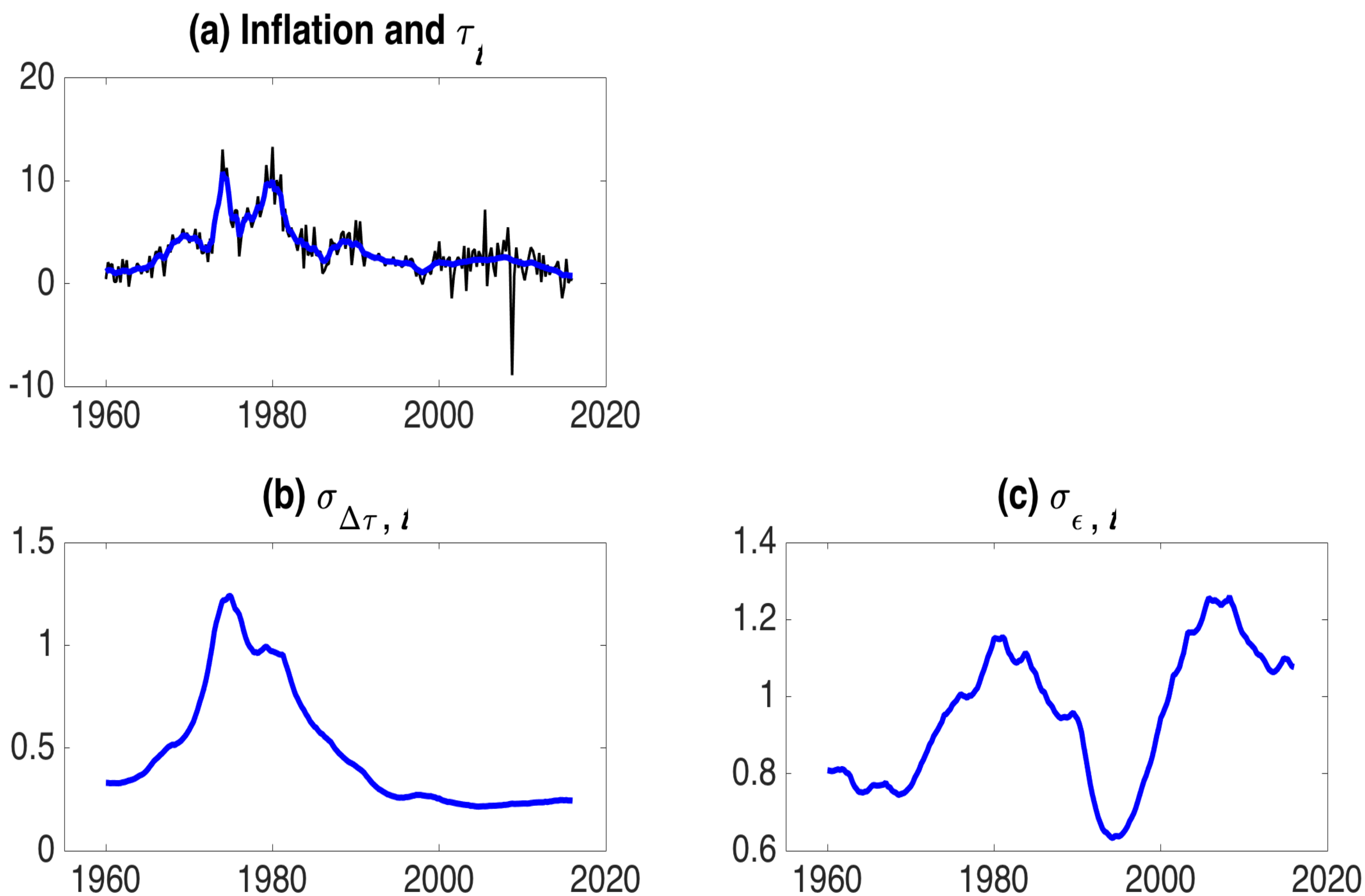
$$\Delta\ln(\sigma^2_{\varepsilon,t}) = \gamma_\varepsilon \nu_{\varepsilon,t}$$

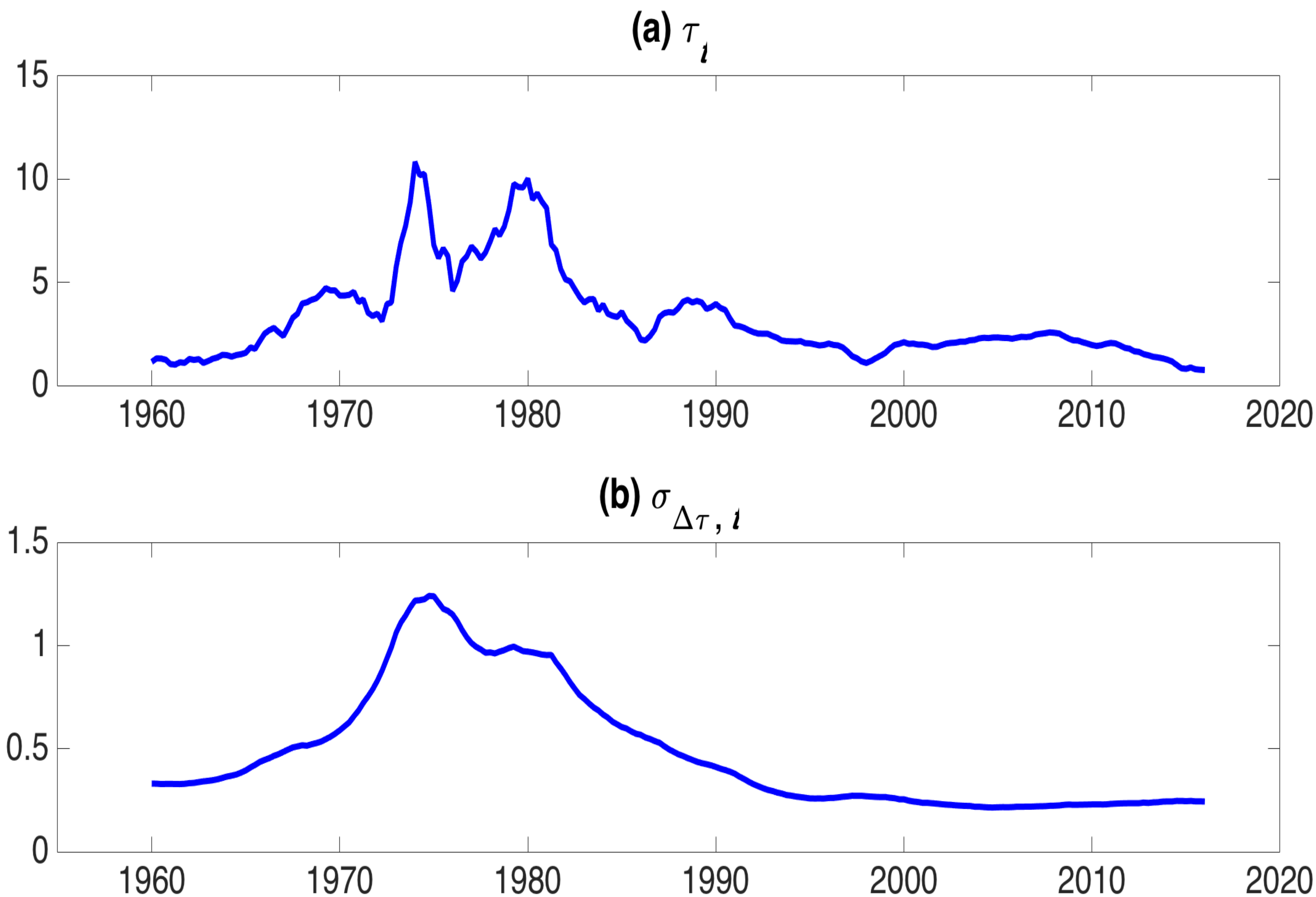
$$\Delta\ln(\sigma^2_{\Delta\tau,t}) = \gamma_{\Delta\tau} \nu_{\Delta\tau,t}$$

$(\eta_\varepsilon, \eta_\tau, \nu_\varepsilon, \nu_{\Delta\tau})$ are iidN(0, I_4)

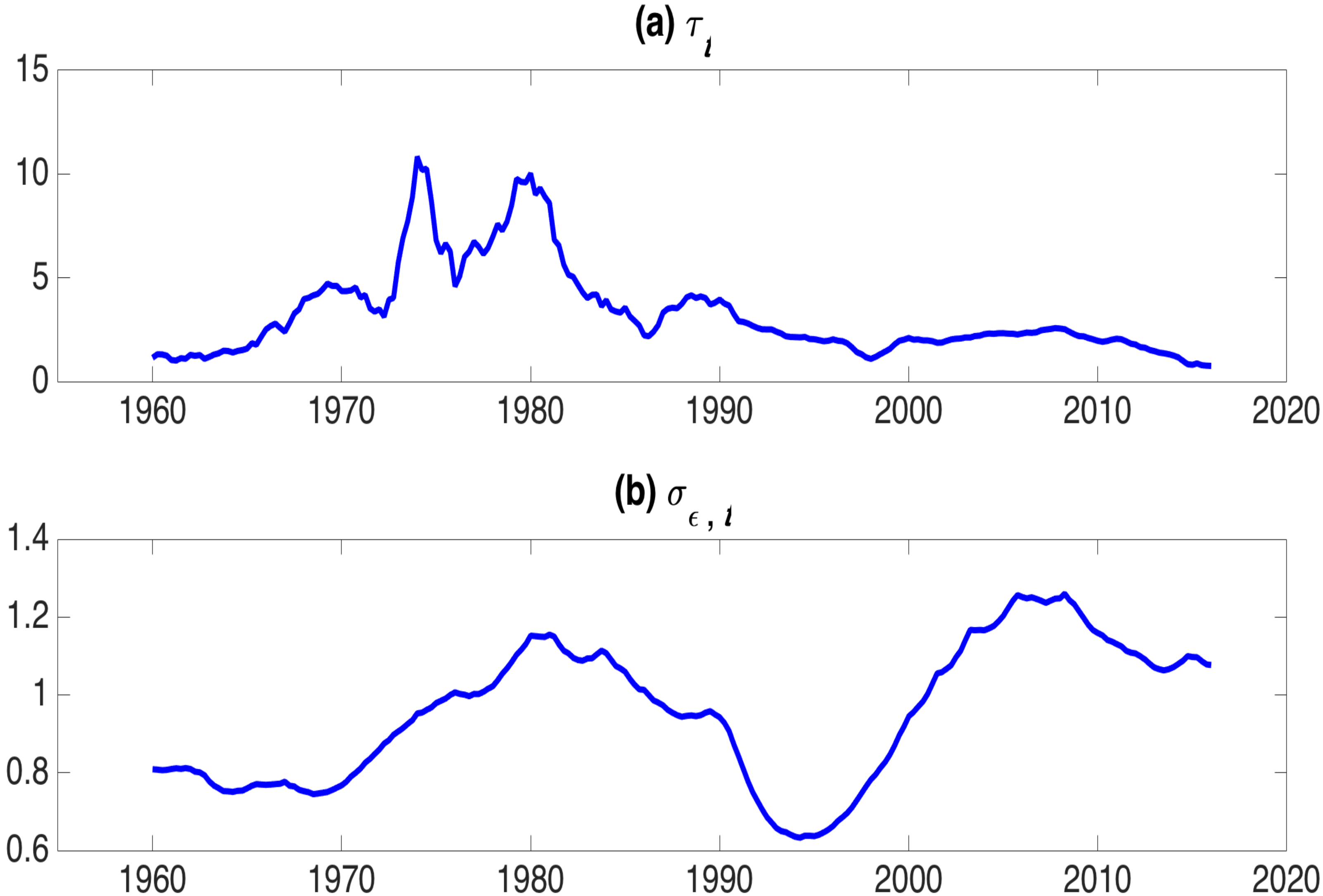
s_t = i.i.d. multinomial with values 1, 5, 10
and probability 0.975, 1/60, and 1/120

US Inflation





(a) τ_t
15
10
5
0
1960
1970
1980
1990
2000
2010
2020
(b) $\sigma_{\Delta\tau,t}$
1.5
1
0.5
0
1960
1970
1980
1990
2000
2010
2020



(a) τ_t
15
10
5
0
1960
1970
1980
1990
2000
2010
2020
(b) $\sigma_{\epsilon,t}$
1.4
1.2
1
0.8
0.6
1960
1970
1980
1990
2000
2010
2020

UCSV model with regressors in volatility process

$$\pi_t = \tau_t + \varepsilon_t$$

$$\tau_t = \tau_{t-1} + \sigma_{\Delta\tau,t} \times \eta_{\tau,t}$$

$$\varepsilon_t = \sigma_{\varepsilon,t} \times s_t \times \eta_{\varepsilon,t}$$

$$\ln(\sigma^2_{\varepsilon,t}) = x_t'\boldsymbol{\delta} + \xi_{\varepsilon,t} \qquad \text{with } \Delta\xi_{\varepsilon,t} = \gamma_\varepsilon \nu_{\varepsilon,t}$$

$$\ln(\sigma^2_{\Delta\tau,t}) = x_t'\boldsymbol{\beta} + \xi_{\Delta\tau,\mathrm{t}} \qquad \text{with } \Delta\xi_{\Delta\tau,t} = \gamma_{\Delta\tau} \nu_{\Delta\tau,t}$$

$(\eta_\varepsilon, \eta_\tau, \nu_\varepsilon, \nu_{\Delta\tau})$ are iidN(0, I_4)

$x_t = \tau_{t-1}$ (Ball-Cecchetti: $\beta > 0$, $\delta \approx 0$?)

Implications for forecasting:

$$\pi_{t+h} = (\tau_{t+h} - \tau_t) + \tau_{t|t} + (\tau_t - \tau_{t|t}) + \varepsilon_{t+h}$$

$$\tau_{t+h} - \tau_t = \sum_{i=1}^{h} \sigma_{\Delta\tau,t+i} \eta_{\tau,t+i}$$

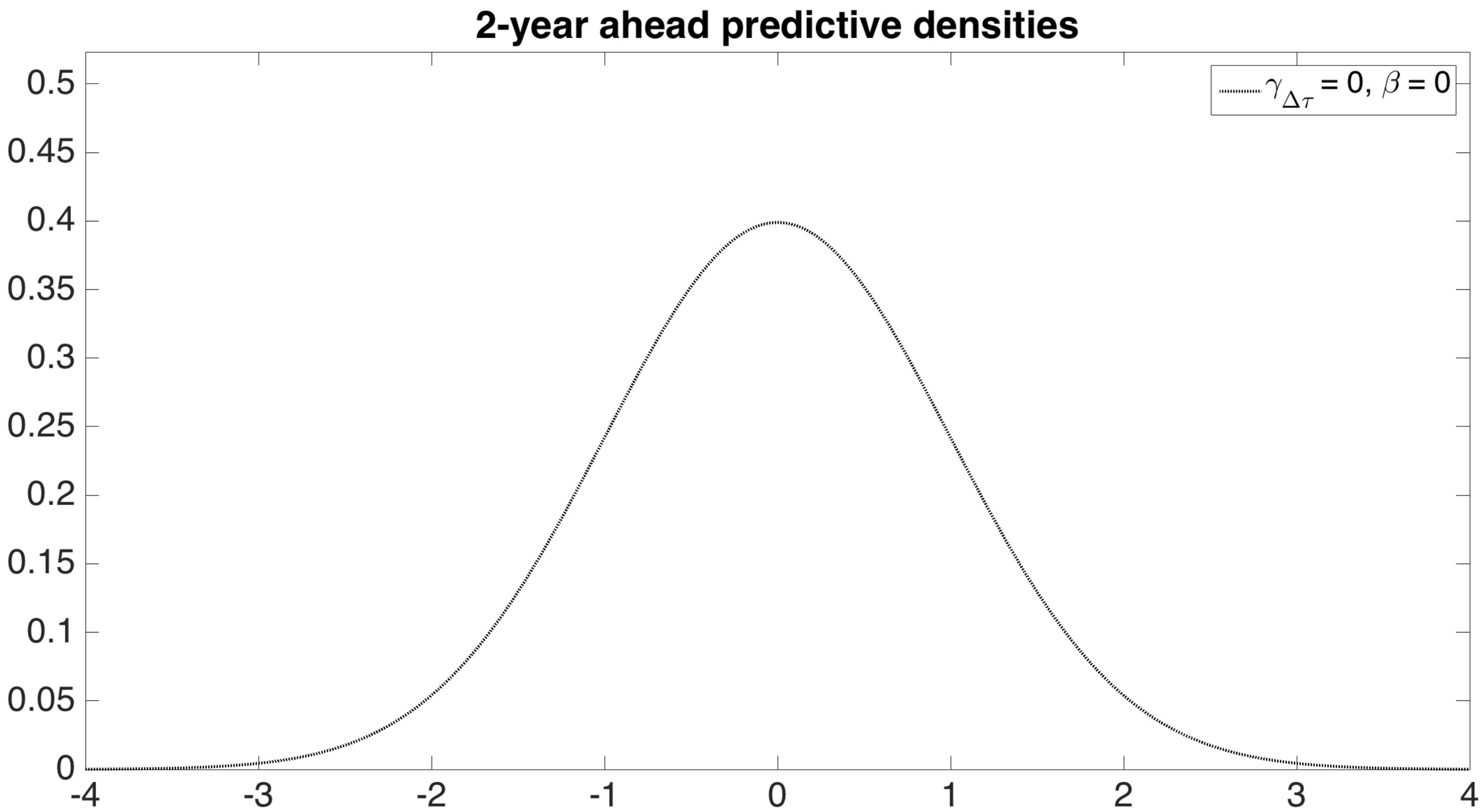
$$\ln(\sigma^2_{\Delta\tau,t}) = x_t'\beta + \xi_{\Delta\tau,\mathrm{t}} \qquad \text{with } \Delta\xi_{\Delta\tau,t} = \gamma_{\Delta\tau} \nu_{\Delta\tau,t}$$

$$(\eta_{\tau,t}, \nu_{\Delta\tau,t}) \sim \mathrm{iidN}(0, \mathrm{I}_2)$$

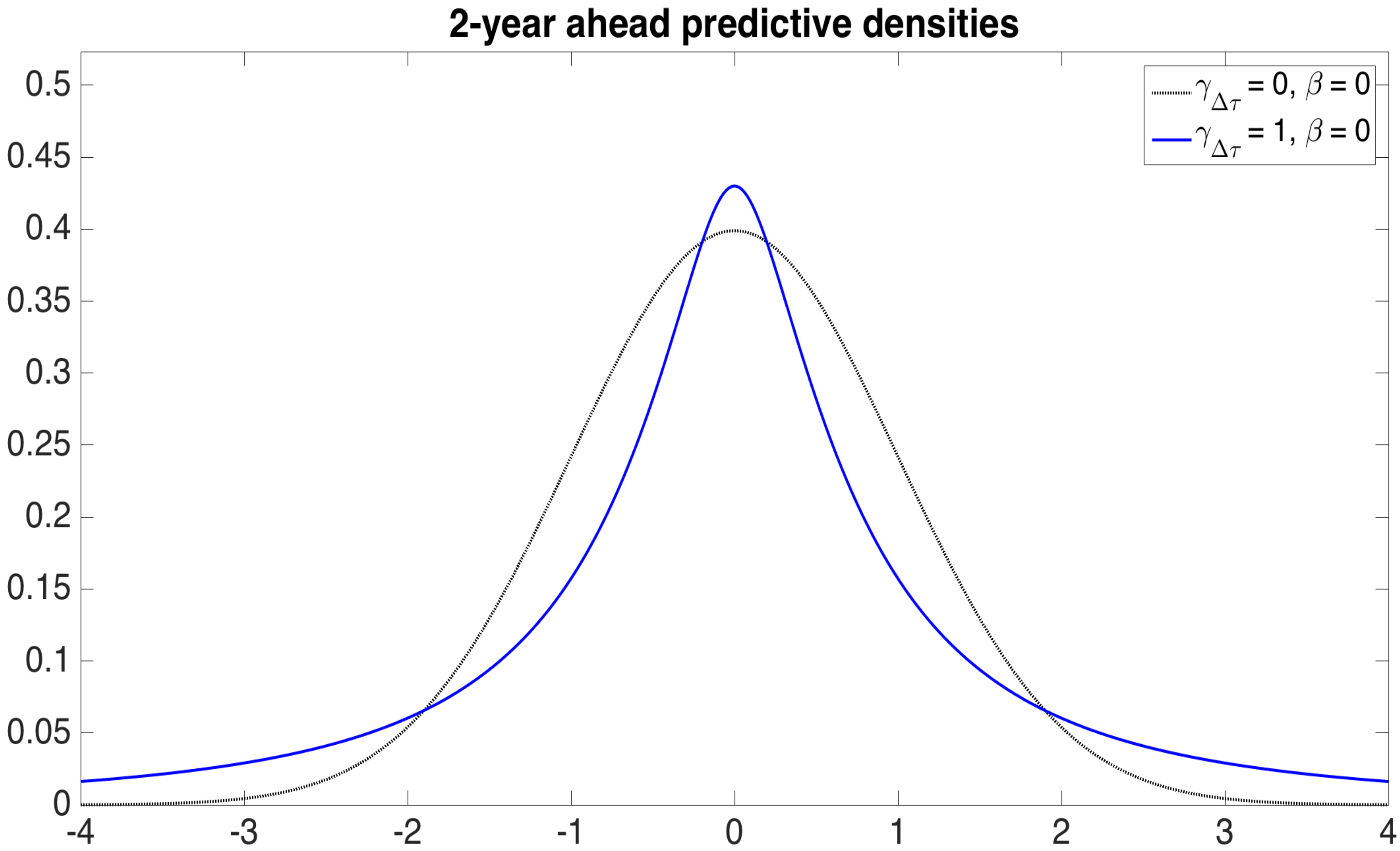
Three cases

- (i) $\gamma_{\Delta\tau} = \beta = 0$
- (ii) $\gamma_{\Delta\tau} > 0,\ \beta = 0$
- (iii) $\gamma_{\Delta\tau},\ \beta > 0$

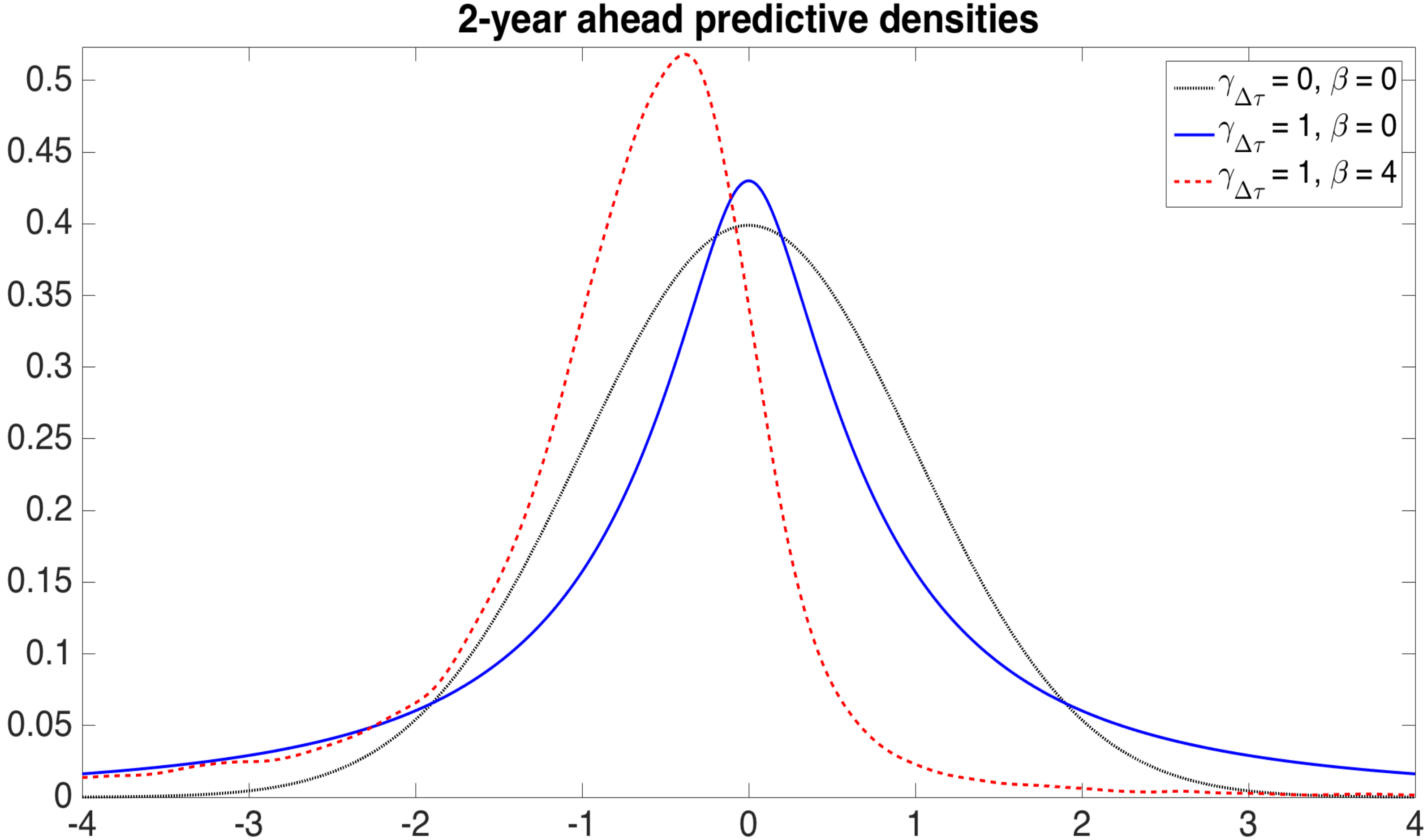
Illustrative Predictive Densities



Illustrative Predictive Densities



Illustrative Predictive Densities



Some empirical results

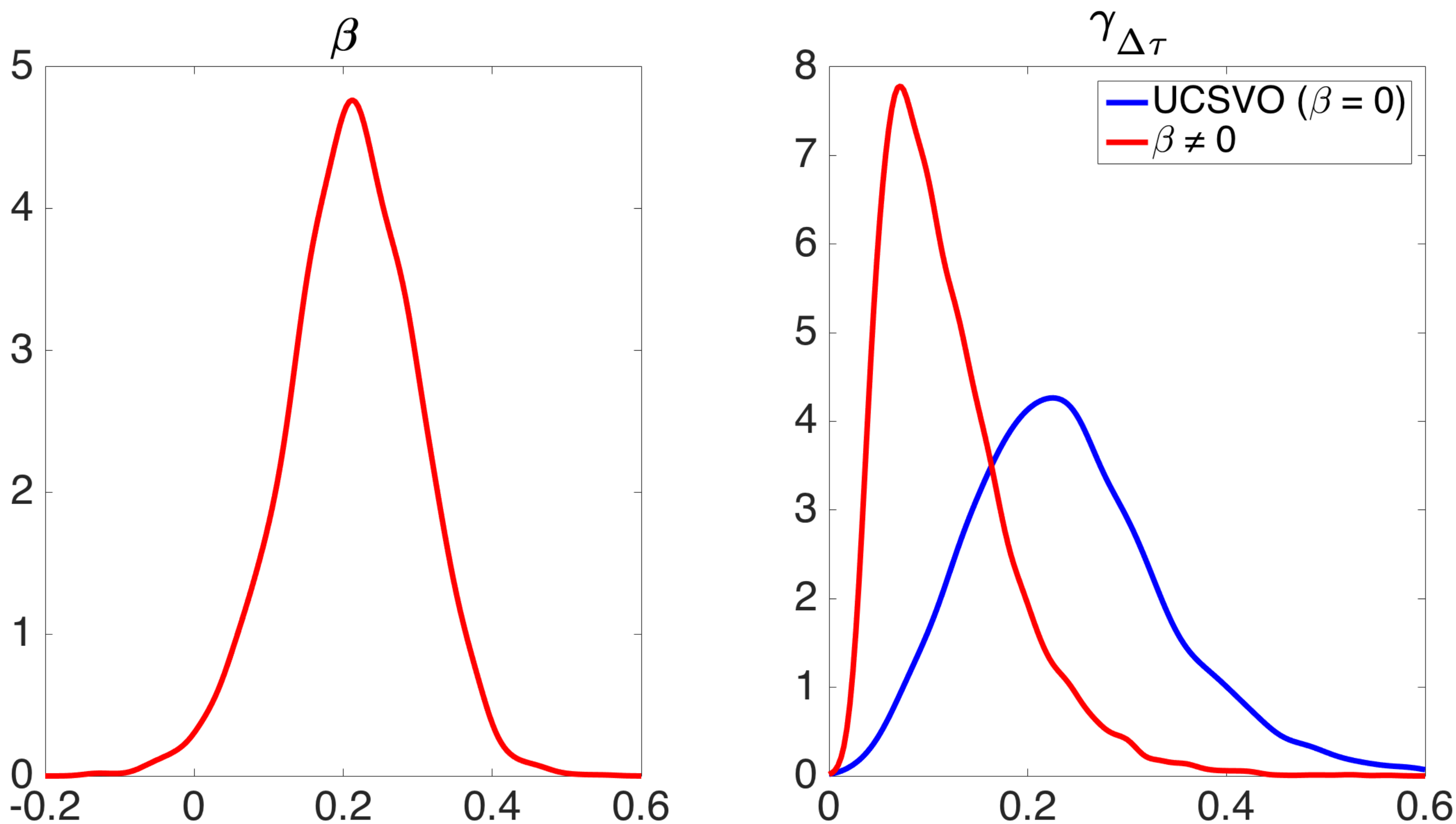
$$\ln(\sigma^2_{\varepsilon,t}) = x_t'\boldsymbol{\delta} + \xi_{\varepsilon,t} \qquad \text{with } \Delta\xi_{\varepsilon,t} = \gamma_\varepsilon \nu_{\varepsilon,t}$$

$$\ln(\sigma^2_{\Delta\tau,t}) = x_t'\boldsymbol{\beta} + \xi_{\Delta\tau,\mathrm{t}} \qquad \text{with } \Delta\xi_{\Delta\tau,t} = \gamma_{\Delta\tau} \nu_{\Delta\tau,t}$$

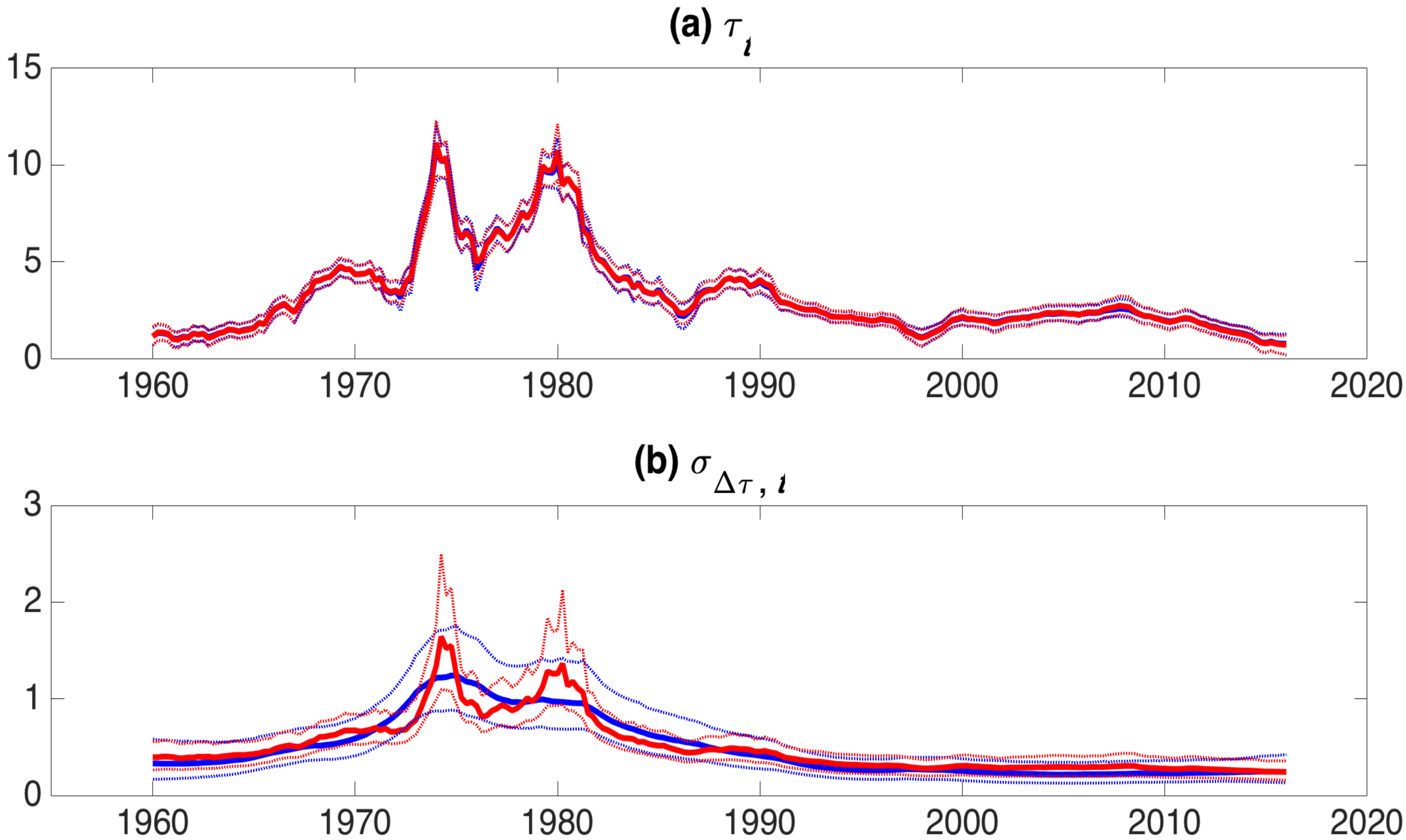
(We'll set $\delta = 0$ for the preliminary results presented here)

$$\ln(\sigma^2_{\Delta\tau,t}) = x_t'\boldsymbol{\beta} + \xi_{\Delta\tau,t} \qquad \text{with} \quad \Delta\xi_{\Delta\tau,t} = \gamma_{\Delta\tau}\nu_{\Delta\tau,t}$$

$$x_t = \tau_{t-1}$$



UCSV and $\beta \neq 0$



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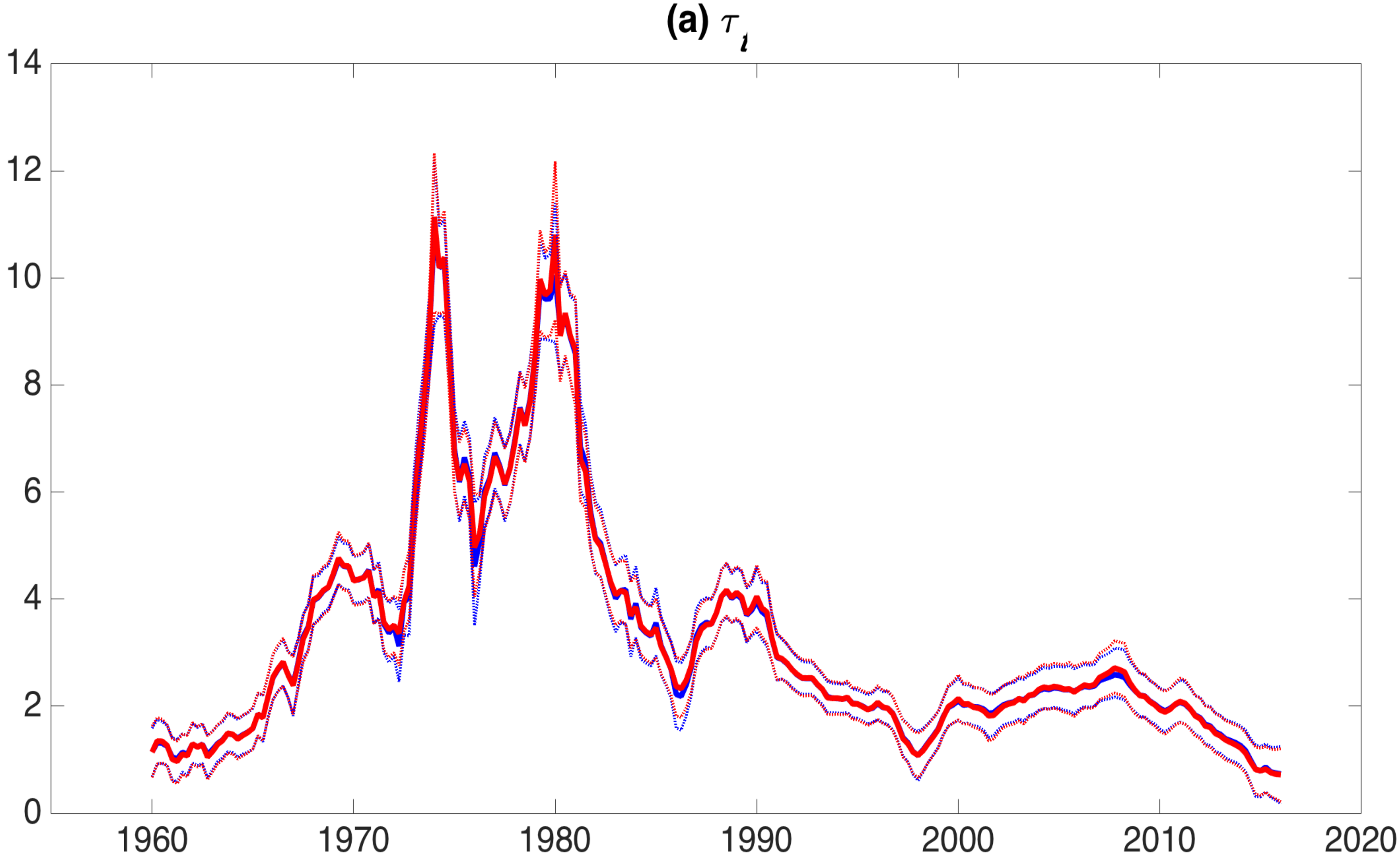


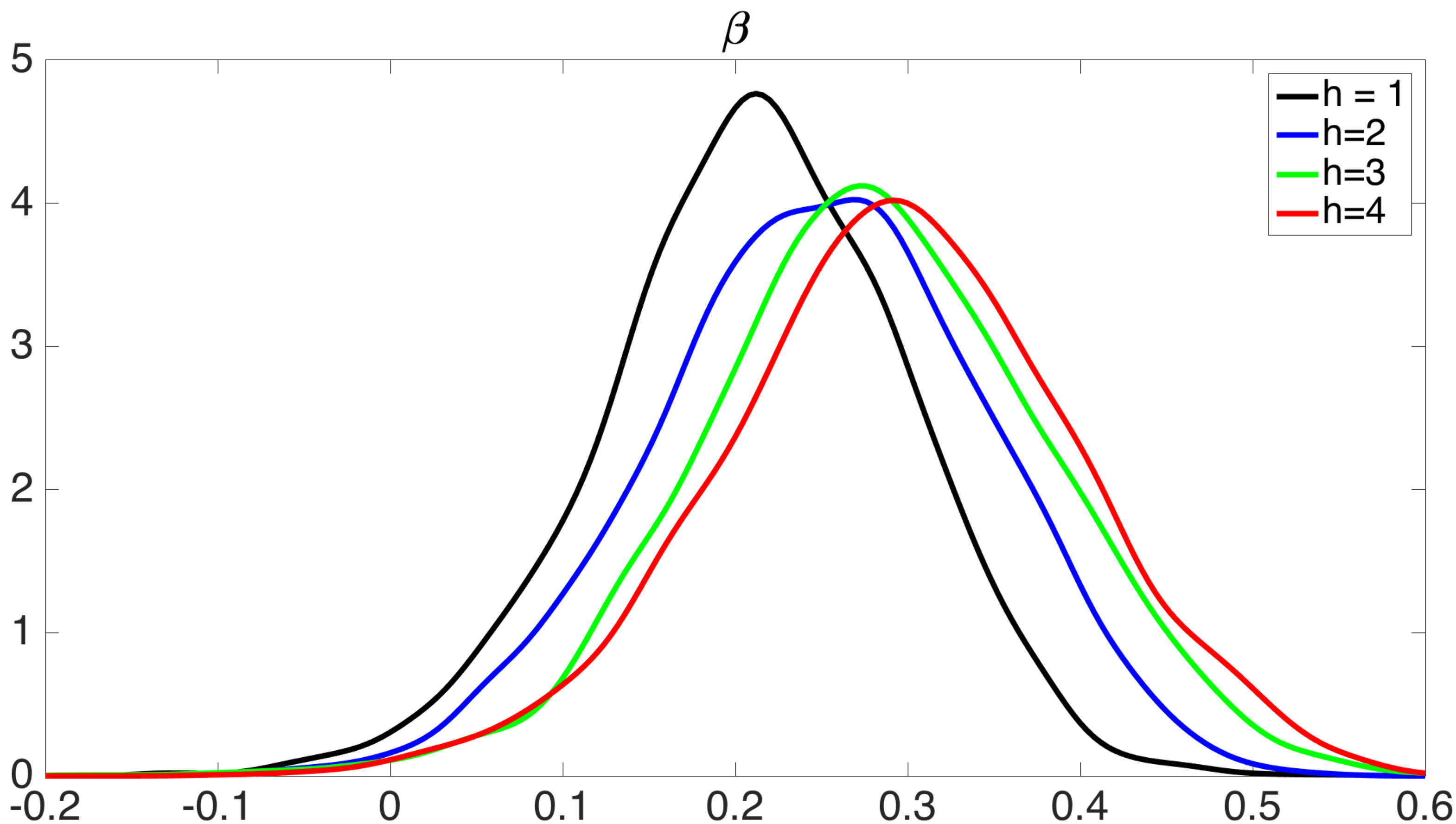
Alternative Regressors

$$\ln(\sigma^2_{\Delta\tau,t}) = x_t'\boldsymbol{\beta} + \xi_{\Delta\tau,\mathrm{t}} \quad \text{with} \quad \Delta\xi_{\Delta\tau,t} = \gamma_{\Delta\tau}\nu_{\Delta\tau,t}$$

$$x_t = (\tau_{t-1} + \tau_{t-2} + ... + \tau_{t-h})/h$$

$$\text{so } \Delta\ln(\sigma^2_{\Delta\tau,t}) = \beta\,(\tau_{t-1} - \tau_{t-h\text{-}1})/h \; + \gamma_{\Delta\tau}\nu_{\Delta\tau,t}$$

Posteriors for β with different values of h



Model Selection

Consider n models:

Common features:

$$\pi_t = \tau_t + \varepsilon_t$$

$$\tau_t = \tau_{t-1} + \sigma_{\Delta\tau,t} \times \eta_{\tau,t}$$

$$\varepsilon_t = \sigma_{\varepsilon,t} \times s_t \times \eta_{\varepsilon,t}$$

$$\Delta\ln(\sigma^2_{\varepsilon,t}) = \gamma_\varepsilon \nu_{\varepsilon,t}$$

Differences:

Model i (M_i): $\ln(\sigma^2_{\Delta\tau,t}) = x_{i,t}'\boldsymbol{\beta}_i + \xi_{\Delta\tau,t}$ with $\Delta\xi_{\Delta\tau,t} = \gamma_{\Delta\tau}\nu_{\Delta\tau,t}$

Let $Y = \left\{\pi_t\right\}_{t=1}^{T}$. We want to calculate $P(M_i|Y)$ for $i = 1, \ldots, n$.

Let $Z = \left\{\tau_t, \ln(\sigma^2_{\Delta\tau,t})\right\}_{t=1}^{T}$ and $Y = \left\{\pi_t\right\}_{t=1}^{T}$

Note:

(1) $P(M_i|Y) = \int P(M_i|Y,Z) f(Z|Y)\, dZ$

(2) $P(M_i|Y,Z) = P(M_i|Z)$

(3) $f(Z|Y) = \sum_{j=1}^{n} f(Z\,|\,Y, M_j) P(M_j\,|\,Y)$

Thus:

$$P(M_i\,|\,Y) = \sum_{j=1}^{n} \left[\int P(M_i\,|\,Z) f(Z\,|\,Y, M_j)\, dZ\right] P(M_j\,|\,Y)$$

$$= \sum_{j=1}^{n} a_{ij} P(M_j\,|\,Y), \text{ where } a_{ij} = E\left[P(M_i\,|\,Z)\,|\,Y, M_j\right]$$

Stacking: $P(M \mid Y) = A\, P(M \mid Y)$

so $P(M \mid Y)$ is an eigenvector of A corresponding to a unit eigenvalue.

Estimation:

$$\widehat{a_{ij}} = \frac{1}{n_j} \sum_{k=1}^{n_j} P(M_i \mid Z_k) \quad \text{with} \quad Z_k \sim f(Z \mid Y, M_j)$$

Model probabilities: $x_t = x_t = (\tau_{t-1} + \tau_{t-2} + ... + \tau_{t-h})/h$

P(M$_h$ | Y)

1	2	3	4
0.13	0.20	0.32	0.35

Compare model M_h (with $h = 4$) to UCSV model

P(M_4 | Y)	P($UCSV$ | Y)
0.74	0.26

Implications for forecasting:

$$\pi_{t+h} = (\tau_{t+h} - \tau_t) + \tau_{t|t} + (\tau_t - \tau_{t|t}) + \varepsilon_{t+h}$$

$$\tau_{t+h} - \tau_t = \sum_{i=1}^{h} \sigma_{\Delta\tau,t+i}\eta_{\tau,t+i}$$

$$\ln(\sigma^2_{\Delta\tau,t}) = x_t'\boldsymbol{\beta} + \xi_{\Delta\tau,\mathrm{t}} \qquad \text{with } \Delta\xi_{\Delta\tau,t} = \gamma_{\Delta\tau}\nu_{\Delta\tau,t}$$

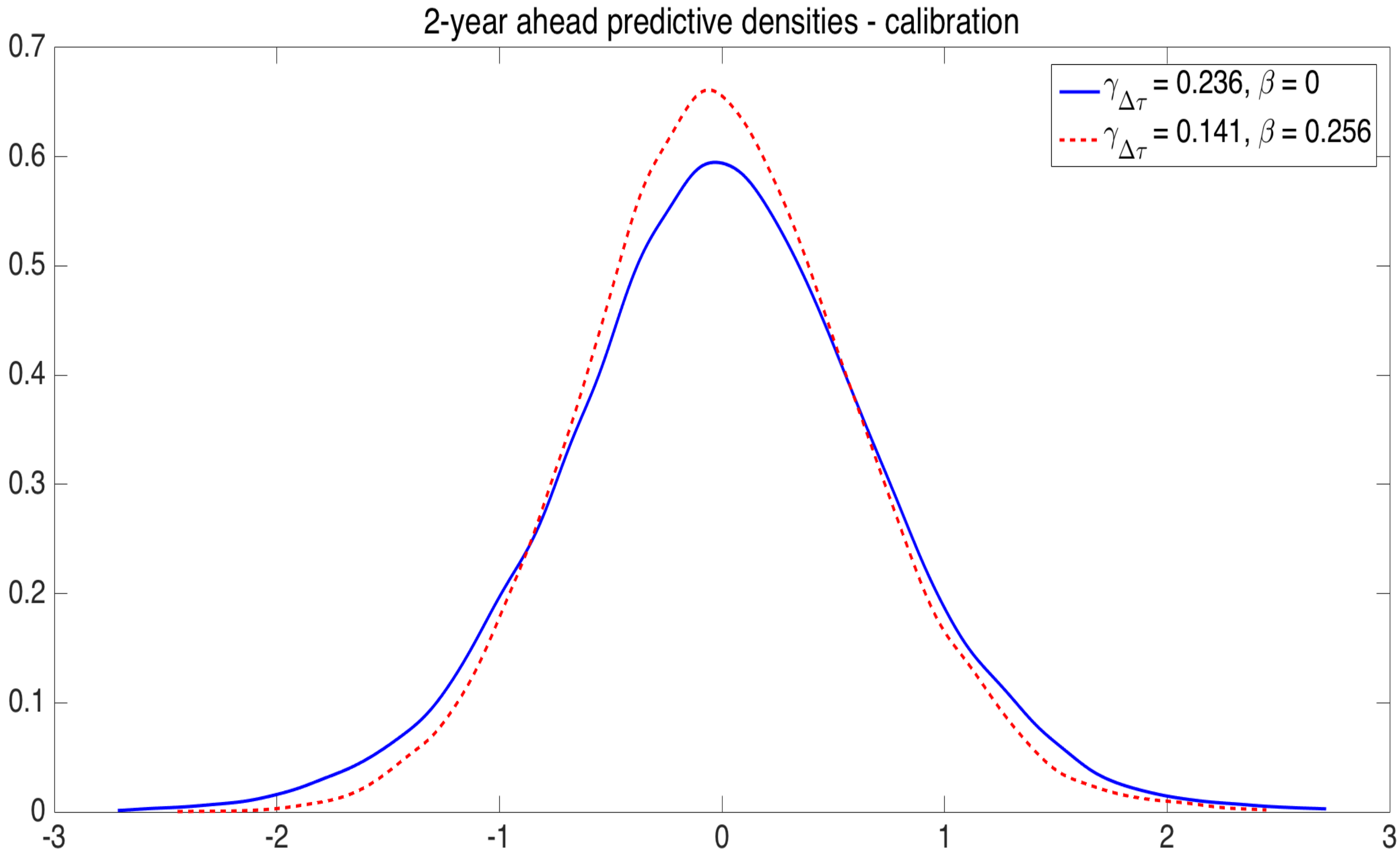
$(\eta_{\tau,t}, \nu_{\Delta\tau,t}) \sim \mathrm{iidN}(0,\mathrm{I}_2)$

~~Three~~ Five cases

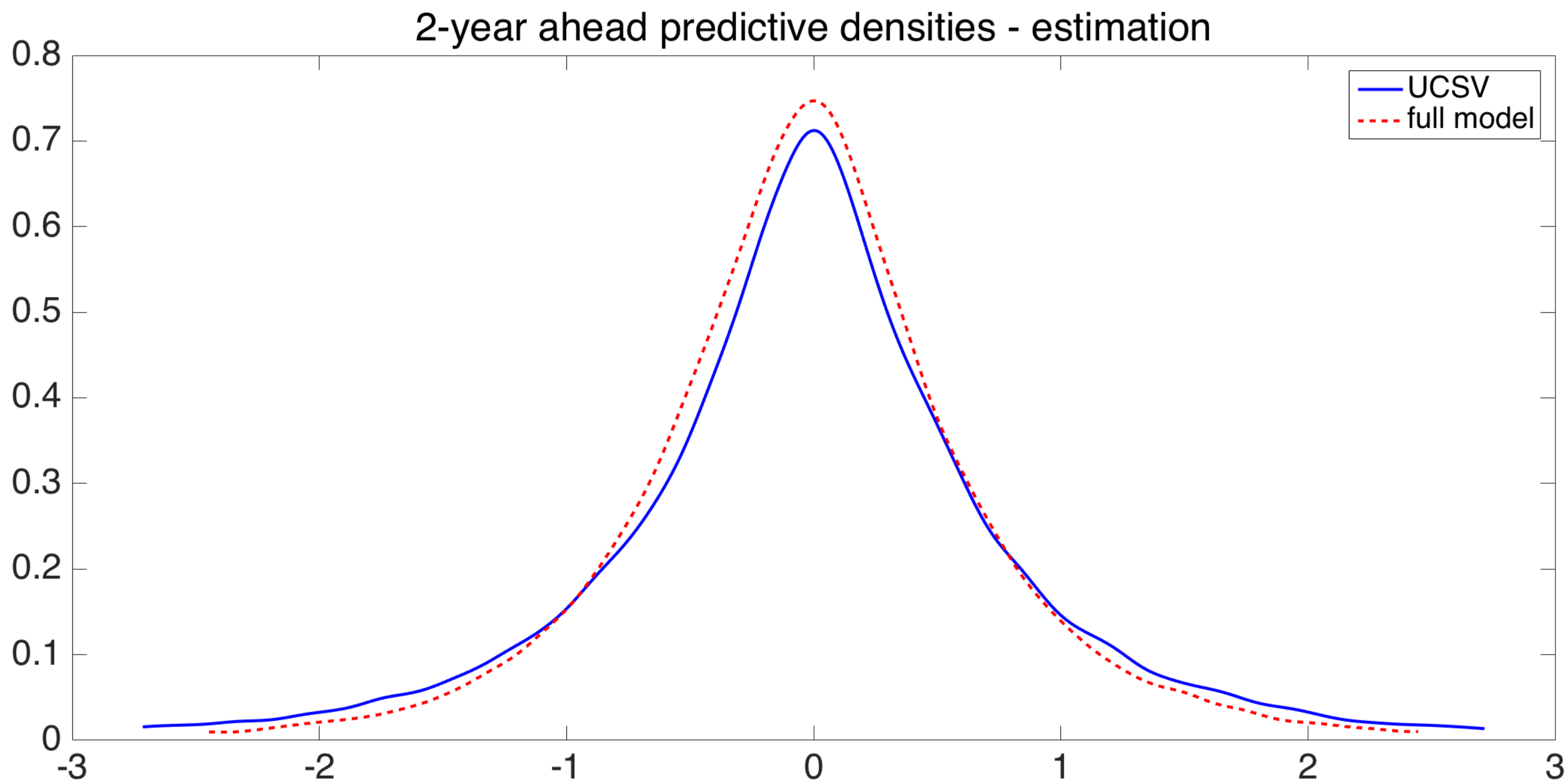
- ~~(i) $\gamma_{\Delta\tau} = \beta = 0$~~
- (ii) $\gamma_{\Delta\tau} > 0$, $\beta = 0$ (Posterior Median from UVSCO)
- (iii) $\gamma_{\Delta\tau}$, $\beta > 0$ (Posterior Median from $h = 4$ model)
- (iv)-(v) Same as (ii) and (iii) but with parameter uncertainty, $[\sigma_{\varepsilon,T}, \sigma_{\Delta\tau,T}, \gamma_\varepsilon, \gamma_{\Delta\tau}, \beta]$.

(ii) $\gamma_{\Delta\tau} > 0$, $\beta = 0$ (Posterior Median from UVSCO)

(iii) $\gamma_{\Delta\tau}$, $\beta > 0$ (Posterior Median from $h = 4$ model)



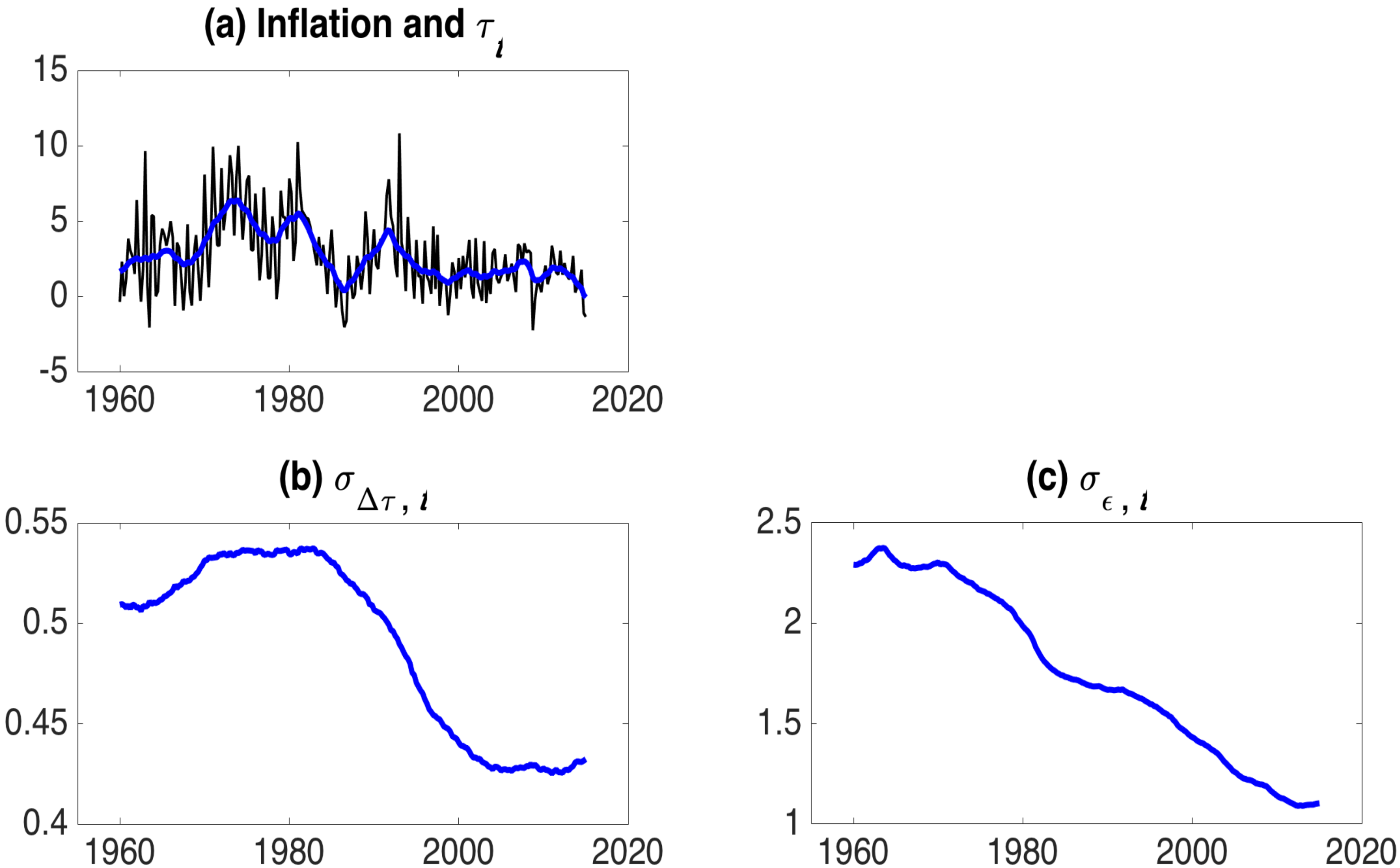
(iv)-(v) Same as (ii) and (iii) but with parameter uncertainty, $[\sigma_{\varepsilon,T}, \sigma_{\Delta\tau,T}, \gamma_{\varepsilon}, \gamma_{\Delta\tau}, \beta]$.



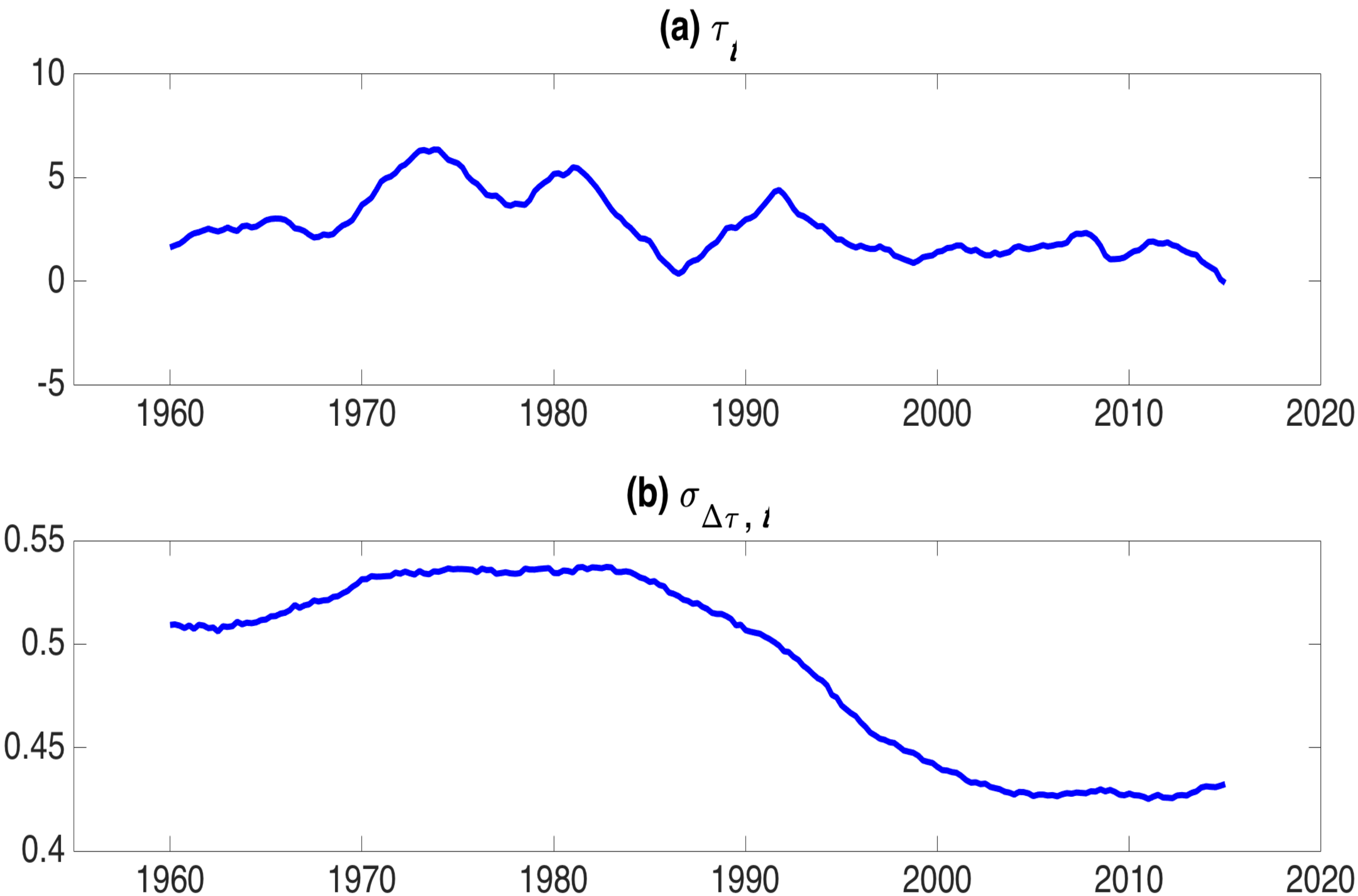
A quick tour of Germany, Italy, and Sweden

(Data 1960 – 2014)

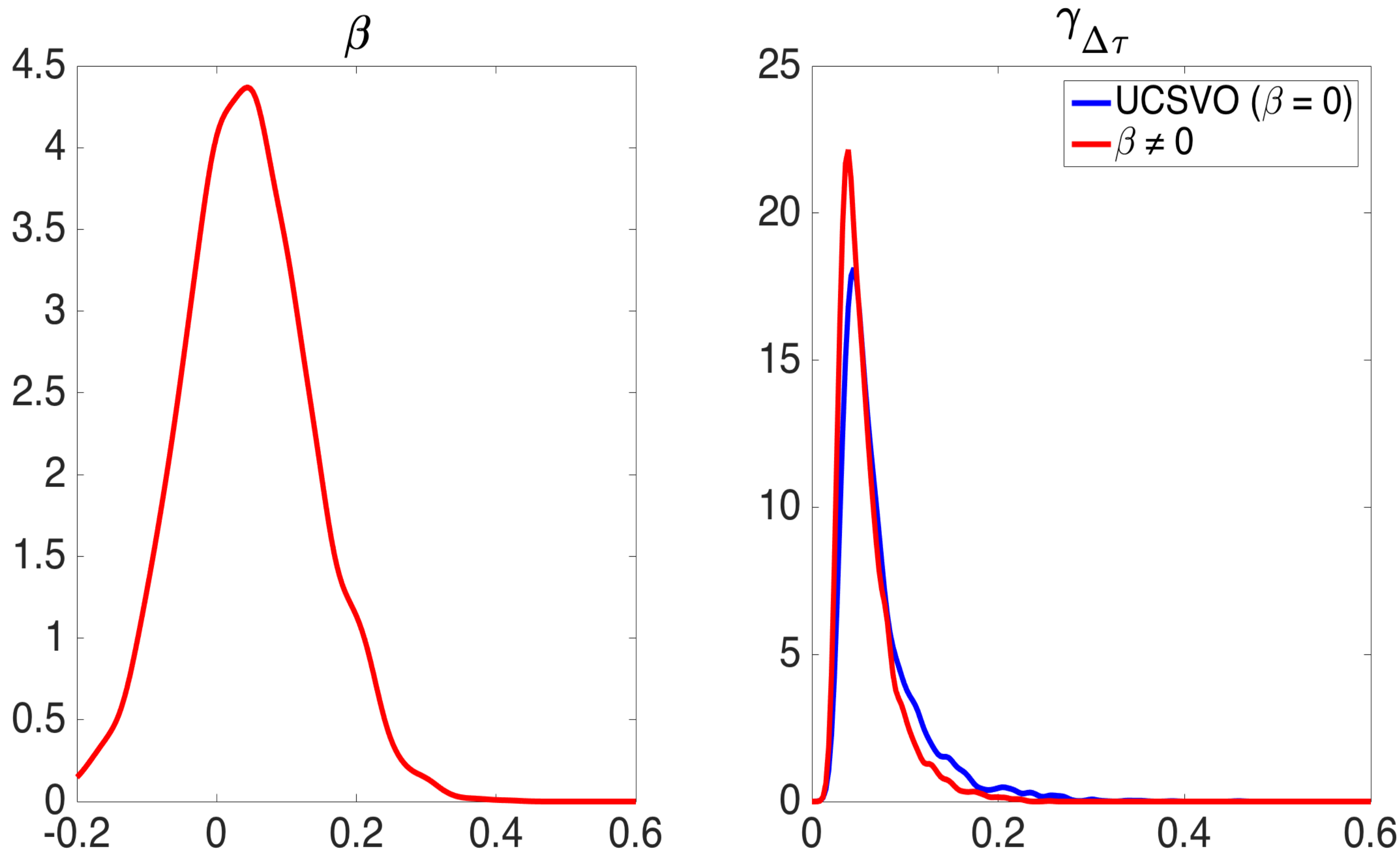
Germany


(a) Inflation and τ_t
15
10
5
0
-5
1960
1980
2000
2020
(b) $\sigma_{\Delta\tau,t}$
0.55
0.5
0.45
0.4
1960
1980
2000
2020
(c) $\sigma_{\epsilon,t}$
2.5
2
1.5
1
1960
1980
2000
2020

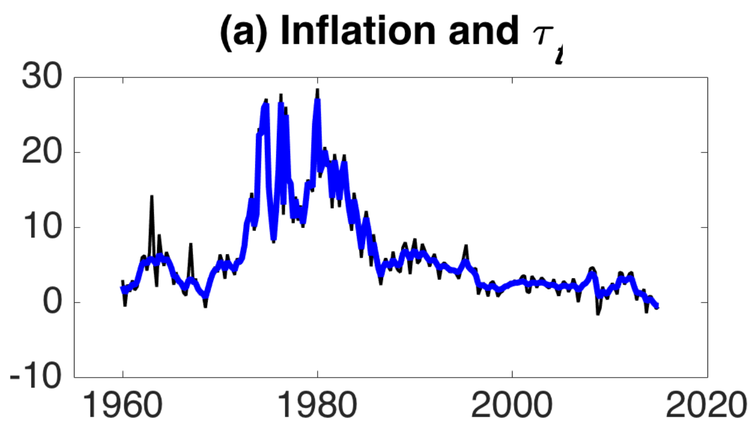
Germany

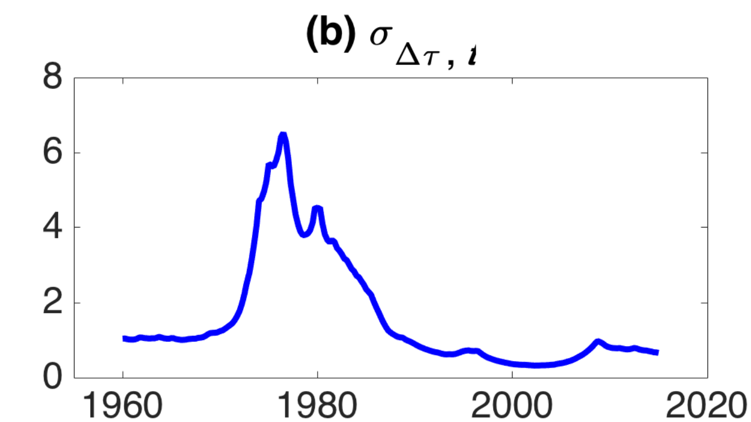

(a) τ_t
10
5
0
-5
1960
1970
1980
1990
2000
2010
2020
(b) $\sigma_{\Delta\tau,t}$
0.55
0.5
0.45
0.4
1960
1970
1980
1990
2000
2010
2020

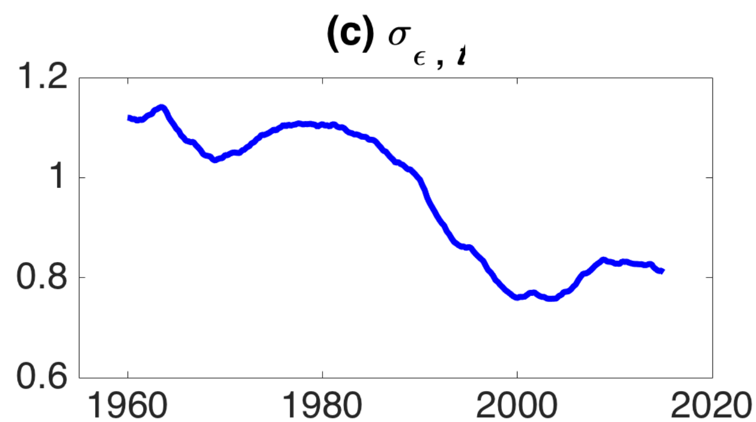
Germany


β
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
-0.2
0
0.2
0.4
0.6
$\gamma_{\Delta\tau}$
25
20
15
10
5
0
0
0.2
0.4
0.6
UCSVO (β = 0)
β ≠ 0

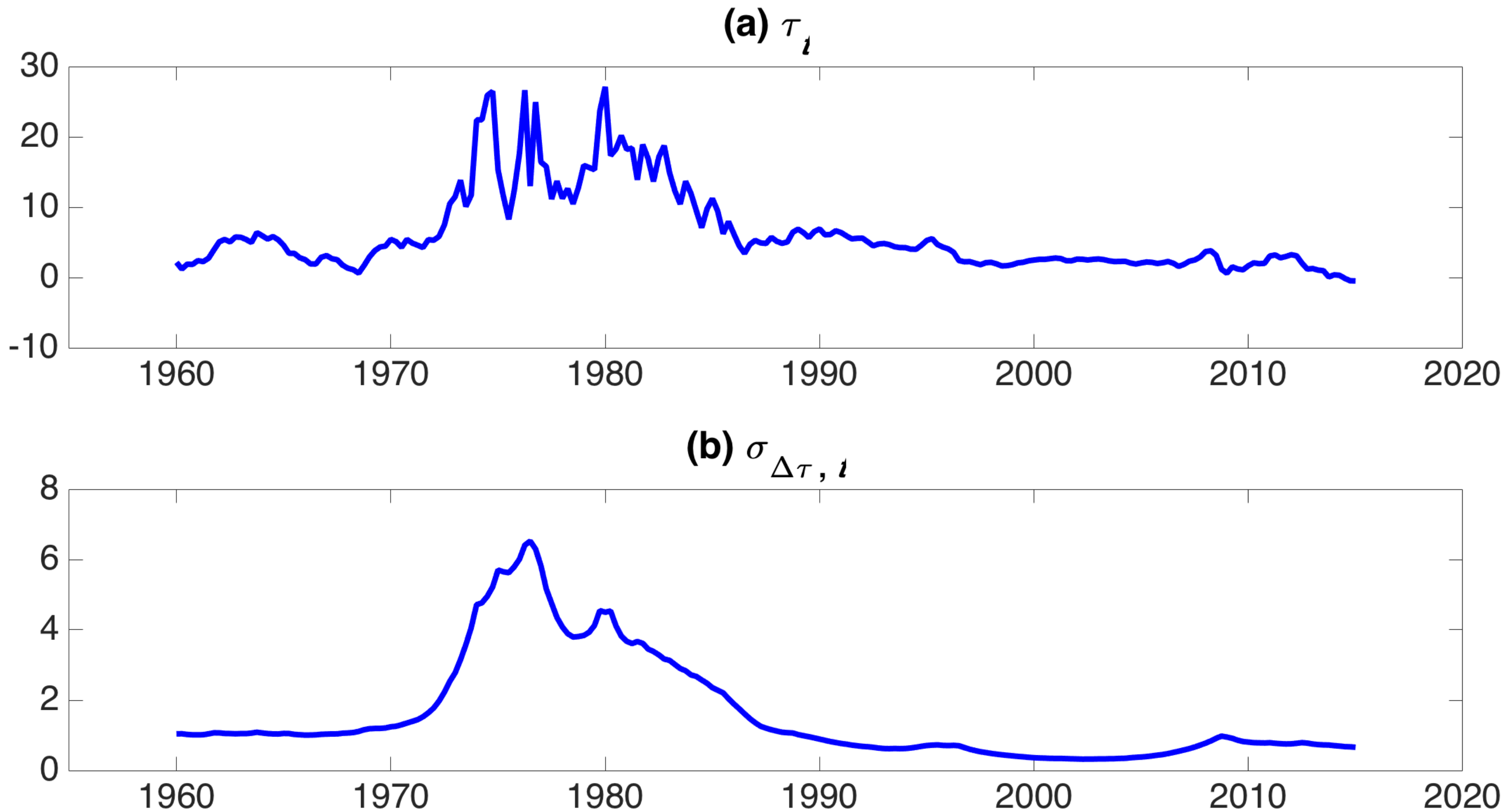
Italy


(a) Inflation and τ_t
30
20
10
0
-10
1960
1980
2000
2020

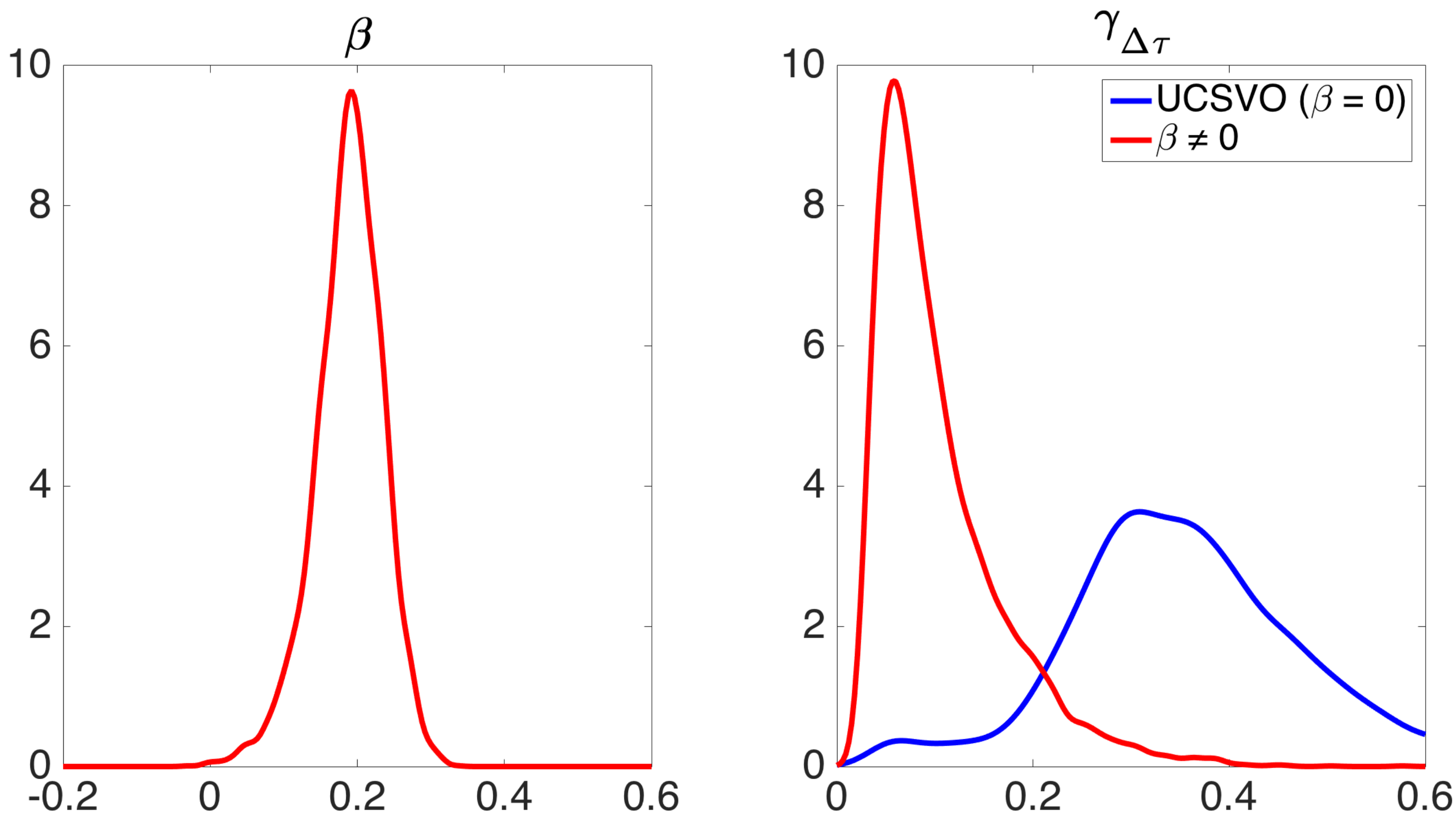

(b) $\sigma_{\Delta\tau,t}$
8
6
4
2
0
1960
1980
2000
2020


(c) $\sigma_{\epsilon,t}$
1.2
1
0.8
0.6
1960
1980
2000
2020

Italy

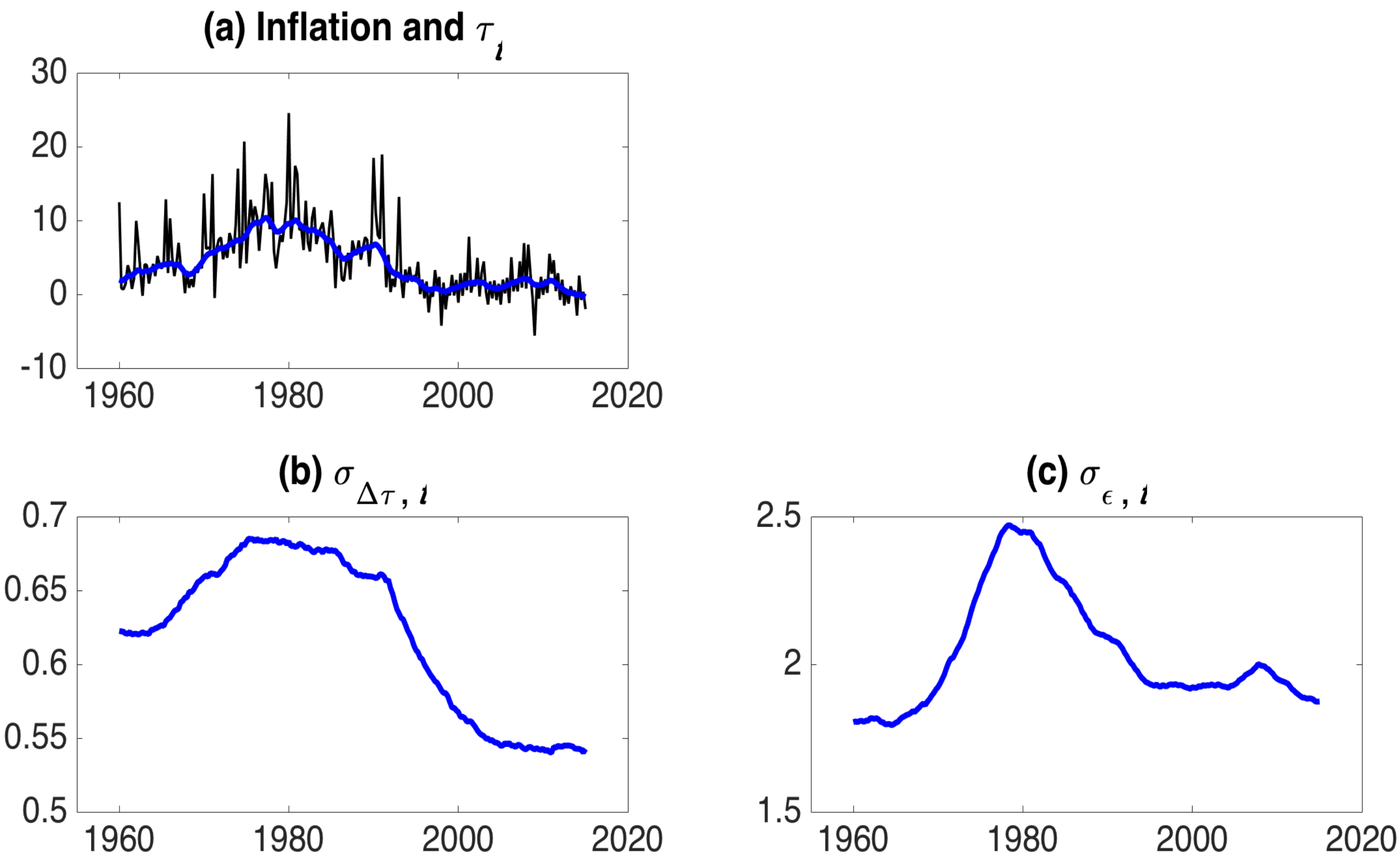

(a) τ_t
30
20
10
0
-10
1960
1970
1980
1990
2000
2010
2020
(b) $\sigma_{\Delta\tau, t}$
8
6
4
2
0
1960
1970
1980
1990
2000
2010
2020

Italy

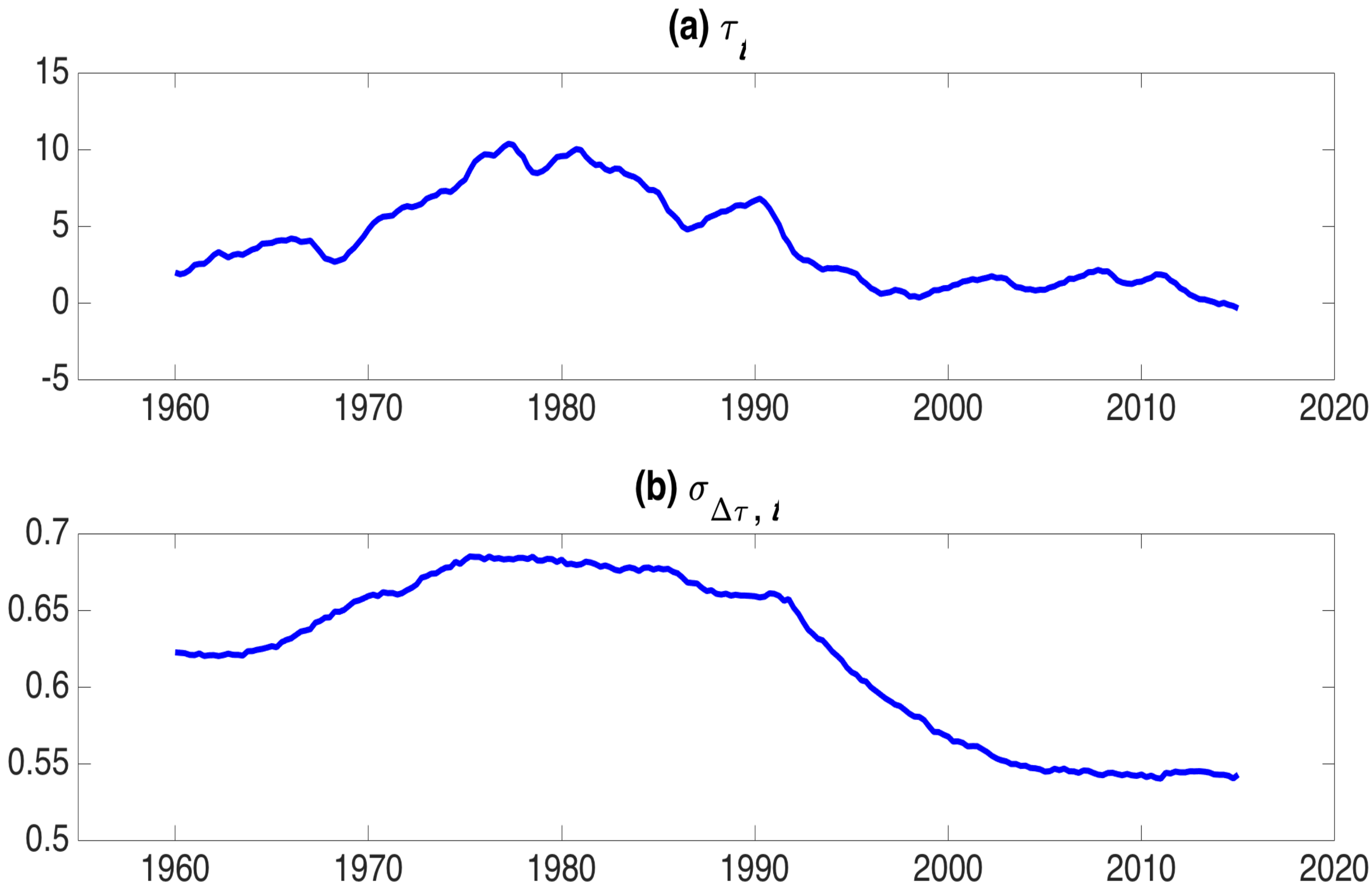


β
$\gamma_{\Delta\tau}$
UCSVO (β = 0)
$\beta \neq 0$
10
8
6
4
2
0
-0.2
0
0.2
0.4
0.6

Sweden

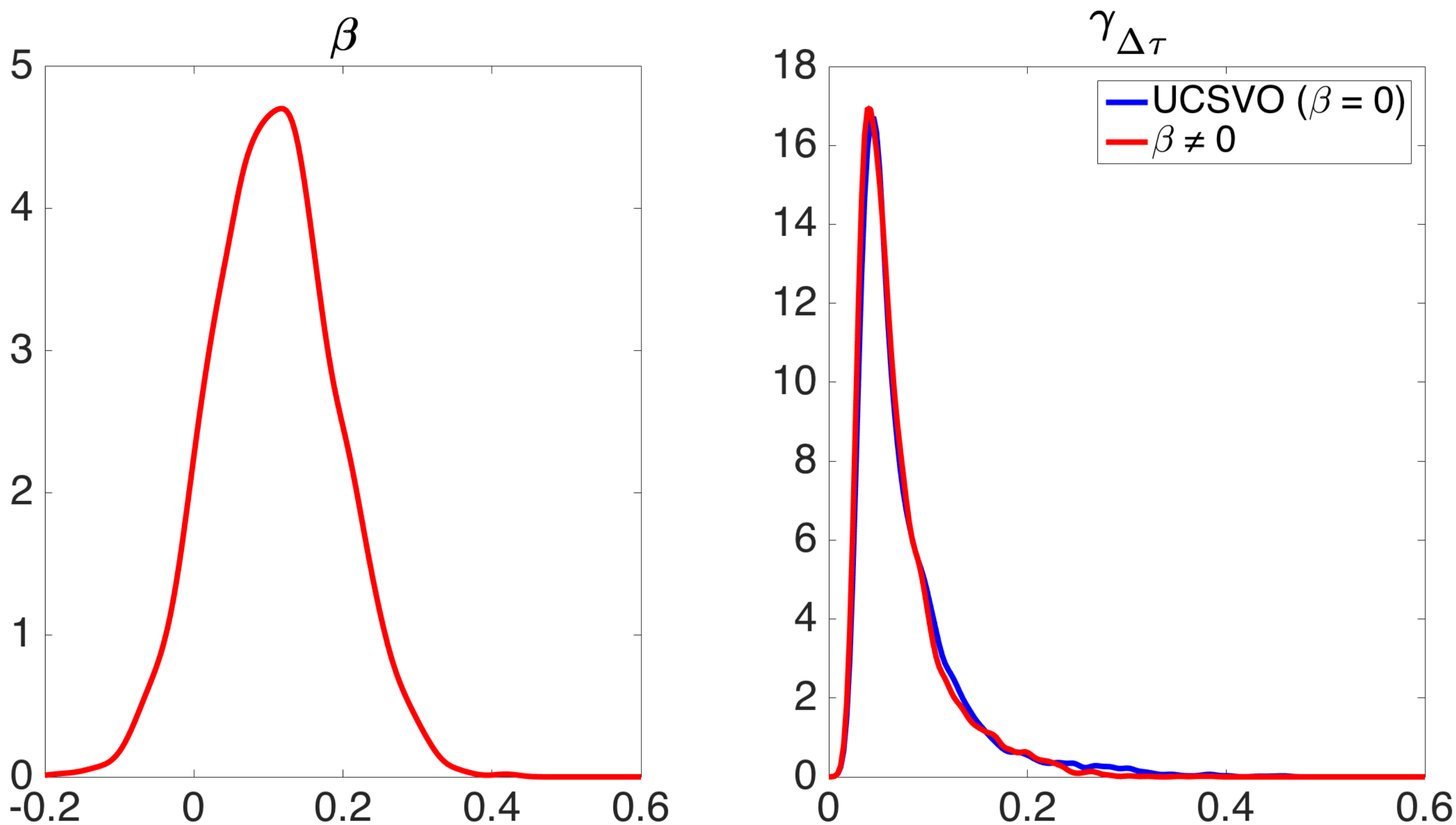

(a) Inflation and τ_t
30
20
10
0
-10
1960
1980
2000
2020
(b) $\sigma_{\Delta\tau, t}$
0.7
0.65
0.6
0.55
0.5
1960
1980
2000
2020
(c) $\sigma_{\epsilon, t}$
2.5
2
1.5
1960
1980
2000
2020

Sweden



(a) τ_t
15
10
5
0
-5
1960
1970
1980
1990
2000
2010
2020
(b) $\sigma_{\Delta\tau,t}$
0.7
0.65
0.6
0.55
0.5
1960
1970
1980
1990
2000
2010
2020

Sweden



β
5
4
3
2
1
0
-0.2
0
0.2
0.4
0.6
$\gamma_{\Delta\tau}$
18
16
14
12
10
8
6
4
2
0
0
0.2
0.4
0.6
UCSVO (β = 0)
$\beta \neq 0$

Ball and Cecchetti (BPEA 1990) "Inflation and Uncertainty at Short and Long Horizons" …

(1) $$\pi_t = \hat{\pi}_t + \eta_t,$$

(2) $$\hat{\pi}_t = \hat{\pi}_{t-1} + \epsilon_t,$$

(10) $$\sigma^2_\epsilon(t) = \beta_0 + \beta_1 \hat{\pi}_{t-1};$$

(11) $$\sigma^2_\eta(t) = \delta_0 + \delta_1 \hat{\pi}_{t-1}.$$

Their findings:

- US: $\beta_1 > 0, \delta_1 \approx 0$
- Different for some other countries

Our contribution (*in progress*): Revisit, augmenting their analysis with (a) more data and (b) improved filtering tools.